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Assessing Korean Middle School Students' Spatial Ability: The Relationship with Mathematics, Gender, and Grade

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Spatial ability has been valued as a talent domain and important skill in mathematics education because it enhanced an intuitive view and an understanding in many areas of mathematic. In addition, spatial ability highly correlates with mathematics achievement, indicating its crucial role in success in mathematics education. Some researchers founded gender differences in mathematics and spatial ability, and indicated that spatial ability served as a mediator of gender difference in mathematics. This study explored the spatial ability of 349 Korean middle school students (Grade 7–9), and investigated the association among students' spatial ability and their mathematics achievement, gender, and grade. The result of this study shows that spatial ability correlates positively with mathematics achievement. While gender difference did not exist in mathematics, significant gender difference existed in spatial ability favoring male students.

Keywords: spatial ability, mental rotation, 3-D mental rotation ability, Revised Purdue Spatial Visualization Tests: Visualization (PSVT:R), mathematics education, gender difference
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INTRODUCTION

Spatial ability is a necessary skill to understand and interpret the world of geometry. Based on this ability, we can understand the features of two-dimensional (2-D) and three-

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dimensional (3-D) objects and recognize the inter-relationship among the geometric figures. Researchers have agreed that spatial ability is essential in learning mathematics because various studies found that:

(a) Statistically significant relationships existed between spatial ability and mathematics achievement (Battista, 1990; McGee, 1979; Sherman, 1979), and
(b) Visual representations of mathematical concepts were beneficial for students to understand these concepts intuitively in various areas of mathematics, even outside of geometry (Krutetskii, 1976; Usiskin, 1987).

In mathematics education, spatial visualization like simple pictorial drawings has been a means to help students understand mathematical problems. In other words, spatial visualization has been used to symbolize mathematical concepts in mathematical situations. Reflecting the recent research results on geometry, According to the National Council of Teachers of Mathematics (NCTM) (2000), spatial sense in mathematics involves the use of visualization and spatial reasoning to solve mathematics problems. NCTM (2000) also emphasized the importance of understanding spatial properties in mathematics education. When considering that geometry is science about space to handle symbolic systems related to space, spatial ability comes to fundamental ability for students to be developed for geometry learning (Freudental, 1973).

There has been disagreement in the research regarding gender difference in mathematics achievement. Some researchers showed that gender difference favouring males did not exist or existed very small in size (Beller & Gafni, 1996; Hedges & Nowell, 1995), whereas others showed that gender related effect sizes of mathematics achievement are small but are consistently positive for favouring males. Recent meta-analyses indicated that gender differences in mathematics achievement were not found (Hyde, Fennema & Lamon, 1990). However, the results of Hyde, Fennema and Lamon (1990) were different by students' grade level. In the case of elementary students, girls had a slight advantage over boys, but in the case of high school students, boys had a slight advantage over girls. In addition, when a content domain was taken into account, the pattern of gender difference in mathematics achievement was different (Hyde, Fennema & Lamon, 1990); sometimes males outperformed females in geometry and word problem solving, sometimes females outperforms males in computational ability and complex arithmetic (Geary, 1996; Hyde, Fennema & Lamon, 1990).

To explain gender difference in mathematics achievement, researchers looked for other mediator components. There are two opposing views around gender differences in mathematics achievement. A group of researchers has indicated that spatial skill as a cognitive factor underlies gender differences in mathematics achievement (Benbow & Lubinski, 1993), whereas others emphasized the internalized attitude about mathematics for the gender-based mathematics differences (Crawford, Chaffin & Fitton, 1995). Particularly, Casey, Nuttall & Pezaris (1997) compared cognitive variables and attitudinal variables and found that mental rotation was the most relevant mediator.

In addition, a body of research showed that spatial skill also served as a mediator of gender difference in mathematics (Casey, Nuttall & Pezaris, 1997; Casey, Nuttall, Pezaris & Benbow, 1995; Masters & Sander, 1993; Voyer, Voyer, & Bryden, 1995). Some researchers argued for biologically based gender difference in mathematics (McGuinness, 1993). Conversely, Crawford, Chaffin & Fitton (1995) claimed that gender differences in spatial ability were primarily due to differential socialization, training and experience between the genders.

A possible role of mental rotation in mathematic achievement tests is related to problem solving strategy. It has been empirically shown that there are two different styles for problem solving: algorithmic process and spatial representation. McGuinness (1993) indicated that algorithmic process is generally less efficient and it is more frequent among male students, which would explain the gender difference.

