Diversity-Multiplexing Tradeoff Analysis of Wireless Multiple-Antenna Cooperative Systems in General Fading Channels

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Abstract

In this paper, diversity-multiplexing tradeoff (DMT) of three-user wireless multiple-antenna cooperative systems is investigated in *general* fading channels when half-duplex and decode-and-forward relay is employed. Three protocols, i.e., adaptive protocol, receive diversity protocol, and dual-hop relaying protocol, are considered. The *general* fading channels may include transmit and/or receive correlation and nonzero channel means, and are extensions of independent and identically distributed Rayleigh or Rician fading channels. Firstly, simple DMT expressions are derived for *general* fading channels with zero channel means and no correlation when users employ arbitrary number of antennas. Explicit DMT expressions are also obtained when all users employ the same number of antennas and the channels between any two users are of the same fading statistics. Finally, the impact of nonzero channel means and/or correlation on DMT is evaluated. It is revealed theoretically that the DMTs depend on the number of antennas at each user, channel means (except for Rayleigh and Rician fading statistics), transmit and/or receive correlation, and the polynomial behavior near zero of the channel gain probability density function. Examples are also provided to illustrate the analysis and results.

Keywords: Cooperative communication, multiple-antenna, diversity-multiplexing tradeoff, general fading channels

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1. Introduction

Cooperative communication is a promising technology to exploit spatial diversity by utilizing antennas belonging to different users in wireless networks. It could be used to enlarge the coverage area of cellular network [1], save power by exploiting spatial diversity of independent links [2], and improve the spectral efficiency (i.e., increasing the capacity of a pair of users by employing relay) [3][4][5]. Cooperative users can employ single antenna or multiple antennas. The multiple-antenna configuration provides additional degrees of freedom and spatial diversity gain and it has been triggering off wide research activities [6][7][8][9][10][11][12][13][14][15][[16].

In this paper, we study the performance of multiple-antenna, half-duplex, and decode-and-forward (DF) cooperative systems in wireless *general* fading channels [17]. The simple three-user case, i.e., one source, one relay, and one destination, is considered. The half-duplex constraint, i.e., the users cannot transmit and receive simultaneously, is imposed on the relay due to the large difference in the transmitting and receiving signal power levels [3]. The diversity-multiplexing tradeoff (DMT) at asymptotically high signal-to-noise ratio (SNR) is adopted as the performance measurement metric. DMT is a comprehensive metric measuring the performance of wireless communication systems. It takes into account both the diversity gain and multiplexing gain (or degree of freedom) simultaneously. Traditional performance metrics, e.g., outage probability, bit error ratio, symbol error ratio, and data rate (or capacity), only focus on one aspect, i.e., diversity gain or multiplexing gain.

The DMT concept was first proposed by Zheng and Tse to study the data rate and pairwise error probability tradeoff in the point-to-point multiple-input multiple-output system (MIMO) [18], and is defined as follows [19].

Definition: Assuming a family of codes $C(\rho)$ with rate $R(\rho)$, operating at SNR ρ, if $P_{out}(\rho)$ is the outage probability of the channel for rate $R(\rho)$, the multiplexing gain r and diversity gain d are defined as $r = \lim_{\rho\to\infty} R(\rho)/\log(\rho)$ and $d = -\lim_{\rho\to\infty} \log(P_{out}(\rho))/\log(\rho)$.

The DMT provides a more comprehensive point of view on the overall performance of cooperative system because it takes into account the impact of data rate (i.e., $r \neq 0$) on outage probability [18] rather than considering outage probability with fixed data rate (i.e., $r = 0$) [20].

Most studies of DMT on multiple-antenna cooperative systems employed the Rayleigh or Rician fading channel model with independent links within the MIMO channel matrix [6], [7], [10][11][12], [21]. The authors in [22] studied the outage probability/capacity for a *general* fading distribution, but the analysis was limited to the low SNR regime and the links within a channel matrix were still required to be independent. The DMT of multiple-antenna relaying systems in *general* fading channels was analyzed in [9], but with the single-antenna relay constraint and only for dual-hop relaying protocol. In this paper, we assume that 1) the source, relay, and destination employ M, K, and N antennas, respectively, with $M, K, N \geq 1$, and 2) the channels between any two users are with *general* fading, i.e., transmit correlation and/or receive correlation, nonzero channel means, and *general* fading statistics[1] (including Rayleigh, Rician, Nakagami-*m*, Weibull, and Nakagami-*q* as special cases) are considered [17]. We consider three protocols, i.e., adaptive protocol (AP) [8], receive diversity protocol (RDP), and dual-hop relaying protocol (DHR) [23], which are the normally adopted protocols in the literature.

The DMT analysis of multiple-antenna cooperative system in *general* fading channels, still an open problem, is of theoretical and practical importance because it takes practical physical environment conditions into consideration and Rayleigh fading channel model is not always appropriate. For example, the line-of-sight microcellular communication and mobile indoor and outdoor communication channels were modeled by Rician distribution [24] and Nakagami-*m* distribution [25], respectively. Indoor and outdoor digital communications were modeled by Weibull distribution [26][27][28]. Furthermore the equipment size-limitation and/or lack of scattering in the environment could lead to transmit and/or receive correlation [29].

It is noteworthy that this paper is an extension of [17], which studied the DMT of point-to-point MIMO system in *general* fading channels, to the cooperative communication scenario. The major contributions are that we analytically obtain the DMTs of half-duplex and DF multiple-antenna cooperative systems with three protocols (i.e., AP, RDP, and DHR), in *general* fading channels. To be specific:

- Instead of formulating the DMT derivation as an optimization problem [21], we derive an iterative algorithm to compute the DMT. The iterative algorithm is an extension of the derivation of DMT in [8] for Rayleigh fading channel to the *general* fading channels. Furthermore, we use the *general* fading channel models while [21] adopted the Rayleigh channel model. Though explicit DMT expressions for *general* cases, including *general* fading statistics and *arbitrary* number of antennas, are not obtained, our expressions are much more simpler and can be more easily computed than the derivation of the solution of an optimization problem. Explicit DMT expressions are also obtained under specific conditions[2].
- We study the DMTs of three different protocols that are normally adopted in the literature with *arbitrary* M, K, and N in *general* fading channels, while [9] studied only the dual-hop relaying protocol and with additional constraint on the number of relay antennas, i.e., $K=1$.
- Compared with the DMT derivation in [8], where the channel between the source and the relay is assumed to be ideal (i.e., without outage), we consider a practical scenario where the source-relay channel may be in outage.
- We find that the DMTs of the *general* fading channels are dependent on the number of antennas, channel means (except for Rayleigh and Rician fading statistics), transmit and/or receive correlation, and the polynomial behavior near zero of the channel gain probability density function (PDF), rather than a specific fading distribution.

The rest of the paper is organized as follows. In Section II, the *general* fading channels are introduced, and three protocols are depicted in details. The DMT analysis is presented in Section III where we assume no transmit or receive correlation and zero channel means, and

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[1] Throughout the paper, general fading statistics only refers to the amplitude distribution of channel gains. General fading channel includes not only general fading statistics but also nonzero channel mean and correlation.

[2] Since the DMTs are all piecewise-linear functions (seen from the ensuing part of the paper), "explicit expressions" means that the connecting points of a piecewise-linear function is expressed explicitly in closed-form while "simple expressions" means the connecting points can be found through some simple operations (though without explicit expressions) throughout the paper.

simple DMT expressions are derived. In Section IV, some special cases with explicit DMT expression are provided, and we also give some examples illustrating the DMT analysis of Section III. The impact of correlation and nonzero channel means on DMT is discussed in Section V. Conclusions are drawn in Section VI.

