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Point In Triangle Testing Based Trilateration Localization Algorithm In Wireless Sensor Networks

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Abstract

Localization of sensor nodes is a key technology in Wireless Sensor Networks(WSNs). Trilateration is an important position determination strategy. To further improve the localization accuracy, a novel Trilateration based on Point In Triangle testing Localization (TPITL)algorithm is proposed in the paper. Unlike the traditional trilateration localization algorithm which randomly selects three neighbor anchors, the proposed TPITL algorithm selects three special neighbor anchors of the unknown node for trilateration. The three anchors construct the smallest anchor triangle which encloses the unknown node. To choose the optimized anchors, we propose Point In Triangle testing based on Distance(PITD) method, which applies the estimated distances for trilateration to reduce the PIT testing errors. Simulation results show that the PIT testing errors of PITD are much lower than Approximation PIT(APIT) method and the proposed TPITL algorithm significantly improves the localization accuracy.

Key words: wireless sensor networks, localization algorithm, localization accuracy, trilateration, point in triangle testing

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1. Introduction

Wireless sensor networks(WSNs) are composed of small, low power, limited computing and communication capacity nodes which sense information in physical word. WSNs have been used in numerous areas such as bush fire surveillance, water quality monitoring, intrusion detection, disaster management and so on. Most of the applications must know the positions of the sensor nodes in the network. The processes of determing the position of the nodes in WSNs are defined as localization [1].

Localization of nodes in WSNs is a key technology attracting considerable research interest in recent years. Existing localization approaches are divided into two categories: range-based methods and range-free methods. The former ones assume the absolute distance and/or the relative directions of neighbors can be estimated by RSSI[2](Received Signal Strength Indictor), TOA[3] (Time Of Arrival) or TDOA[4] (Time Difference Of Arrival).The representative algorithms include Euclidean[5], N-hop-multilateration[6] and primitive cooperative ranging[7] and so on. The accuracy of such algorithms is high. But it is subject to the surrounding environment and relies on complex hardware. In contrast, range-free approaches needn't directly measure the distance or angles between nodes but depend on the connectivity information of the network. Thus these localization algorithms have the advantages of low cost, low power and simple hardware while low accuracy compared with range-based ones. The representative algorithms are APIT[8](Approximate Point In Triangle Testing), DV-Hop[9], MDS-MAP[10], Amorphous Localization[11]and Centroid Localization[12] and so on.

In both range-based and range-free algorithms, the processes of localization usually include two steps: range or distances estimation and coordinates determination. After the distances or the range from multiple references are estimated by range-based or range-free methods, the coordinates of the nodes are calculated by trilateration or multilateration. To further improve the localization accuracy of trilateration algorithm, the characterstics of trilateraltion are studied deeply in the paper.It is found that the anchor nodes which are selected for trilateration greatly affect the localization accuracy. So Trilateration based on Point In Triangle testing Localization(TPITL) algorithm is presented in the paper to improve the localization accuracy. Without loss of generality, we consider localization of a stationary network in a 2-D plane. In the proposed TPITL algorithm, point in triangle testing(PIT) is introduced to select the anchor nodes(position known ahead) for trilateration.The selected anchor nodes construct the smallest triangle which encloses the unknown node. Meanwhile distances estimated for the trilateration are also used in the PIT testing to reduce the judgment errors compared with the traditional method APIT.

The rest of the paper is organized as follows. In section 2, the related work are reviewed. In section 3, we discuss the localization accuracy of different anchor triangles with different geometric location and size. Point In Triangle testing based on Distances(PITD) is proposed in section 4. Then, Trilateration based on Point In Triangle testing Localization(TPITL) algorithm is presented in section 5. The simulation results and performances of the algorithm are analyzed in section 6. We conclude this work in section 7.

2. Related work

Many efforts have been made for promoting the accuracy of localization. The contribution can be concluded as two aspects: reducing the distances estimation errors and improving the coordinates determination strategy.