Spatial ability has been used together with spatial orientation, spatial perception, and spatial visualization. In the Seventh Mathematics Curriculum of Korea, spatial sense was frequently used in discussing about mathematics. The curriculum says that "spatial sense is a part of intuition about environment and objects. To raise spatial sense, students need experience focused on geometric relations, orientations of objects in space, and differences in shapes of geometric figures or objects. Here, spatial sense is a newly included concept to learn geometry in the Seventh Mathematics Curriculum of Korea. This reflects that school education is now focused on cultivation of spatial ability to learn dynamic features of geometry beyond static and flat Euclidian geometry.

Spatial ability, as one component of intelligence, has been investigated from the early history of research on intelligence. Particularly, a series of studies on spatial ability has reported that spatial ability, as a talent domain, is associated with achievements in science, technology, engineering, and mathematics (STEM) (Lubinski & Benbow, 2006; Wai, Lubinski & Benbow, 2009; Webb, Lubinski & Benbow, 2007). Researchers spotlighted spatial ability for various reasons. First, spatial ability is one dimension of intelligence as grounded by the multiple intelligence theory (Gardner, 1993). Second, spatial ability can be a predictor of STEM performance (Humphreys, Lubinski & Yao, 1993; Lubinski & Benbow, 2006; Wai, Lubinski & Benbow, 2009; Webb, Lubinski & Benbow, 2007; Wheatley, 1983). Third, spatial ability, as a non-verbal ability, can be independent from the influence of culture, language, and socioeconomic status (Chan, 2010; Lohman, 2005; Naglieri & Ford, 2003).

Based on the research results showing the role of spatial ability, measuring the spatial ability of students becomes important in the practice of education. Particularly, to cultivate human resources to procure national competitiveness in the future, researchers in the developing and developed countries currently include assessment of spatial ability in tests, as one way to identify gifted students talented in STEM areas (Chan, 2010; Lohman, 2005; Naglieri & Ford, 2003). For example, Chan (2010) recognized spatial ability as a 21st century skill and developed the Impossible Figures Task (IFT) with data from 492 gifted elementary, middle, and high school students in Hong Kong, as a way to identify talents in spatial ability. Before then, Naglieri & Ford (2003) and Lohman (2005), developed nonverbal instruments, NNAT and CogAT, respectively, considering spatial ability as one of the components in the instruments. Now, the authors of the instruments have focused on the standardization of the instruments, including evaluation of the instruments with diverse learners (Lohman, Korb & Lakin, 2008; Naglieri & Ford, 2005).

According to the literature, researchers have defined spatial ability in various ways and categorized factors of spatial ability into different ways. For example, McGee (1979) defined spatial ability as an ability to mentally manipulate, rotate, or orient the given image of objects and suggested two factors, spatial visualization and spatial orientation. Meanwhile, Lohman (1979) included one more factor, spatial relation in addition to the two factors suggested by McGee (1979). Linn & Peterson (1985) defined spatial ability as an ability to mentally visualize the given spatial information and categorized spatial ability to three factors, spatial perception, mental rotation, and spatial visualization.

Hwang (1984) defined spatial ability as an ability to symbolize, create, or remember an image situated in space, and relate the image to the different objects or relocate it in the space. In summary, spatial ability can be broadly defined, as an ability to mentally form an image when stimulated by external factors, an ability to mentally manipulate the formed image, an ability to visually recognize an object, or an ability to mentally produce, rotate, reorient, or rearrange an image of object.

Even though, there have been varied definitions and distinctions of factors of spatial ability, it was spatial visualization ability that researchers commonly defined and considered as an important factor. According to Lohman (1979), who classified spatial ability into three major spatial factors (spatial relation, spatial orientation, and spatial visualization), spatial visualization is an ability that shares characteristics from both spatial relation and spatial orientation. Fennema (1975) defined spatial ability as an ability to visualize the image of an object or its movement, or change of the object. Similarly, Clements (1981) defined spatial visualization ability as an ability to imagine the movement of 3-D objects or mentally manipulate visual images of the objects. Tarte (1990) and Kersh & Cook (1979) considered mental rotation and mental transformation as two subfactors of spatial visualization. Here, mental rotation is an ability to match an image of 2-D or 3-D object, which is rotated into a different direction, to the original mage of the object. Mental transformation means an ability to combine different parts of visual images of an object to form a whole image of the object. In summary, spatial visualization indicates an ability to mentally reconstruct a 2-D or 3-D object through manipulation or rotation.