Notation: Bold uppercase letters represent matrices and bold italic letters denote vectors. $\log$ is the logarithm function with base 2, $x^{+} = \max(0, x)$. $\mathbf{I}_p$ denotes the identity matrix with size $p \times p$ unless specified with other meanings. $\det(\cdot)$ denotes the determinant of a matrix. $(\cdot)^{\dagger}$ is the conjugate transposition of a matrix. $|\cdot|$ is the absolute value of a variable. Exponential equality is represented by $f(x) \doteq x^{q}$, which means $\lim_{x\to\infty} \log f(x)/\log x = q$, and q is called the exponential order of $f(x)$. $P_r(\cdot)$ is the probability of a random event. $P_r(\cdot|\cdot)$ is the conditional probability.

2. System and Channel Model

2.1. Three Protocols

The cooperative protocols under consideration are AP, RDP, and DHR[3]. The source, relay, and destination are denoted by S, R, and D, each employing M, K, and N antennas, respectively. The channel matrices corresponding to the links $S \to R$, $R \to D$, and $S \to D$ are represented by $\mathbf{H}_{SR}$, $\mathbf{H}_{RD}$, and $\mathbf{H}_{SD}$, respectively. All relays in the three protocols operate with half-duplex and DF mode.

The three protocols are illustrated in **Fig. 1**.

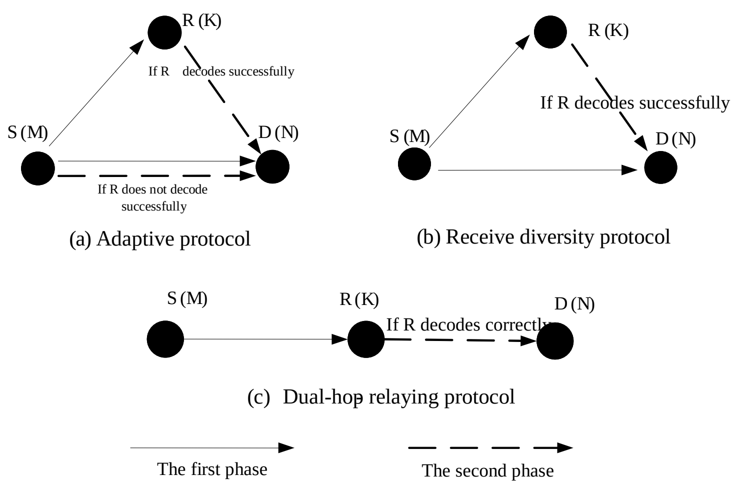

Fig. 1. The three cooperative relaying protocols

For AP, the transmitted information is firstly encoded to a codeword. The source broadcasts the first half of the codeword to the relay and destination in the first phase. In the

[3] There are other half-duplex DF protocols, e.g., transmit diversity protocol and simplified transmit diversity protocol [19]. These protocols can be analyzed in the same way provided in this paper but the analysis is more complicated, which is tedious and beyond the interest of this paper. The three protocols in this paper represent the most used DF protocols in the literature.

second phase, the relay re-encodes the received information (i.e., recovers the whole codeword from the source) and forwards the second half of the codeword to the destination with the source keeping silent if it decodes the information from the source correctly. Otherwise, the relay sends a non-acknowledge (NACK) feedback to the source, then the source continues to transmit the second half of the codeword to the destination. The destination receives information from the relay link (i.e., the $S \rightarrow R \rightarrow D$ link) if the relay decodes the information correctly or from the direct link (i.e., the $S \rightarrow D$ link) while the relay fails decoding. Without loss of generality, the NACK is assumed to be correctly received by the source with negligible spectral efficiency sacrifice [19].

For RDP, the first phase is the same as AP. In the second phase, the relay retransmits the second half of the codeword to the destination if it decodes the information from the source correctly, or else, keeps silent. The relay does not send NACK feedback if it fails decoding the information and the source keeps silent in the second phase. If the relay decodes successfully, the destination combines the signals from both the direct link and the relay link with a code-combing method. The destination receives information from the direct link if the relay cannot decode the information correctly or from both links (i.e., direct and relay links) while the relay is successful in decoding the information. Since RDP needs no NACK feedback, it is simpler than AP.

For DHR, there is no direct link between the source and the destination, corresponding to the scenario where the source is far from the destination or the channel between them is blocked by obstacles. In the first phase, the source broadcasts the first half of the codeword to the relay. In the second phase, the relay operates in the same way with the second phase of RDP. In this protocol, the destination only receives information from the relay if the relay makes a successful decoding or does not receive any otherwise.

It is noted that in all three protocols the coded cooperation scheme is employed rather than the repetition-based scheme and Gaussian random codebook is assumed throughout the paper.

2.2. Channel Model

The *general* fading channel models adopted here are the same as that of [17]. We restate it here for completeness. The channels between any two antennas of any two users are assumed to be quasi-static, flat fading with *general* fading statistics. *General* fading statistics means the following three properties hold for PDF (denoted as $p(h)$) of any element h in any channel matrix (i.e., $\mathbf{H}_{SR}$, $\mathbf{H}_{RD}$, or $\mathbf{H}_{SD}$).

1) polynomial behavior near 0: $|h| \rightarrow 0$, $p(h) = \Xi\left(|h|^{t}\right)$, $t \geq 0$;
2) fast decaying behavior near ∞: $|h| \rightarrow \infty$, $p(h) = O\left(e^{-b|h|^{\beta}}\right)$, $b > 0$, and $\beta > 0$;
3) $p(h)$ is upper bounded by a constant K;

where $f(x) = \Xi(g(x))$ denotes that there exists two positive numbers a_1 and a_2 such that $a_1 g(x) \leq f(x) \leq a_2 g(x)$ holds, and $f(x) = O(g(x))$ means that there exists a positive number a_3 such that $f(x) \leq a_3 g(x)$. The *general* fading channel models are suitable for many practical channel models, including complex Gaussian channels (their amplitudes follow Rayleigh or Rician distribution), Nakagami-*m* ($m \geq 1$), Weibull ($\eta \geq 2$), and Nakagami-*q* ($0 < q \leq 1$) with the assumption that the phase is uniformly distributed over $[0, 2\pi)$ and independent of the amplitude. The corresponding parameters t, b, and β are listed in [17, TABLE I] and omitted here for brevity.

In Section III, all the elements of all channel matrices are mutually independent and with zero mean, but not necessarily identically distributed. Impact of transmit and/or receive correlation and non-zero mean on DMT is studied in Section V. It should be emphasized that correlation only means the elements in the same channel matrix are no longer mutually independent (or uncorrelated) and all channel matrices, i.e., $\mathbf{H}_{SR}$, $\mathbf{H}_{RD}$, and $\mathbf{H}_{SD}$, are still mutually independent (or uncorrelated).