2.1 Distances Estimation Schemes

There are two ways in estimating the distances between nodes in WSNs: directly and indirectly schemes. The former schemes are completed by physical module. The technologies usually used are TOA, TDOA and RSSI. In TOA technique, all sensors transmit a signal with a predefined velocity to their neighbors. Then, the nodes each send a signal back to their neighbors and the transmission and received times are recorded. Thus the distances between nodes can be calculated. TOA requires rigid time synchronization. Like TOA, TDOA is also a time-based technique. This technique requires the nodes to transmit two signals that travel at different speeds. The nodes can compute the distance between nodes by using the time differences that arrived. TDOA needs expensive hardware devices.

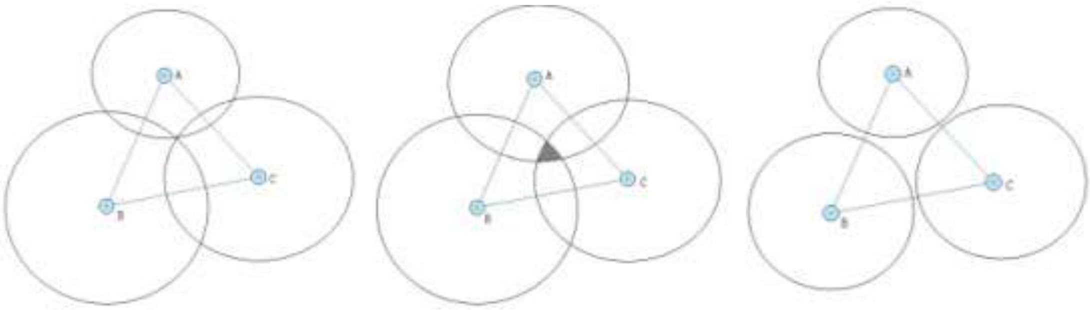
Unlike the time-based techniques, RSSI technique uses radio propagation model to translate the signal strength into distance. As most of the sensors have the function of RSSI, this method needn't extra hardware. But problems such as multi-path fading, background interference and irregular signal propagation affect the accuracy of distance estimation. Considerable work have been done to mitigate the errors. In [13]Santiago Mazuelas et al. present a novel method which dynamically estimates the propagation models. The models best fit the propagation environment by RSS obtained in real time. Reference [14] proposes a sigma-Kalman smoother based localization for indoor positioning. In [15], Ken-Ichi Itoh analyses the performances of a distance-based handoff algorithm with additional criterion of relative signal strength in a log-normal environment.

Indirectly estimating schemes rely on the connectivity of nodes. The cost is low while the errors are high compared with the directly schemes. Among these methods, DV-Hop is a typical indirectly estimating algorithm. The minimum hop counts between nodes are recorded and the average hopping distance is calculated in the algorithm. Thus the distances between nodes can be estimated by simple multiply. People have been trying to spare no effort on researching on the algorithm to reduce the estimation errors. In [16], Radhika Nagpal degrades the hop counts between nodes with gradient neighbors information to promote the distances estimation resolution. In [17], the average hopping distance is modified based on minimum average square principle. In [18], different power levels are used to divide the hopping counts into different degrees to improve the hopping information.

2.2 Coordinates Determination Strategy

There are three basic coordinates determination techniques in WSNs: Triangulation, trilateration and multilateration. Triangulation is used when the direction of the node rather than the distances is estimated. This method requires an additional antenna arry and has poor ant-interference. So it is not usually used in WSNs.

Trilateration is based on the fact that the position of a point on a two dimensional plane can be determined by its distances from three non-colinear reference points. In WSNs, the reference point is the anchor node. The three non-conlinear anchors construct the anchor triangle. **Fig. 1** shows the principle of trilateration. Supposing the vertexes of the anchor triangle are A, B and C. The distances from the unknown node to the three anchors are d_A, d_B and d_C. In theory, the circles intersect at a single point, as shown in **Fig. 1(a)**. The intersection point is the position of the unknown node.



(a) intersect at a single point (b) intersect at three points (c) don't intersect

Fig. 1. Trilateration localization

Actually, due to the estimation errors of the distances, the three circles don't intersect at a single point or don't intersect at all. **Fig. 1(b)** and **Fig. 1(c)** plot the circumstances.