However, in Korea, spatial ability has not received enough attention from researchers and educators. Particularly, there has been a lack of research regarding 3-D mental rotation ability of middle school students. Under these circumstances, this study attempted to measure Korean middle school students' spatial ability and to examine its difference by gender and grade. Furthermore, we investigated the association between spatial ability and students' achievement in three subject areas, mathematics, science, and Korean language arts. To measure students' 3-D mental rotation ability, we utilized the Korean version of the Revised Purdue Spatial Visualization Tests: Visualization of Rotations (the Revised PSVT:R) (Yoon, 2011).

As an exploratory study, this study conducted:

(a) An investigation of the association between spatial ability and students' achievement in different academic domains, such as mathematics, science, and Korean language arts; and

(b) An exploration of the students' spatial ability according to their demographic variables (gender and grade).

METHOD

Samples

During the fall of 2010(October, 5–November, 15), a stratified purposive sampling was conducted through cooperation of school personnel of three middle schools in Gangwon province and two middle schools in Gyeonggi province in Korea. Each five middle schools were belonged in different school districts of two provinces. Among 364 participants, participants who have insufficient data and missing demographic information were excluded. In total, 349 Korean middle school students (7th through 9th graders) participated in the study: 229 students were female and 120 students were male students.

Measures

Spatial ability.

Among various spatial abilities, this study focused on the assessment of spatial visualization ability in three-dimensional (3-D) mental rotation measured by the Korean version of the *Revised Purdue Spatial Visualization Tests: Visualization of Rotations* (the Revised

PSVT:R). Originally, Guay (1976) developed the PSVT:R to measure spatial ability of individuals aged 13 or older to rotate 3-D objects. Since then, the PSVT:R (Guay, 1976) has been commonly used in science, technology, engineering, and mathematics (STEM) fields to investigate the relationship between students' academic performance and their spatial ability. It is because the test seemed to be relevant tasks aligned with problem solving skills necessary to perform well in STEM education. The PSVT:R has 2 practice items followed by 30 test items which consist of 13 symmetrical and 17 asymmetrical figures of 3-D objects, which are drawn with a 2-D isometric format. All the figures contain shapes of cubes or cylinders with varied truncated slots (See Figure 1 for an example). In each item, respondents need to find the same figure of a given example among five choices, which are rotated in different directions and shown at different angles.

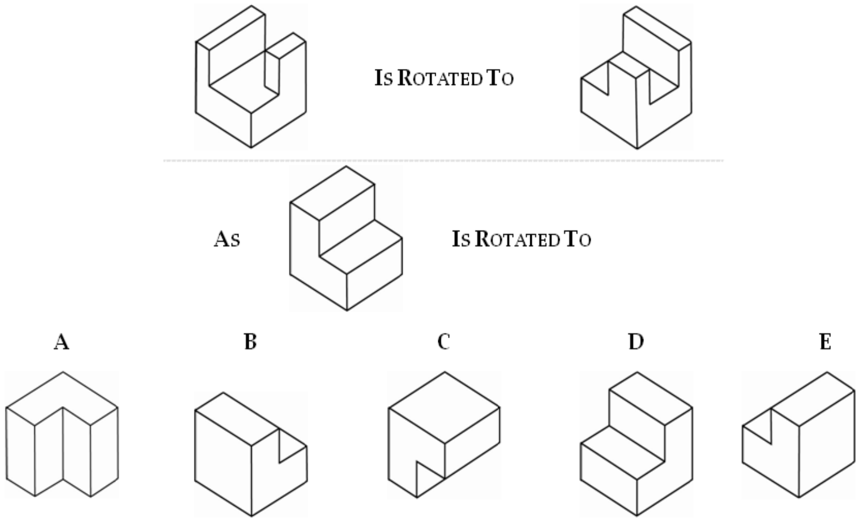

Figure 1. Example of the Revised PSVT:R (Yoon, 2011)

This study used the revised version of the PSVT:R (the Revised PSVT:R) that corrected several figural errors on the test (Yoon, 2011). The reliability and validity evidence of the PSVT:R data and the Revised PSVT:R is summarized in Table 1. The factor analyses with data from college students have demonstrated construct validity of one-dimensional factor structure (Maeda & Yoon, 2011; Yoon, 2011).