3. Diversity-Multiplexing Tradeoff Analysis

Before proceeding, we adopt the following notations. The channel outage events of links $S \to R$, $R \to D$, and $S \to D$ are denoted by Θ_i s($i = SR, RD$, and SD) and $\overline{\Theta}_i$ is the corresponding complementary event. All transmitters are assumed to transmit with the same total power P (different power constraints of transmitters have no effect on DMT since they are all treated as ρ^0 in DMT analysis [18]). The complex additive white Gaussian noises (AWGNs) at all receivers are mutually independent with zero mean and power σ^2, and independent of all channel elements. The data rate is assumed to be $C = r\log\rho$ (bits/s/Hz) with r being the multiplexing gain, $\rho = P/\sigma^2$ is the total transmit SNR. The instantaneous channel capacity (in bits/s/Hz), between the links $S \to R$, $R \to D$, and $S \to D$, are written as $I_{SR} = \log\det\left(\mathbf{I}_K + P\mathbf{H}_{SR}\mathbf{H}_{SR}^{\dagger}/\left(M\sigma^2\right)\right)$, $I_{RD} = \log\det\left(\mathbf{I}_N + P\mathbf{H}_{RD}\mathbf{H}_{RD}^{\dagger}/\left(K\sigma^2\right)\right)$, and $I_{SD} = \log\det\left(\mathbf{I}_N + P\mathbf{H}_{SD}\mathbf{H}_{SD}^{\dagger}/\left(M\sigma^2\right)\right)$, respectively, assuming that only the receivers of the transmit-receive pairs have channel state information (CSI) while the corresponding transmitters do not have [30]. According to [18],

$$I_{SR} \triangleq \log\det\left(\mathbf{I}_K + \rho\mathbf{H}_{SR}\mathbf{H}_{SR}^{\dagger}\right) = \log\prod_{i=1}^{\min(M,K)}\left(1+\rho c_i\right) = \log\prod_{i=1}^{\min(M,K)}\rho^{(1-\alpha_i)^+}, \quad (1)$$

$$I_{RD} \triangleq \log\det\left(\mathbf{I}_N + \rho\mathbf{H}_{RD}\mathbf{H}_{RD}^{\dagger}\right) = \log\prod_{j=1}^{\min(N,K)}\left(1+\rho d_j\right) = \log\prod_{j=1}^{\min(N,K)}\rho^{(1-\beta_j)^+}, \quad (2)$$

$$I_{SD} \triangleq \log\det\left(\mathbf{I}_N + \rho\mathbf{H}_{SD}\mathbf{H}_{SD}^{\dagger}\right) = \log\prod_{k=1}^{\min(M,N)}\left(1+\rho l_k\right) = \log\prod_{k=1}^{\min(M,N)}\rho^{(1-\gamma_k)^+}, \quad (3)$$

where $\left(c_1, c_2, \cdots, c_{\min(M,K)}\right)$, $\left(d_1, d_2, \cdots, d_{\min(N,K)}\right)$, and $\left(l_1, l_2, \cdots, l_{\min(M,N)}\right)$ represent the eigenvalues of $\mathbf{H}_{SR}\mathbf{H}_{SR}^{\dagger}$, $\mathbf{H}_{RD}\mathbf{H}_{RD}^{\dagger}$, and $\mathbf{H}_{SD}\mathbf{H}_{SD}^{\dagger}$, respectively, with $c_i = \rho^{-\alpha_i}$, $d_j = \rho^{-\beta_j}$, and $l_k = \rho^{-\gamma_k}$, $\forall\, i \in \{1, 2, \ldots, \min(M,K)\}$, $j \in \{1, 2, \ldots, \min(N,K)\}$, and $k \in \{1, 2, \ldots, \min(M,N)\}$.

3.1. Receive Diversity Protocol

Considering Gaussian random codebook, the outage probability of the receive diversity protocol with data rate $C = r\log\rho$ (bits/s/Hz) is expressed as

$$\begin{aligned} P_r^{RDP}(C) &= P_r^{RDP}\left(\Theta_{SR}\right)P_r^{RDP}\left(\Theta_{SD} \mid \Theta_{SR}\right) + P_r^{RDP}\left(\overline{\Theta_{SR}}\right)P_r^{RDP}\left(\Theta_{SD} \mid \overline{\Theta_{SR}}\right) \\ &= P_r\left(I_{SR}/2 < C\right)P_r\left(I_{SD}/2 < C\right) + \left(1 - P_r\left(I_{SR}/2 < C\right)\right)P_r\left(\left(I_{SD} + I_{RD}\right)/2 < C\right) \\ &\doteq P_r\left(\sum_{i=1}^{\min(M,K)}\left(1-\alpha_i\right)^+ < 2r\right)P_r\left(\sum_{k=1}^{\min(M,N)}\left(1-\gamma_k\right)^+ < 2r\right) + \\ &\left(1 - P_r\left(\sum_{i=1}^{\min(M,K)}\left(1-\alpha_i\right)^+ < 2r\right)\right)P_r\left(\sum_{j=1}^{\min(N,K)}\left(1-\beta_j\right)^+ + \sum_{k=1}^{\min(M,N)}\left(1-\gamma_k\right)^+ < 2r\right) \end{aligned} \quad .(4)$$

To compute (4), we first introduce the following theorem.

Theorem 1*[17]*: For the MIMO channel with independent and *general* fading paths, let $m=\min\{n_R,n_T\}$, where n_R and n_T are the number of receive and transmit antennas, respectively, and the data rate be $C=r\log\rho$ with $0\le r\le m$, the outage probability satisfies $P_{out}(C) \doteq \rho^{-d(r)}$ where $d(r)=\inf_{\boldsymbol{\theta}\in A(r)} f(\boldsymbol{\theta})$, with $f(\boldsymbol{\theta})=\sum_{i=1}^{m}(|n_T-n_R|+2i-1)\theta_i+\theta_m\sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\frac{t_{ij}}{2}$, $\boldsymbol{\theta}=(\theta_1,\theta_2,\ldots,\theta_m)$, and t_{ij} is the parameter reflecting the polynomial behavior near zero of the channel gain PDF corresponding to the j – th transmit antenna and i – th receive antenna link, and $A(r)=\left\{\boldsymbol{\theta}:\theta_1\ge\theta_2\ge\cdots\theta_m\ge 0,\sum_{i=1}^{m}(1-\theta_i)^+<r\right\}$. The DMT curve, represented by $d(r)$, is given by the piecewise-linear function connecting the points $(k,d(k))$, for $k=0,1,\cdots,m$, where

$$d(k)=\begin{cases}\sum_{i=1}^{n_R}\sum_{j=1}^{n_T}(1+t_{ij}/2), & \text{if } k=0\\ (n_R-k)(n_T-k), & \text{if } k=1,2,\cdots,m\end{cases}. \qquad \blacksquare$$

It is seen that the DMT of point-to-point MIMO system is decided by the number of transmit and receive antennas and t_{ij} s. The following part will show how these parameters affect the DMT of the multiple-antenna cooperative communication systems.

By *Theorem* 1, if we denote $P_r\left(\sum_{i=1}^{\min(M,K)}(1-\alpha_i)^+<2r\right)\doteq\rho^{-d_{SR}(r)}$, then $d_{SR}(r)$ is a piecewise-linear function connecting the points $(r_i,d_{SR}(r_i))$, where $r_i=i/2$, $i=0,1,2,\cdots,\min(M,K)$, and

$$d_{SR}(r_i)=\begin{cases}\sum_{i_{SR}=1}^{M}\sum_{j_{SR}=1}^{K}(1+t_{i_{SR}j_{SR}}/2), & \text{if } i=0\\ (M-2r_i)(K-2r_i), & \text{if } i=1,2,\cdots,\min(M,K)\end{cases}. \qquad (5)$$

With the same way, denoting $P_r\left(\sum_{k=1}^{\min(M,N)}(1-\gamma_k)^+<2r\right)\doteq\rho^{-d_{SD}(r)}$, then $d_{SD}(r)$ is a piecewise-linear function connecting the points $(r_k,d_{SD}(r_k))$, where $r_k=k/2$, $k=0,1,2,\cdots,\min(M,N)$, and[4]

$$d_{SD}(r_k)=\begin{cases}\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}(1+t_{i_{SD}j_{SD}}/2), & \text{if } k=0\\ (M-2r_k)(N-2r_k), & \text{if } k=1,2,\cdots,\min(M,N)\end{cases}. \qquad (6)$$

The computation of the term $P_r\left(\sum_{j=1}^{\min(N,K)}(1-\beta_j)^++\sum_{k=1}^{\min(M,N)}(1-\gamma_k)^+<2r\right)$ is rather complex than that of the above two, and is vital in obtaining simple or explicit DMT expressions. To compute the term, we extend the proof of [8], which assumes Rayleigh fading channel, to the *general* fading channels and obtain the following *Lemma*.