In order to improve the localization accuracy of trilateration and multilateration, researchers in [19][20][21][22][23][24][25][26] have done much work. In [19], the iterative approach has been studied. To reduce errors accumulation caused by iteratively localization, reference [20] introduces an error control and robust formulation for the localization problems. In [21] time round mechanism and anchor nodes triangle placement scheme are presented to reduce localization errors and prevent abnormal phenomena caused by iterative localization. In [22], quality of trilateration(QoT) that quantify the geometric relationship of objects and the ranging noise is proposed. Based on QoT, the authors design a confidence based iterative localization scheme, in which nodes dynamically select trilateration with highest quantity for location.

In [23], the authors put forward an alternating combination trilateration(ACT) algorithm. In ACT, a position is calculated out to serve as an apex of a weighted polygon by any three anchors. Then an m-side weighted polygon is formed and the location of the unknown node is calculated out by centroid algorithm. To reduce the computation complexity, reference [24] studies the value of m further and proposed different m to be defined with different number of anchors.

In [25], an anchor node weighted compensated localization algorithm based on trilateration and centroid localization algorithm is advanced. When calculating the coordinates of the unknown node, the errors can be compensated by using the anchors' own locating capabilities. In [26], anchors are optimized and chosen to calculate the positions of the unknown nodes by trilateration.

In this paper, we consider improving the accuracy of trilateration by using the distances information estimated in trilateration.

3. Anchor Triangle And Localization Accuracy

In [27], it is observed that the position estimation errors for the unknown nodes are dependent on relative position of the anchor nodes.

Definition 1: anchor triangle. Supposing an unknown node's neighbor anchors(anchor nodes in the unknown node's communication radius) constitute a set A={A_1, A_2, …A_N}. Any three anchors in set A compose an anchor triangle for the unknown node. There are C_N^3 anchor triangles for the unknown node.

The following subsections discuss localization accuracy of nodes in and out of anchor triangles as well as in big and small anchor triangles.

3.1 Localization Accuracy Of Nodes Inside And Outside Of Triangle

In trilateration, the distances estimation errors between the unknown node and the neighbor anchor nodes have great impact on the localization accuracy in algorithm based on anchors. Supposing the distances measurement errors are modeled as an indepent Gaussian random distribution with zero mean and variance Er. For example, if the real distance between an unknown node and an anchor node is dr. The estimation error variance is Ed. Then the estimation distance de=dr+dr*Ed*N(0, 1).

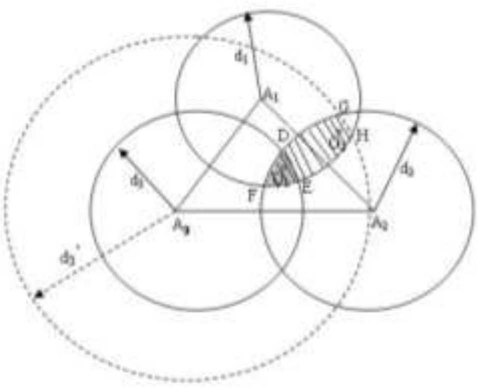


Fig. 2. Trilateration principle of nodes inside and outside of anchor triangle.

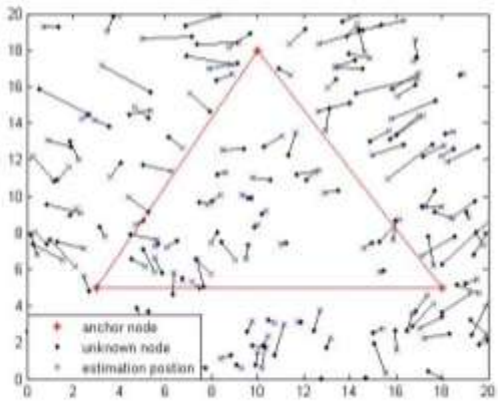


Fig. 3. Localization errors of nodes in and out of anchor triangle.

In **Fig. 2**, △$A_1A_2A_3$ is the anchor triangle of the nodes O_1 and O_2. O_1 is inside of the anchor triangle while O_2 is outside. Supposing O_2 is symmetric point of O_1 about line A_1A_2, and the distances estimation errors are the same for both O_1 and O_2. The shadow area DEF and FGH represent the possible position that the nodes O_1 and O_2 locate in respectively. It is obviously that larger area means larger error. It is proved in Appendix A that the area of DEF is smaller than FGH. It means that the error is lower if the node is inside of its anchor triangle under the same distances estimation error.