To administer the instrument to Korean students, the Revised PSVT:R was translated into a Korean version by the authors of this study. As a nonverbal test, the Revised PSVT:R does not involve any significant vocabulary except the first two pages of directions. Thus, we assumed that the content validity of the Korean version is equivalent to the Revised PSVT:R. A Korean linguistic scholar checked the readability of directions in Korean, and then three Korean middle school students took the test as a pilot test and their feedback from the pilot test was reflected on the final Korean version of the Revised PSVT:R.

Table 1. Reliability and Validity Evidence of the PSVT:R and the Revised PSVT:R

Literatures	Sample (Numbers of Participants)	Internal Consistency Reliability	Concurrent Validity
Alkhateeb (2004)	College Students (180)	r_a = 0.81	
Battista (1990)[1]	High School Students (Grades 10–12) (128)	r_a = 0.77	
Branoff (2000)[2]	College Students (277)	$r_{KR\text{-}20}$ = 0.83	MRT and r_P = 0.67, 0.65
Chae, Chae & Mann (2008)[1]	Grades 5–12 (106)	$r_{KR\text{-}20}$, r_{SH} = 0.78~ 0.85	
Guay (1980)	College Students (217)	$r_{KR\text{-}20}$ = 0.87	
Guay & McDaniel (1978)	College Students (101)	r_{SH} = 0.90 $r_{KR\text{-}20}$ = 0.86	Shepard-Metzler Rotations Tests and r_P = 0.61 RMPFBT and r_P = 0.25
Maeda & Yoon (2011)[3]	585 first year engineering students	r_α = 0.84	
Sorby & Baartmans (1996)	College Students, Engineering Major, Freshmen (492)	$r_{KR\text{-}20}$ = 0.82	

Note.

[1] Used a shortened version of the PSVT:R with 20 questions;

[2] This study used a computer-based form of the PSVT:R;

[3] This study used the Revised PSVT:R; r_α = Cronbach's alpha reliability coefficient; r_{SH} = Split-Half reliability coefficient; $r_{KR\text{-}20}$ = Kuder-Richardson reliability coefficient; r_P = Pearson product-moment correlation coefficient; RMPFBT = Revised Minnesota Paper Form Board Test; MCT = the Mental Cutting Test (MCT) (College Entrance Examination Board, 1939); Differential Aptitude Tests: Spatial Relations (DAT:SR) (Bennett, Seashore, & Wesman, 1973).

Academic Performance (Mathematics, Science, Korean language).

Participants' performance on three subjects, mathematics, science, and Korean language arts were measured using a national academic achievement test set. These tests were developed by Educational Research Center of Gyeonggi province in 2009. The test set was designed to evaluate the 7th grade students' performance at the end of school year, and consists of 25 questions for each subject. While 25 questions of mathematics and science areas were administered, the five listening problems were excluded in the Korean language arts test.

Demographic Information.

Participants completed a demographic survey that consisted of questions such as gender, age, and grade level.

Procedure

The spatial ability test was administered to participants in a group. Participants were given a paper-and-pencil version of the Korean Revised PSVT: R, a Scantron sheet, and the demographic survey. After solving two example questions in the directions on the test, participants started the test on the same time given no time limit to complete the test. When they finished the test, they could start the demographic survey. Since the Revised PSVT:R consists of 30 multiple choice items, Scantron sheets was used to record participants' responses on the test and facilitate accurate and fast scoring. The academic performance tests were also administered to participants in a group. The Three subjects, mathematics, science, and Korean language arts were tested separately by the subject teacher of each school. The test time for each subject was 50 minutes.

Data Analysis

For a descriptive data analysis, first, we obtained simple statistics, such as means, standard deviations, and score distributions, of the participants' test scores on the four measures (spatial, mathematics, science, and Korean language arts tests), and checked whether or not the data meet the assumptions (independent observation, normal distribution, and equal variance) for the application of the statistical methods to answer each research questions. The reliability coefficient of internal consistency, Cronbach's α, of each measure was also estimated. To obtain correlations among the four measures and the other demographic variables (gender and grade), Pearson r correlation coefficients were obtained. All statistical results were evaluated with $\alpha = 0.05$ and their associated effect sizes (Cohen, 1988) were reported. As statistical methods to answer the gender and grade differences in spatial ability, mathematics, science, and Korean language arts achievement, this study has included t-tests and analyses of variance (ANOVA). Prior to statistical analyses, assumption checking for each statistical method was conducted. Post-hoc analyses were also conducted for ANOVA. For inferential statistics, IBM-SPSS 18.0

(2010) was utilized.