Lemma: Let $P_r\left(\sum_{j=1}^{\min(N,K)}(1-\beta_j)^++\sum_{k=1}^{\min(M,N)}(1-\gamma_k)^+<2r\right)\doteq\rho^{-d_{out}(r)}$, then $d_{out}(r)$ is a piecewise-linear function connecting points $(r_t,d_{out}(r_t))$, where $r_t=t/2$, $t=0,1,2,\cdots,\min(N,K)+\min(M,N)$, and

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4 $t_{i_{SD}j_{SD}}$ is the parameter reflecting the polynomial behavior near zero of the channel gain PDF of the j – th transmit antenna and i – th receive antenna link in the channel matrix $\mathbf{H}_{SD}$. The same notation is used for the channel matrices $\mathbf{H}_{RD}$ and $\mathbf{H}_{SR}$.

$$d_{out}\left(r_t\right)=\left(N-a_t\right)\left(K-a_t\right)+Q\left(b_t\right)\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}t_{i_{SD}j_{SD}}/2+\left(M-b_t\right)\left(N-b_t\right)+P\left(a_t\right)\sum_{i_{RD}=1}^{K}\sum_{j_{RD}=1}^{N}t_{i_{RD}j_{RD}}/2 \quad (7)$$

where $Q\left(b_t\right)\triangleq\begin{cases}1, & \text{if } b_t=0\\ 0, & \text{if } b_t\neq 0\end{cases}$ and $P\left(a_t\right)\triangleq\begin{cases}1, & \text{if } a_t=0\\ 0, & \text{if } a_t\neq 0\end{cases}$. The parameters a_t and b_t are determined by the following algorithm.

Algorithm 1

`Initialization` $t=0,\ a_0=b_0=0,$

`for` $t=1:\min\left(K,N\right)+\min\left(M,N\right)$

  `if` $\left(M-b_{t-1}\right)\left(N-b_{t-1}\right)-\left(M-b_{t-1}-1\right)\left(N-b_{t-1}-1\right)+Q\left(b_{t-1}\right)\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}t_{i_{SD}j_{SD}}/2$

   $<\left(N-a_{t-1}\right)\left(K-a_{t-1}\right)-\left(N-a_{t-1}-1\right)\left(K-a_{t-1}-1\right)+P\left(a_{t-1}\right)\sum_{i_{RD}=1}^{K}\sum_{j_{RD}=1}^{N}t_{i_{RD}j_{RD}}/2$

   $a_t=a_{t-1}+1,\ \ b_t=b_{t-1}$

  `else`

   $a_t=a_{t-1},\ \ b_t=b_{t-1}+1$

  `end`

`end`

Proof : See APPENDIX A.

The algorithm, including $\min\left(K,N\right)+\min\left(M,N\right)$ iterations (each with simple comparison and addition operation), provides a much simple way to compute the term $P_r\left(\sum_{j=1}^{\min(N,K)}\left(1-\beta_j\right)^{+}+\sum_{k=1}^{\min(M,N)}\left(1-\gamma_k\right)^{+}<2r\right)$. An explicit expression is also obtained as per (7). It is emphasized that the computation of the term is a hard problem, but with vital importance, in the derivation of DMT for *arbitrary* number of antennas and *general* fading statistics. In [21] it is formulated as an optimization problem for *arbitrary* number of antennas (but not *general* fading statistics) with no explicit or simple expression[5]. With those results, the DMT of RDP is easily obtained in *Theorem* 2, the proof of which is omitted because it is straightforward.

Theorem 2: Let $P_r^{RDP}\left(C\right)\doteq\rho^{-d_{RDP}(r)}$, then the DMT curve of the receive diversity protocol, denoted by $d_{RDP}\left(r\right)$, is $d_{RDP}\left(r\right)=\min\left(d_{SR}\left(r\right)+d_{SD}\left(r\right),d_{out}\left(r\right)\right)$, where $d_{SR}\left(r\right)$, $d_{SD}\left(r\right)$, and $d_{out}\left(r\right)$ are piecewise-linear functions with their connecting points given by (5), (6) and the *Lemma*, respectively .

3.2. Adaptive Protocol

Following the same way as that of RDP, the outage probability of the adaptive protocol with data rate $C=r\log\rho$ (bits/s/Hz) can be expressed as

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[5] Even though the optimization problem in [21] had (actually had no except for some specific scenarios) simple expressions, the derivation of the solution still is of higher computation complexity than the simple $\min\left(K,N\right)+\min\left(M,N\right)$ iterations in *Algorithm* 1 in the paper.

$$\begin{aligned} P_r^{AP}(C) &= P_r^{AP}(\Theta_{SR}) P_r^{AP}(\Theta_{SD} \mid \Theta_{SR}) + P_r^{AP}\left(\overline{\Theta_{SR}}\right) P_r^{AP}\left(\Theta_{SD} \mid \overline{\Theta_{SR}}\right) \\ &= P_r(I_{SR}/2 < C) P_r(I_{SD} < C) + \left(1 - P_r(I_{SR}/2 < C)\right) P_r\left((I_{SD} + I_{RD})/2 < C\right) \\ &\doteq P_r\left(\sum_{i=1}^{\min(M,K)} (1-\alpha_i)^+ < 2r\right) P_r\left(\sum_{k=1}^{\min(M,N)} (1-\gamma_k)^+ < r\right) + \\ &\left(1 - P_r\left(\sum_{i=1}^{\min(M,K)} (1-\alpha_i)^+ < 2r\right)\right) P_r\left(\sum_{j=1}^{\min(N,K)} (1-\beta_j)^+ + \sum_{k=1}^{\min(M,N)} (1-\gamma_k)^+ < 2r\right) \end{aligned} \quad (8)$$

The difference between (8) and (4) is that $P_r^{AP}(O_{SD} \mid O_{SR}) = \Pr(I_{SD} < C) \doteq \rho^{-d'_{SD}(r)}$, where $d'_{SD}(r)$ is a piecewise-linear function connecting points $(r_k, d'_{SD}(r_k))$, with $r_k = k$, $k = 0,1,2,\cdots,\min(M, N)$, and

$$d'_{SD}(r_k) = \begin{cases} \sum_{i_{SD}=1}^{M} \sum_{j_{SD}=1}^{N} \left(1 + t_{i_{SD} j_{SD}}/2\right), & \text{if } k = 0 \\ (M - r_k)(N - r_k), & \text{if } k = 1,2,\cdots,\min(M,N) \end{cases}. \quad (9)$$

This difference results from the fact that the source continues to transmit to the destination if the relay cannot decode successfully in AP while it does not transmit in RDP in the second phase. The DMT of AP is thus obtained in *Theorem* 3, whose proof is omitted because of its straightforwardness.