To validate the conclusion, simulation is done on Matlab7.0 platform. **Fig. 3** shows the localization results of nodes in and out of the anchor triangle. The length of the line represents the localization error. It can be seen from **Fig. 3** that positioning of nodes in anchor triangle outperforms that of nodes outside.

3. 2 Anchor Triangle Size And Localization Error

The subsection 3.1 validates anchor triangle which embodies the unknown node has more effective localization results than that uncloses the unknown node. While the size of the anchor triangle which the unknown node locates in also has great impact on the localization accuracy.

In **Fig. 4**, the area of anchor triangle $\triangle A_1'A_2'A_3'$ is bigger than the anchor triangle $\triangle A_1A_2A_3$. We define that $\triangle A_1'A_2'A_3'$ is the big anchor triangle while $\triangle A_1A_2A_3$ is the small one. The unknown node O is located in both anchor triangles.Supposing the distances estimation error variance is Er, the same for both anchor triangles. The estimation distances between O and A_1, A_2, A_3 are d_1, d_2, d_3. While the estimation distances between O and A_1', A_2', A_3' are d_1', d_2', d_3'. It is easy to prove that $d_1' > d_1$, $d_2' > d_2$, $d_3' > d_3$ under the same distances estimation error variance. The cross area of real circle DEF represents the possible area the point O locates in by trilateration of anchor triangle $\triangle A_1A_2A_3$, while the cross area of dashed circle GHI is the location result with $\triangle A_1'A_2'A_3'$. It is obvious the shadow area of GHI is bigger than DEF, which means the bigger the anchor triangle the node locates in, the larger the localization error is. **Fig. 5** shows the simulation results of the nodes in big and small anchor triangle respectively.

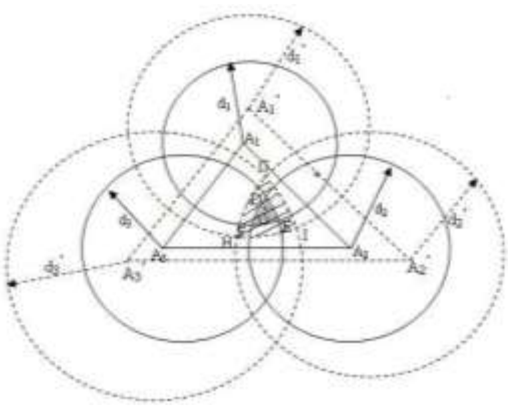


Fig. 4. Trilateration principle of nodes in big and small anchor triangles.

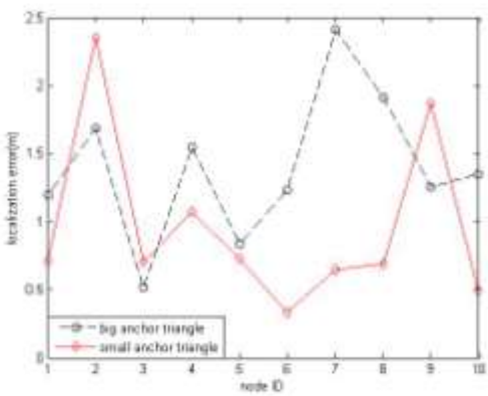


Fig. 5. Localization errors of nodes in big and small anchor triangles

3.3 Effective Anchor Triangle

From section 3.1 and 3.2, we can find that choosing anchors is very important in trilateration.

Definition 2: effective anchor triangle. The smallest anchor triangle of the anchor triangles which embody the unknown node is defined as its effective anchor triangle.

In trilateration algorithm, in order to reduce the localization errors, the effective anchor triangle should be chosen to locate the position of the unknown node. We note that effective anchor triangle is a triangle that the unknown node locates in. The problem is how to judge whether a point with unknown position is inside an anchor triangle. In the next section, we describe point in triangle testing based on distances algorithm and discuss the performances of this PIT testing method in WSNs.

4. Point In Triangle Testing Based On Distance(PITD)

A traditional way to check if a point is in a triangle is to check if the point and the centroid of the triangle △ABC are at the same side of line AB, AC and BC simultaneously. If they satisfy the condition synchronously, the point is inside the triangle. Otherwise it is not. It works, but it requires the coordinates of the point. While in WSNs, the point to check is the unknown node whose coordinates are unknown.