RESULTS

Descriptive Statistics

Table 2 delineates characteristics of the four measures (spatial, mathematics, science, and Korean language arts tests), such as the reliability coefficients of internal consistency, Cronbach's α, and maximum points available, score distribution of participants, and the number of perfect scorers. The reliability coefficients of all the four measures were ranged from 0.843 to 0.877, which present appropriate consistency in the measures. The high percentage of perfect scorers implies that a ceiling effect might occur on the Korean language arts and mathematics, so that the tests did not function well to differentiate the performance of high ability students. While more female students achieved perfect scores than male students on Korean language arts and mathematics, more male students scored perfect on science. Regarding the spatial ability test, one male student scored perfect on the spatial test and no female student. In addition, relatively less number of students (0.83%) scored perfect on the spatial test compared to the numbers on the other subjects (6.75% on mathematics, 2.93 on science, and 13.57% on Korean language arts).

Table 2. Characteristics of the Four Measures (Spatial, Mathematics, Science and Korean language arts Tests) and Score Distribution of Participants

	Spatial Ability	Mathematics	Science	Korean Language Arts
Participants (*N*)	349	341	341	339
Female	229	225	225	224
Male	120	116	116	115
Reliability				
Cronbach's α	0.843	0.877	0.852	0.866
Maximum score	30	25	25	20
Score range				
Female	4–29	2–25	2–25	4–20
Male	4–30	5–25	4–25	0–20
Perfect scorers				
Female (%)	0 (0.00 %)	13 (5.78 %)	2 (0.89 %)	30 (13.99%)
Male (%)	1 (.83%)	10 (8.62%)	8 (6.90%)	16 (13.91%)

Table 3 presents descriptive statistics, such as means and standard deviations of four test scores (spatial ability, mathematics, science, and Korean language arts) by gender, and grade level.

Table 3. Descriptive Statistics Results from Korean Middle School Participants

	Spatial Ability			Mathematics			Science			Korean Language Arts		
	N	*M*	*SD*	*N*	*M*	*SD*	*N*	*M*	*SD*	*N*	*M*	*SD*
Gender												
Female	229	16.50	5.54	225	18.16	5.59	225	15.94	5.09	224	16.21	3.62
Male	120	19.58	6.28	116	18.44	5.65	116	16.34	5.62	115	14.90	4.80
Grade												
7	39	13.51	5.85	37	13.84	6.99	36	15.81	6.61	37	10.76	4.47
8	200	18.38	5.86	196	18.72	5.03	197	18.29	4.98	195	16.70	3.51
9	110	17.52	5.70	108	18.92	5.45	108	17.84	5.24	107	15.80	3.73
Total	349	17.56	5.98	341	18.26	5.60	341	16.08	5.27	339	15.77	4.10

Note. Due to missing data, the numbers of participants are inconsistent.

Correlations

Table 4 shows intercorrelations among the four test scores, gender (female vs. male), and grade level (7th, 8th, and 9th). Overall, the correlations among the variables were ranged from –0.152 to 0.614, meaning that no multicollinearity over 0.85 existed (Kline, 2011). The correlations of the Revised PSVT:R score with three subject test scores and gender were positive and significant at $\alpha = 0.01$ level. Particularly, the magnitudes of correlations of the spatial ability with three subject test scores (mathematics, science, and Korean language arts) were moderate with Pearson r correlation coefficient, $\rho_r = 0.482$, 0.422, and 0.316, respectively.

Gender Differences

Table 5 shows that male students outperformed female students on the spatial test and the gender difference was significant ($t\,(347) = -\,4.71, p < 0.001$) and large (Cohen's $d =$ 0.531). There was no gender differences in mathematics ($t\,(339) = -\,0.44, p = 0.663$) and science ($t\,(339) = -\,0.68, p = 0.500$), and also the magnitude of the effect was small with Cohen's $d = 0.045$ and 0.076 respectively. Contrary to male students' better performance on spatial, mathematics and science tests, female students significantly scored better on Korean language arts ($t\,(182.25)= 2.81, p = 0.011$) with a small effect size of Cohen's $d =$ $-\,0.323$.