Theorem 3: Denoting $P_r^{AP}(C) \doteq \rho^{-d_{AP}(r)}$, then the DMT curve of the adaptive protocol, denoted by $d_{AP}(r)$, is $d_{AP}(r) = \min\left(d_{SR}(r) + d'_{SD}(r), d_{out}(r)\right)$, where $d_{SR}(r)$, $d'_{SD}(r)$, and $d_{out}(r)$ are piecewise-linear functions with their connecting points given by (5), (9) and the *Lemma*, respectively .

3.3. Dual-Hop Relaying Protocol

With the same reasoning as that of the above two protocols, the outage probability of the dual-hop relaying protocol with data rate $C = r\log\rho$ (bits/s/Hz) can be expressed as

$$\begin{aligned} P_r^{DHR}(C) &= P_r^{DHR}(\Theta_{SR}) + P_r^{DHR}\left(\overline{\Theta_{SR}}\right) P_r^{DHR}\left(\Theta_{RD} \mid \overline{\Theta_{SR}}\right) \\ &= P_r(I_{SR}/2 < C) + \left(1 - P_r(I_{SR}/2 < C)\right) P_r(I_{RD}/2 < C) \\ &\doteq P_r\left(\sum_{i=1}^{\min(M,K)} (1-\alpha_i)^+ < 2r\right) + P_r\left(\sum_{j=1}^{\min(N,K)} (1-\beta_j)^+ < 2r\right) \end{aligned} . \quad (10)$$

The first term $P_r^{DHR}(\Theta_{SR})$ states that if the $S \to R$ links is in outage then the DHR is in outage. According to *Theorem* 1, $P_r\left(\sum_{j=1}^{\min(N,K)} (1-\beta_j)^+ < 2r\right) \doteq \rho^{-d_{RD}(r)}$, where $d_{RD}(r)$ is a piecewise-linear function connecting points $(r_j, d_{RD}(r_j))$, with $r_j = j/2$, $j = 0,1,2,\cdots,\min(K,N)$, and

$$d_{RD}(r_j) = \begin{cases} \sum_{i_{RD}=1}^{K} \sum_{j_{RD}=1}^{N} \left(1 + t_{i_{RD} j_{RD}}/2\right), & \text{if } j = 0 \\ (M - 2r_j)(N - 2r_j), & \text{if } j = 1,2,\cdots,\min(K,N) \end{cases}, \quad (11)$$

then the DMT of DHR is obtained in *Theorem* 4, whose proof is omitted for its straightforwardness.

Theorem 4: Let $P_r^{DHR}(C) \doteq \rho^{-d_{DHR}(r)}$, then the DMT curve of the dual-hop relaying protocol, denoted by $d_{DHR}(r)$, is $d_{DHR}(r) = \min\left(d_{SR}(r), d_{RD}(r)\right)$, where both $d_{SR}(r)$

and $d_{RD}(r)$ are piecewise-linear functions with their connecting points given by (5) and (11), respectively .

It is noted that this DMT result has been reported in [7, *Proposition* 1] in Rayleigh fading channel. Our result is an extension to the *general* fading channels[6]. It can be seen from the above theorems that the DMT depends on M, K, N, and channel parameter t (i.e., polynomial behavior near zero of the channel probability density function). Channel parameters b, β, and channel gain variance have no impact on DMT.

Though explicit DMT expressions are not obtained for the three protocols, the expressions of $d_{SR}(r)$, $d_{SD}(r)$, $d'_{SD}(r)$, and $d_{RD}(r)$ are explicit, and $d_{out}(r)$ can be computed by the *Lemma* (all are piecewise-linear functions). The DMTs are obtained finally by a min operation shown in the above theorems, and are also piecewise-linear functions since the piecewise-linear property would not alter with the addition and min operations. Our computation is much more simpler than that of [21] where the DMT computation was formulated as an optimization problem and no explicit or simple expressions were provided for *arbitrary* number of antennas. That is why we say simple (not explicit) expressions are obtained in our paper[7].

4. Special Cases With Explicit DMT Expressions

The DMT expressions of the three protocols can be computed by the results in Section III for *general* fading channels with *arbitrary* M, K, and N, but explicit expressions cannot be obtained except for some special cases. In this section we give the explicit DMT expressions for the special case when all users employ the same number of antennas (i.e., $M = K = N$) and the channels between any two users are of the same fading statistics.

For the case when $M = K = N$ and $\sum_{i_{SR}=1}^{M}\sum_{j_{SR}=1}^{K}\left(1+\frac{t_{i_{SR}j_{SR}}}{2}\right)=\sum_{i_{RD}=1}^{K}\sum_{j_{RD}=1}^{N}\left(1+\frac{t_{i_{RD}j_{RD}}}{2}\right)=\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}\left(1+\frac{t_{i_{SD}j_{SD}}}{2}\right)$, there is the following corollary.

Corollary : (i) The DMT curve of RDP is $d'_{RDP}(r)$, a piecewise-linear function connecting points $\left(r_{i_{RDP}}, d'_{RDP}\left(r_{i_{RDP}}\right)\right)$ with $r_{i_{RDP}} = i_{RDP}/2$, $i_{RDP} = 0,1,2,\cdots,M$, and

$$d'_{RDP}\left(r_{i_{RDP}}\right)=\begin{cases}\sum_{i_{RD}=1}^{K}\sum_{j_{RD}=1}^{N}\left(1+t_{i_{RD}j_{RD}}/2\right)+\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}\left(1+t_{i_{SD}j_{SD}}/2\right), & \text{if } i_{RDP}=0\\ 2\left(M-2r_{i_{RDP}}\right)^2, & \text{if } i_{RDP}=1,2,\ldots,M\end{cases}. \quad (12)$$

(ii) The DMT curve of AP is $d'_{AP}(r)$, a piecewise-linear function connecting points $\left(r_{i_{AP}}, d'_{AP}\left(r_{i_{AP}}\right)\right)$ with $r_{i_{AP}} = i_{AP}/2$, $i_{AP} = 0,1,2,\cdots,2M$, and

[6] Though *Proposition* 1 in [7] stated a general expression with no explicit DMT expression, the Rayleigh fading channel is assumed implicitly there. After extending the general fading channel to the DHR, the form of DMT expression in the paper is the same as that of [7, *Proposition* 1] regardless of channel statistics.

[7] Though the same expressions with the min operation form as those in *Theorems* 2, 3, and 4 are also obtained in [6], [21]. The operands there in the min operation have no simple or explicit expressions.

$$d'_{AP}\left(r_{i_{AP}}\right)=\begin{cases}\sum_{i_{SR}=1}^{M}\sum_{j_{SR}=1}^{K}\left(1+t_{i_{SR}j_{SR}}/2\right)+\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}\left(1+t_{i_{SD}j_{SD}}/2\right), & \text{if } i_{AP}=0\\ \frac{1}{2}\left(3\left(M-1\right)^2+\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}\left(1+t_{i_{SD}j_{SD}}/2\right)\right), & \text{if } i_{AP}=1\\ \left(\left(M-2r_{i_{AP}}\right)^+\right)^2+\left(M-r_{i_{AP}}\right)^2, & \text{if } i_{AP}=2,4,\cdots,2M\\ \left(\left(M-2r_{i_{AP}}\right)^+\right)^2+\left(M-r_{i_{AP}}\right)^2+1/4, & \text{if } i_{AP}=3,5,\cdots,2M-1\end{cases}. \quad (13)$$

(iii) The DMT curve of DHR is $d'_{DHR}(r)$. It is also a piecewise-linear function connecting points $\left(r_{i_{MHP}}, d'_{DHR}\left(r_{i_{MHP}}\right)\right)$ with $r_{i_{DHR}}=i_{DHR}/2$, $i_{DHR}=0,1,2,\cdots,M$, and

$$d'_{DHR}\left(r_{i_{DHR}}\right)=\begin{cases}\sum_{i_{SR}=1}^{M}\sum_{j_{SR}=1}^{K}\left(1+t_{i_{SR}j_{SR}}/2\right), & \text{if } i_{DHR}=0\\ \left(M-2r_{i_{DHR}}\right)^2, & \text{if } i_{DHR}=1,2,\cdots,M\end{cases}. \quad (14)$$

Proof: See APPENDIX B.