To perform the PIT testing under this circumstance, Tian He proposes Approximation PIT(APIT) in [8]. The basic idea behind APIT is to use the neighbor information to emulate the node movement. The algorithm introduces RSSI technique to decide whether a node is further away from an anchor node compared with its neighbor nodes. The accuracy of APIT largely depends on the average neighbors(ANB, Average number of neighbor nodes for all sensor nodes. It is also defined as connectivity).To reduce out-to-in and in-to-out errors of APIT, Zhou et al. in [28] analyze the reasons for APIT testing errors and present two improvements. In [29], an improved APIT based on direction searching is proposed. The improved algorithm solve the problem of low accuracy in sparse network for APIT but it introduces an extra searching direction node. We further study the characteristics of trilateration and find that the distances information which are estimated for trilateration can be used in PIT to reduce the testing error. So, Point In Triangle testing based on Distance(PITD) algorithm is presented in the paper.

In the following subsections, the base principle of PITD is analyzed and the pseudo code of PITD is described firstly. And then the performances of PITD are compared with APIT.

4. 1 PITD Algorithm

It is obvious that the geometric relation of anchors significantly affect the localization accuracy. In [22], quality of trilateration, a fine grained method, is presented to provide a quantitative evaluation of different forms of trilateration. It is proved in the paper the estimation positions are separated around the node to be located in trilateration under noisy distances measurement. From this conclusion, we can conclude that the estimation position and the real coordinates of the node have almost the same geometric relationship with the anchor triangle. Thus, in order to check whether the unknown node is in the anchor triangle, the coordinates of the node can be estimated by trilateration firstly. Then the estimated coordinates are used to perform the PIT testing with the traditional way. The test results can be considered as the results of the unknown node. Unlike APIT, the testing errors of which largely depend on the node density of the network, the testing accuracy of PITD is greatly affected by the distances estimation errors.

The pseudo code of the PITD algorithm is as follows:

Algorithm 1:Point In Triangle testing based on Distances(PITD)

Input:

1) The coordinates of vertexes for the anchor triangle.
2) The estimated distances from the unknown node to each vertex of the anchor triangle.

Output:

The flag whether the unknown node is in or out of the anchor triangle.

Procedure:

1:for (each anchor triangle)

2:Estimate the position of the unknown node with trilateration. Supposing the coordinates of the estimated position is Pxy.

3:Compute the centroid Cxy of the anchor triangle.

4: if Pxy and Cxy are at the same side of line AB, BC and AC simultaneously

5: Point is inside of the triangle.

6: else

7: Point is outside of the triangle.

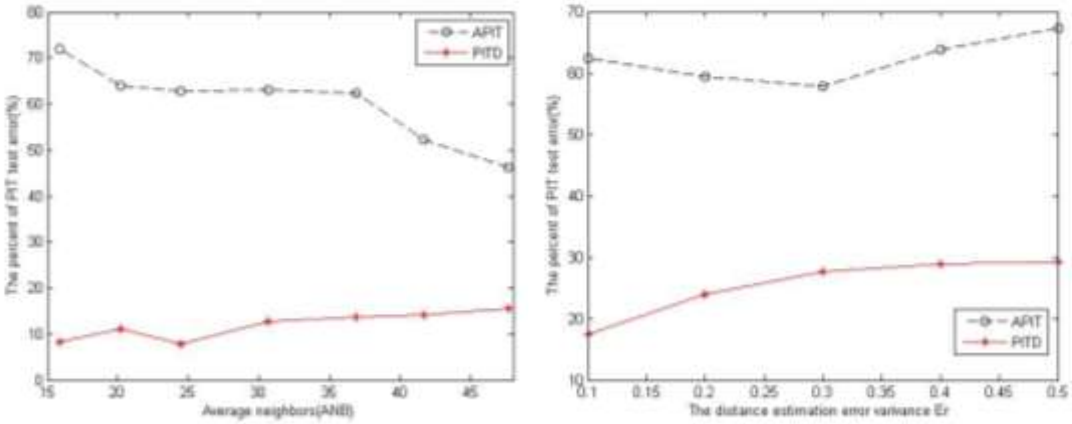
8: endif

9:end for

4. 2 Performance Evaluation Of PITD

We consider a static network with 3 anchors and 100 nodes which are randomly placed in a 20x20 rectangular terrain. The simulation scene is the same as the scene displayed in Fig. 3. All the nodes are in the communication radius of each anchor. The communication radius is 30. Supposing the distances measurement errors are still modeled as an indepent Gaussian random distribution with zero mean and variance Er. Point in triangle testing is completed by the algorithm PITD and APIT both.