Table 4. Correlations among the four measures (spatial, Korean language arts, mathematics, and science test scores) and the other observed variables

	2	3	4	5	6
1. Spatial Ability	0.482**	0.422**	0.316**	0.245**	0.118*
2. Mathematics	—	0.614**	0.542**	0.024	0.199**
3. Science		—	0.538**	0.037	0.175**
4. Korean Language Arts			—	0.152**	0.220**
5. Gender[a]				—	–0.004
6. Grade					—

Note. Intercorrelations among the Revised PSVT:R scores and the other variables are presented above the diagonal. Missing cases were excluded pairwise.
[a] Girls were coded as 0 and boys as 1. $^{**}p < 0.01$. $^{*}p < 0.05$.

Table 5. Mean comparisons of students' performance by gender in spatial ability, mathematics, science, and Korean language arts.

	Statistics	*p*	Effect Size (Cohen's)
Spatial ability	$t(347) = -4.71$	< 0.001	Cohen's $d = 0.531$
Mathematics	$t(339) = -0.44$	0.663	Cohen's $d = 0.045$
Science	$t(339) = -0.68$	0.500	Cohen's $d = 0.076$
Korean language arts	$t(182.25) = 2.81$	0.011	Cohen's $d = -0.323$

Grade Level Differences

One-way ANOVA (Table 6) revealed significant grade level difference in spatial ability, mathematics, science, and Korean language arts test scores with magnitude of effect sizes, respectively. There were significant effects of grade level in spatial ability (Welch's $F(2, 346) = 11.448$, $p < 0.001$; $\omega = 0.24$) and mathematics (Welch's $F(2, 89.83) = 8.665$, $p < 0.001$; $\omega = 0.27$), science (Welch's $F(2, 338) = 32.301$, $p < 0.001$; $\omega = 0.39$), and Korean language arts(Welch's $F(2, 336) = 40.349$, $p < 0.001$; $\omega = 0.43$).

Here, relatively conservative and robust Welch's F test was derived because homogeneity of variance assumption was broken and group sizes were unequal (Field, 2009). Particularly, post-hoc analyses revealed that eighth and ninth grade students performed significantly better than seventh grade students in mathematics, but no grade level differences existed between eighth grade and ninth grade students. Similar results occurred in Korean language arts.

Table 6. Mean Comparisons of Students' Performance by Grade in Spatial ability, Mathematics, Science, and Korean Language arts.

	Statistics	p	Effect Size(ω)
Spatial ability	$F(2, 346) = 11.448$	< 0.001	0.24
Mathematics	$^{a}F(2, 89.83) = 8.665$	< 0.001	0.27
Science	$F(2,338) = 32.301$	< 0.001	0.39
Korean Language arts	$F(2,336) = 40.349$	< 0.001	0.43

Note. [a]Welch's F statistics

CONCLUSION

Spatial ability has been valued as one component of intelligence and as a talented domain. In addition, it is generally acknowledged that spatial ability associated with achievements in science, technology, engineering, and mathematics (STEM) disciplines. Particularly, researchers agree that spatial ability is important in mathematics education because it enhances an intuitive view and an understanding in many areas of mathematic, like geometry. Among various types of spatial ability, this study explored 3-D mental rotation ability of Korean middle school students (Grades 7–9), and investigated its association with their achievement in three subjects, mathematics, science, and Korean language arts, by gender and grade level.

The results of this study revealed the positive and significant correlation between the 3-D mental rotation ability and each subject performance. Among the three subjects, the magnitude of correlation with mathematics was larger than science and Korean language arts. This result supports the significant correlation between spatial ability and mathematical achievement in the literature (*e.g.*, Battista, 1990; McGee, 1979; Sherman, 1979). However, this study found that gender difference did not exist in mathematics. While this result is consistent with some research (e.g., Hyde, Fennema & Lamon, 1990), showing not gender difference in mathematics achievement but gender difference in spatial ability favoring male students. This might be contributed to the ceiling effect due to relatively easy mathematics questions for the participants of this study. In other words, the mathematics test was relatively easy, so it did not function well to differentiate students' mathematics ability.

There are many studies examining spatial ability, relationships between spatial ability and mathematics achievement, and gender difference in mathematics and spatial ability. However, the results of each study have not been consistent across the studies because of different instruments used to measure spatial ability and mathematics performance. Therefore, it is necessary to carefully develop and select both spatial and math measures when conducting research.

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