It is seen from the corollary that the AP has larger multiplexing gain range than RDP, resulting from the NACK feedback [31]. The NACK feedback enables AP to transmit for two phases for the worst case (i.e., the $S\rightarrow D$ link transmission with two phases) whereas the worst of RDP is the $S\rightarrow D$ link transmission with only one transmission phase.

For *arbitrary* number of antennas and *general* fading statistics, some examples are also given for illustration. In the following examples, all elements in one channel matrix are assumed to have the same parameter t for simplicity and without any loss of generality.

Here we use $d_{(M,K,N)(t_1,t_2,t_3)}$ to denote the DMT of RDP or AP, with M, K, and N being the number of antennas employed by the source, relay, and destination, respectively. t_1, t_2, and t_3 are the channel parameters (i.e., the polynomial behavior near zero of the channel gain PDF) of the $S\rightarrow R$ link, $R\rightarrow D$ link and $S\rightarrow R$ link, respectively, and the system is denoted as $(M,K,N)(t_1,t_2,t_3)$. We note that the notation is not suitable for DHR because DHR has only two links. We also use the notation $\left[(r_0,d_0),(r_1,d_1),\ldots,(r_n,d_n)\right]$ to denote the connecting points of a piecewise-linear function.

Example 1: For $M=K=N=2$, and all channels between any two users have the same fading statistics, i.e., with Rayleigh ($t=0$), Rician ($t=0$), Nakagami-*q* ($t=0$), Nakagami-*m* ($t=2$) or Weibull ($t=4$) [17]. The DMT expressions of the three protocols can be computed by the *corollary* in Section IV or the theorems in Section III, and are shown in **Fig. 2**, **Fig. 3**, and **Fig. 4** respectively. In **Fig. 2**, the connecting points of the RDP in the Rayleigh (Rician or Nakagami-*q*), Nakagami-*m*, and Weibull fading channels are $\left[(0,8),(0.5,2),(1,0)\right]$, $\left[(0,16),(0.5,2),(1,0)\right]$, and $\left[(0,24),(0.5,2),(1,0)\right]$, respectively. The connecting points of the AP (see **Fig. 3**) in the Rayleigh (Rician or Nakagami-*q*), Nakagami-*m*, and Weibull fading channels are $\left[(0,8),(0.5,3.5),(1,1),(1.5,0.5),(2,0)\right]$, $\left[(0,16),(0.5,5.5),(1,1),(1.5,0.5),(2,0)\right]$, and $\left[(0,24),(0.5,7.5),(1,1),(1.5,0.5),(2,0)\right]$, respectively. The connecting points of the DHR in **Fig. 4** in the above fading channels are $\left[(0,4),(0.5,1),(1,0)\right]$, $\left[(0,8),(0.5,1),(1,0)\right]$, and $\left[(0,12),(0.5,1),(1,0)\right]$, respectively.

It is easily from these figures that the AP has larger multiplexing gain range than the RDP, resulting from the NACK feedback gain. We also observe that the AP obtains a larger (or not less) diversity gain than the RDP for any given multiplexing gain because of the

NACK feedback gain again. Thus the AP is superior to the RDP in terms of DMT. It is also seen that the RDP is superior to DHR in terms of DMT because it fully utilizes the signal received from the source at the first phase.

Example 2: For system $(4,2,3)(0,0,0)$ with RDP and system $(2,2,3)(2,0,4)$ with AP, their DMT curves, computed by *Theorem* 2 and *Theorem* 3, are presented in **Fig. 5** and **Fig. 6**, respectively. In those figures, the min operation is illustrated between $d_{SR}(r)+d_{SD}(r)$ (or $d_{SR}(r)+d'_{SD}(r)$) and $d_{out}(r)$. In **Fig. 5**, the connecting points of $d_{SR}(r)+d_{SD}(r)$ are $[(0,20),(0.5,9),(1,2),(1.5,0),(2,0),(2.5,0)]$ while that of $d_{out}(r)$ are $[(0,18),(0.5,12),(1,8),(1.5,4),(2,2),(2.5,0)]$. The two piecewise-linear functions coincide at point $(0.2,15.6)$, thus the DMT of the system $(4,2,3)(0,0,0)$ with RDP is also a piece-wise linear function connecting the points $[(0,18),(0.2,15.6),(0.5,9),(1,2),(1.5,0),(2,0),(2.5,0)]$. In **Fig. 6**, the connecting points of $d_{SR}(r)+d'_{SD}(r)$ and $d_{out}(r)$ are $[(0,26),(0.5,11),(1,2),(1.5,1),(2,0)]$ and $[(0,24),(0.5,8),(1,4),(1.5,2),(2,0)]$. The two piecewise-linear functions coincide at point $(0.8,5.6)$ and the DMT of the system $(2,2,3)(2,0,4)$ with AP is a piece-wise linear function connecting the points $[(0,24),(0.5,8),(0.8,5.6),(1,2),(1.5,1),(2,0)]$.

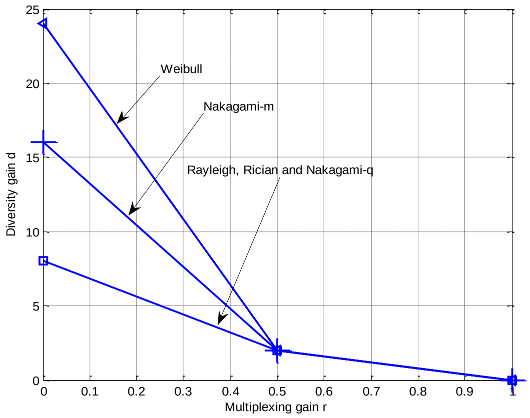

Fig. 2 The DMT curves of RDP with identical fading channels between any two users.

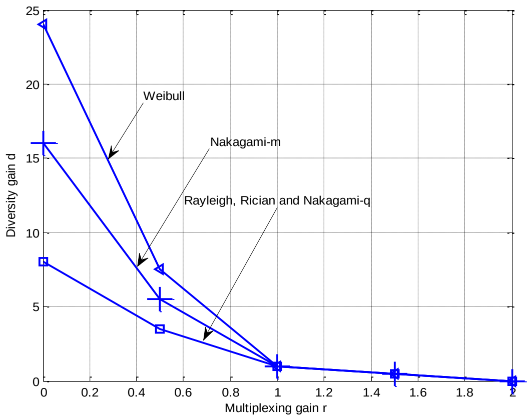

Fig. 3 The DMT curves of AP with identical fading channels between any two users.

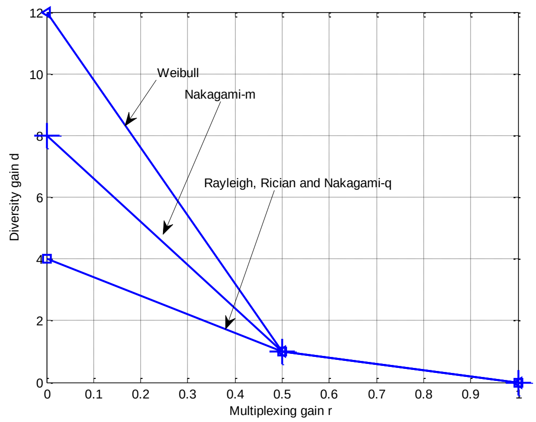

Fig. 4 The DMT curves of DHR with identical fading channels between any two users.