The simulation results are shown in **Fig. 6**. The PIT testing errors include out-to-in and in-to-out errors. **Fig. 6(a)** shows the results with the average neighbors(ANB)varying from 16 to 48 under the distances estimation error variance Er=0.1. As for PITD algorithm, The error percentage stays almost constant, around 10%, under the same distances estimation errors because the accuracy of PITD does not depend on the node density. While the testing errors of APIT reduce with the increase of nodes, which is consistent with the characteristics of APIT analyzed in section 4.1. Obviously, PITD outperforms APIT. In **Fig. 6(b)**, the number of nodes is a constant while the distances estimation error variable Er varies from 0.1 to 0.5. From the figure, we can conclude that APIT is not sensitive to the distances estimation errors. While for the PITD algorithm, the testing errors increase with the increase of the distances estimation errors. But the testing errors are still lower than that of APIT.



(a) (b)

Fig. 6. (a)PIT errors vs. ANB(Er=0.1) (b)PIT errors vs. Er

5. TPITL Algorithm

In this section, we describe our novel TPITL algorithm. The network assumptions for this algorithm are as follows:

(1)All sensor nodes are randomly placed. Each node supports a distance estimation measurement technique, RSSI for example, to estimate the distances to their neighbor nodes.

(2)All sensor nodes have the same communication radius R.

5.1 Algorithm Description

In TPITL algorithm, the key procedure is to find the effective anchor triangle of the unknown node for trilateration. In order to find the effective anchor triangle, all anchor triangles for the node should be developed at first. The point in triangle testing is then performed for the unknown node to select the anchor triangles which the node locates in. After the anchor triangles the node locates in are found, the smallest area one of them is the effective anchor triangle. The detailed processes are as follows:

(1)Distances estimation for the unknown nodes

In trilateration, distances estimation is necessary. Comparing with indirectly distances estimation schemes, RSSI technique is more precise. Meanwhile unlike other directly estimation schemes it needn't extra hardware. Thus RSSI technique is used to estimate the distances from the unknown nodes to their neighbor anchor nodes in the algorithm.

Supposing the channel fading model complies to Okumura-Hata model. Assuming the power at a node d_0 meters away from the transmitter is PL_0 dBm, the power where d meters to the transmitter is[30]:

$$PL_d = PL_0 - 10\alpha \lg \frac{d}{d_0} + X_\sigma \qquad (1)$$

Where α is the RF channel attenuation exponent. The value of α equals 2 in free space and varies from 3 to 6 in town and suburb. X_σ is an random variable with the standard variance σ.

(2)Anchor triangles composition and PIT testing

Supposing there are N neighbor anchors for the unknown node. Any three of them are combined to compose an anchor triangle. Thus there are C_N^3 anchor triangles. Then the unknown node is used to test which anchor triangles it locates in. PITD is introduced to perform the PIT testing. Supposing there are T anchor triangles the node locates in.

(3)Determination of effective anchor triangle

Among the T anchor triangles, the smallest area one of them is the effective anchor triangle. To reduce the compute complexity of the algorithm, the sums of the distances from the unknown node to the vertexes of each anchor triangle are saved. Then the smallest sum is considered to reflect the area of the corresponding anchor triangle as the smallest one.

(4)Positioning for the unknown node

The unknown node estimates its position using the vertexes of the effective anchor triangle with trilateration.

It is necessary to mention that maybe the effective anchor triangle for some node does not exit at all. Node of this kind is defined as orphan node. The orphan node may be caused by two reasons. One is that there are less than three neighbor anchors so that there is no anchor triangle for the unkown node at all. The other reason is that the unknown node is outside of all anchor triangles. In TPITL, for the former case, the position is estimated with the average coordinates of the neighbor anchors. For the latter one, the position is determined by multilateration instead of trilateration. The pseudo code for the algorithm is as follows:

Algorithm 2: Trilateration based on Point In Triangle testing Localization algorithm(TPITL)

Input:

1) The coordinates matrix for the neighbor anchors of the unknown node.
2) The estimated distances matrix from the unknown node to the neighbor anchors.