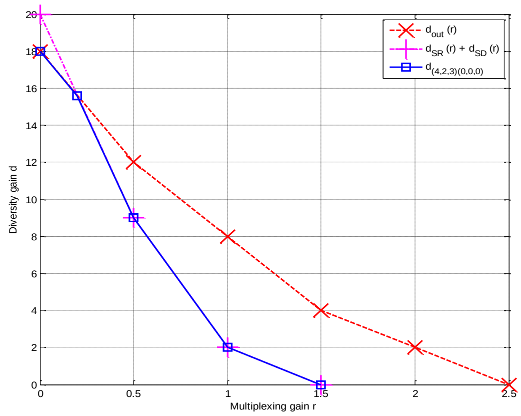

Fig. 5 The DMT curve of RDP for system $(4,2,3)(0,0,0)$.

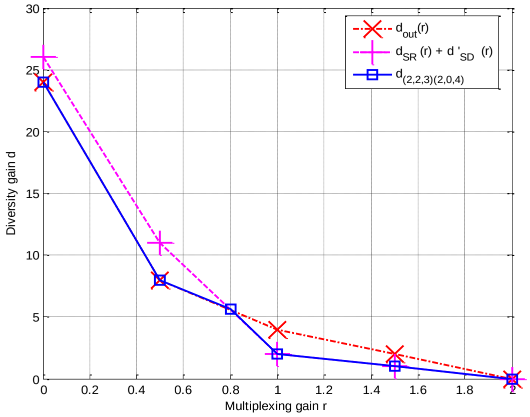

Fig. 6 The DMT curve of AP for system $(2,2,3)(2,0,4)$.

From **Fig. 5** and **Fig. 6**, it is shown that DMT for *general* fading channels with *arbitrary* number of antennas, though is a piecewise-linear function, is hard to expressed in explicit form, due to its dependency on M, K, N, and channel parameter t, but can be calculated in a simple way shown above.

It is noted that [9, *Theorem* 2] found the DMT expression for DHR in *general* fading channels with single-antenna relay constraint. The result therein is a specific example of our general results here.

5. Discussions

In this section, the impact of correlation and/or nonzero channel means on DMT is investigated. The results here are extensions of the results of the point-to-point MIMO system to the multiple-antenna cooperative communication scenario.

According to [17, *corollary* 1], it is easy to show that the DMT of a cooperative system with correlation in *general* fading channels is the same as that system with no correlation if all channel matrices have full rank transmit and receive covariance matrices. If one channel matrix has no full rank transmit or receive covariance matrix, its impact on DMT is difficult to analyze. The reason is the same as the difficulty in derivation of explicit DMT expressions in *general* fading channels. Even with the special conditions in the corollary in Section IV, it is difficult to analyze its impact because the relationship between $d_{SR}(r)+d_{SD}(r)$ and $d_{out}(r)$ is not clear in *general* channels. For example, in RDP, if the $R \rightarrow D$ link channel matrix has non-full rank transmit covariance matrix (thus the corresponding $d_{out}(r)$ is not larger than $d_{out}(r)$ corresponding to the non-correlation case), the DMT result is not affected by this non-full rank correction property as long as $d_{SR}(r)+d_{SD}(r)$ is not larger than the $d_{out}(r)$ corresponding to the non-full rank correction case.

Nonzero channel means have no effects on the DMT if channel matrices $\mathbf{H}_{SR}$, $\mathbf{H}_{RD}$, and $\mathbf{H}_{SD}$ are of Rayleigh or Rician distribution because their parameters ts are all zero (according to [17, *corollary* 2]). For other fading statistics, it is difficult to analyze the effects. The reason is the same as that of non-full rank correlation impact on DMT. The combined effects of correlation and nonzero channel means are straightforward based on the aforementioned analysis, and omitted here.

6. Conclusion

In this paper, we derive the diversity-multiplexing tradeoff of three-user half-duplex and decode-and-forward cooperative systems for three protocols in *general* fading channels, with *arbitrary* number of antennas. Our derivation is an extension of the point-to-point MIMO system to the multiple-antenna cooperative communication scenario. Simple expressions are obtained by an iterative algorithm and comparison operation. Explicit expressions are also derived for specific conditions when all users employ the same number of antennas and channels between any two users have the same statistics (with the same parameter t as a specific example). DMTs of some *general* fading channels with arbitrary number of antennas are also provided as examples to illustrate the analysis and results in Section III. Transmit and/or receive correlation and nonzero channel means effects on DMT are also investigated. Analysis reveals that the DMT is not affected if all transmit and receive covariance matrices are of full rank. It is shown that the DMT only depends on the number of antennas, channel means (except for Rayleigh and Rician fading statistics), transmit and/or receive correlation, and the polynomial behavior near zero of the channel gain probability density function, i.e., channel parameter t, rather than a specific fading distribution. Our analysis of DMT takes into account the physical environment, thus is of theoretical and practical meanings.

APPENDIX A

The proof of the *Lemma* follows the same way as that of [8, *Theorem* 4], but is an extension from Rayleigh fading channel to *general* fading channels.

Since $P_r\left(\sum_{j=1}^{\min(N,K)}\left(1-\beta_j\right)^+ + \sum_{k=1}^{\min(M,N)}\left(1-\gamma_k\right)^+ < 2r\right) \doteq \rho^{-d_{out}(r)}$, then $d_{out}(r) = \inf_{(\beta,\gamma)\in B(r)} g(\boldsymbol{\beta},\boldsymbol{\gamma})$, where (according to *Theorem* 1),

$$g(\boldsymbol{\beta},\boldsymbol{\gamma}) = \sum_{j=1}^{\min(N,K)}\left(|N-K|+2j-1\right)\beta_j + \frac{\sum_{i_{SR}=1}^{K}\sum_{j_{SR}=1}^{N} t_{i_{SR}j_{SR}}}{2}\beta_{\min(N,K)} + \sum_{k=1}^{\min(N,M)}\left(|N-M|+2k-1\right)\gamma_k + \frac{\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N} t_{i_{SD}j_{SD}}}{2}\gamma_{\min(N,M)}, \quad \text{(A.1)}$$

with $(\boldsymbol{\beta},\boldsymbol{\gamma}) = \left(\beta_1,\beta_2,\ldots\beta_{\min(N,K)},\gamma_1,\gamma_2,\ldots\gamma_{\min(M,N)}\right)$, and

$$B(r) = \left\{ \begin{array}{c} (\boldsymbol{\beta},\boldsymbol{\gamma}):\ \beta_1 \ge \beta_2 \ge \cdots \ge \beta_{\min(N,K)},\ \gamma_1 \ge \gamma_2 \ge \cdots \ge \gamma_{\min(N,M)}, \\ \text{and} \sum_{j=1}^{\min(N,K)}\left(1-\beta_j\right)^+ + \sum_{k=1}^{\min(M,N)}\left(1-\gamma_k\right)^+ < 2r \end{array} \right\}. \quad \text{(A.2)}$$

When $r=0$, all $\beta_j=1$ and all $\gamma_k=1$, thus $d_{out}(r)$ obtains its largest value $\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}\left(1+t_{i_{SD}j_{SD}}/2\right)+\sum_{i_{RD}=1}^{K}\sum_{j_{RD}=1}^{N}\left(1+t_{i_{RD}j_{RD}}/2\right)$.

When $r=1/2$, the element in $(\boldsymbol{\beta},\boldsymbol{\gamma})$, corresponding to the largest term of the summations in (A. 1), is set to zero to obtain the infimum value of $g(\boldsymbol{\beta},\boldsymbol{\gamma})$. If

$MN-(M-1)(N-1)+\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}\left(1+t_{i_{SD}j_{SD}}/2\right)<NK-(N-1)(K-1)+\sum_{i_{RD}=1}^{K}\sum_{j_{RD}=1}^{N}\left(1+t_{i_{RD}j_{RD}}/2\right)$, $\beta_{\min(N,K)}=0$, and $d_{out}(r)=(N-1)(K-1)+\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}\left(1+t_{i_{SD}j_{SD}}/2\right)$, otherwise $\gamma_{\min(N,M)}=0$, $d_{out}(r)=(N-1)(M-1)+\sum_{i_{RD}=1}^{K}\sum_{j_{RD}=1}^{N}\left(1+t_{i_{RD}j_{RD}}/2\right)$. It is easily seen that the curve $d_{out}(r)$ is linear in the range $0\le r\le 1/2$. The same process continues (with r increased by $1/2$ each time) until r reaches $\left[\min(N,K)+\min(M,N)\right]/2$, where all β_j and all γ_k equal to zero and $d_{out}(r)=0$. The pseudocode in *Algorithm 1* is used to execute the process.

APPENDIX B

According to *Theorem* 1 and with the special case in the corollary, $d_{SR}(r_i)$, $d_{SD}(r_k)$, $d_{out}(r_t)$, $d'_{SD}(r_k)$, and $d_{RD}(r_j)$ in (5), (6), (7), (9), and (11) can be rewritten as (B.1), (B.2), (B.3), (B.4) , and (B.5), respectively as follows

$$d_{SR}(r_i)=\begin{cases}\sum_{i_{SR}=1}^{M}\sum_{j_{SR}=1}^{K}\left(1+t_{i_{SR}j_{SR}}/2\right), & \text{if } i=0\\ (M-2r_i)^2, & \text{if } i=1,2,\cdots,M\end{cases},\tag{B.1}$$

$$d_{SD}(r_k)=\begin{cases}\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}\left(1+t_{i_{SD}j_{SD}}/2\right), & \text{if } k=0\\ (M-2r_k)^2, & \text{if } k=1,2,\cdots,M\end{cases},\tag{B.2}$$

$$d_{out}(r_t)=\begin{cases}\sum_{i_{RD}=1}^{K}\sum_{j_{RD}=1}^{N}\left(1+t_{i_{RD}j_{RD}}/2\right)+\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}\left(1+t_{i_{SD}j_{SD}}/2\right), & \text{if } t=0\\ \sum_{i_{RD}=1}^{K}\sum_{j_{RD}=1}^{N}\left(1+t_{i_{RD}j_{RD}}/2\right)+(M-1)^2, & \text{if } t=1\\ 2(M-r_t)^2, & \text{if } t=2,4,\cdots,2M\\ (M-r_t-1/2)^2+(M-r_t+1/2)^2, & \text{if } t=3,5,\cdots,2M-1\end{cases},\tag{B.3}$$

$$d'_{SD}(r_k)=\begin{cases}\sum_{i_{SD}=1}^{M}\sum_{j_{SD}=1}^{N}\left(1+t_{i_{SD}j_{SD}}/2\right), & \text{if } r_k=0 \text{ and } k=0\\ (M-r_k)^2, & \text{if } r_k=k \text{ and } k=1,2,\cdots,M\end{cases},\tag{B.4}$$

and

$$d_{RD}(r_j)=\begin{cases}\sum_{i_{RD}=1}^{K}\sum_{j_{RD}=1}^{N}\left(1+t_{i_{RD}j_{RD}}/2\right), & \text{if } j=0\\ (M-2r_j)^2, & \text{if } j=1,2,\cdots,M\end{cases}.\tag{B.5}$$

It is emphasized that all expressions (except (B.4)) have the implicit relationship $r_n=n/2$ ($n=i,j,k,$ or t). According to the above expressions, it is easy to observe that $d_{SR}(r)+d_{SD}(r)\le d_{out}(r)$, with the equality holding at $r=0$ or $r=M$. Thus $d_{RDP}(r)=\min\left(d_{SR}(r)+d_{SD}(r),d_{out}(r)\right)=d_{SR}(r)+d_{SD}(r)$, and is expressed explicitly in (12), finishing the proof of statement (i).

For adaptive protocol, consider firstly when $r_{i_{AP}} = i_{AP}/2$ and i_{AP} is odd. If $2 < i_{AP} \le M$, $d_{SR}\left(r_{i_{AP}}\right) + d'_{SD}\left(r_{i_{AP}}\right) = \left(M - 2r_{i_{AP}}\right)^2 + \frac{1}{2}\left(\left(M - r_{i_{AP}} - 1/2\right)^2 + \left(M - r_{i_{AP}} + 1/2\right)^2\right)$. It is noted that for $d'_{SD}(r)$ the corresponding point is the middle point of the point $\left(r_{i_{AP}} - 1/2,\ \left(M - r_{i_{AP}} + 1/2\right)^2\right)$ and the point $\left(r_{i_{AP}} + 1/2,\ \left(M - r_{i_{AP}} - 1/2\right)^2\right)$. Therefore, $d_{out}\left(r_{i_{AP}}\right) - \left(d_{SR}\left(r_{i_{AP}}\right) + d'_{SD}\left(r_{i_{AP}}\right)\right) = -3r_{i_{AP}}^{\ 2} + 2Mr_{i_{AP}} + \frac{1}{4}$, this quadratic polynomial is always larger than zero for $r_{i_{AP}} \le M/2$ (treating $r_{i_{AP}}$ as variable), thus $d_{SR}\left(r_{i_{AP}}\right) + d'_{SD}\left(r_{i_{AP}}\right) < d_{out}\left(r_{i_{AP}}\right)$; if $M \le i_{AP} \le 2M$, $d_{SR}\left(r_{i_{AP}}\right) = 0$, and $d'_{SD}\left(r_{i_{AP}}\right) = \frac{1}{2}\left(\left(M - r_{i_{AP}} - 1/2\right)^2 + \left(M - r_{i_{AP}} + 1/2\right)^2\right)$, thus $d_{SR}\left(r_{i_{AP}}\right) + d'_{SD}\left(r_{i_{AP}}\right) = d_{out}\left(r_{i_{AP}}\right)$; finally if $r_{i_{AP}} = 1/2$, $d_{SR}\left(r_{i_{AP}}\right) + d'_{SD}\left(r_{i_{AP}}\right) < d_{out}\left(r_{i_{AP}}\right)$.

It is obvious that $d_{SR}\left(r_{i_{AP}}\right) + d'_{SD}\left(r_{i_{AP}}\right) \le d_{out}\left(r_{i_{AP}}\right)$ for $r_{i_{AP}} = i_{AP}/2$ and i_{AP} is even, with the equality holding at $r = 0$ or $r = M$. Concluding those arguments, the statement (ii) is obtained. The proof of the statement (iii) is obvious, thus omitted for brevity.

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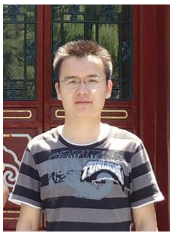

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