Output:

The estimation position for the unknown node.

Procedure:

```
1:if the number of Neighbor anchors for the unknown node N>=3
2: Choose any three of the N anchors as the vertexes of the anchor triangle Ti.
3:    for (each anchor triangle Ti)
4:    Test whether the unknown node is in the anchor triangle via proposed PITD.
5:        if (the unknown node is in the anchor triangle)
6:        Add Ti to the anchor triangle set ET.
7:        endif
8:    endfor
9:        if( ET is not empty)
10:       Find the smallest triangle Te in the anchor triangle set ET.
11:       Estimate the position for the unknown node with trilateration by using the vertexes of
Te.
12:       else
13:       Estimate the position with multilateration.
```

14: endif
15: else
16: The average coordinates of the anchors are the estimation coordinates for the node.
17:endif

5.2 Localization Refinement

To enhance the localization accuracy of orphan nodes, TPITL is improved by introducing pseudo-anchors. Pseudo-anchor node is the unknown node that is located with effective anchor triangle before locating the orphan nodes. In localization refinement step, all neighbor nodes of the orphan node, including neighbor anchor nodes and neighbor pseudo-anchor nodes are collected to compose anchor triangles. Thus the effective anchor triangle may be found out among the anchor triangles. If not, the step is iterated until the effective anchor triangle for the orphan node is constructed.

Fig. 7 shows the principle of the localization refinement. Nodes 1, 2 and 3 are orphan nodes. Take orphan node 1 for example. The black broken line represents the localization error before refinement and the real one represents the localization error after refinement. In refinement, the neighbor unknown nodes 4, 5 and 6 are upgraded as the pseudo-anchor. Then nodes 4, 5 and 6 are added to the neighbor anchors for node 1 to compose anchor triangles. Thus effective anchor triangle is constructed to locate node 1. Results show than the localization errors for orphan nodes are reduced after refinement.

Localization refinement increases the localization accuracy but it introduces additional communication and computation cost.

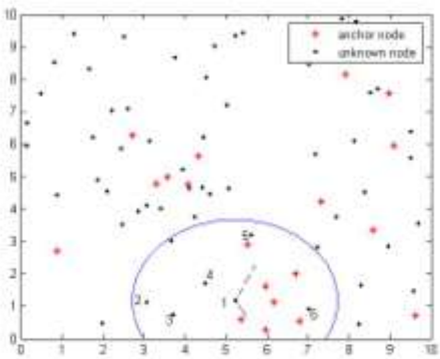


Fig. 7. Refinement

6. Performances Evaluation

Simulation is implemented on Matlab7.0 platform to verify the effectiveness of the proposed algorithm.

6.1 Simulation Settings And System Parameters

This section describes the simulation settings and system parameters used in our evaluation.The performances of algorithms Centroid Localization(CL), Multilateration Localization(ML) and TPITL are compared in the experiments. Both CL and ML use all neighbor anchors of the unknown node for localization. The proposed TPITL only uses the three optimized anchors. CL is range-free while TPITL and ML are range-based. The comparison among them are representative.

6.1.1 Radio Model

Equation 1 in subsection 5.1 assumes that a perfect circular radio model exists. But in real WSNs environment this assumption is invalid. In [31], Zhou ea al. introduce a more accuracy model, Radio Irregularity Model(RIM). The model is defined by equation 2.

$$K_i = \begin{cases} 1 & ,i = 0 \\ K_{i-1} + Rand \times DOI, 0 < i < 360, \left| K_0 - K_{359} \right| \le DOI, i \in N \end{cases} \quad (2)$$

Thus modified radio model is

$$PL_d = PL_0 - 10\alpha \lg \frac{dK_i}{d_0} + X_\sigma \quad (3)$$

The parameter DOI(Degree Of Irregularity) is used to denote the irregularity of the radio pattern. When DOI is zero, there is no rang variation, resulting in a perfectly circular radio model, as shown in equation 1. To make sure that our simulation is close to real WSNs scene, the modified model equation 3 is used in our evaluation.

6.1.2 System Parameters

In our simulation, all nodes are randomly placed in a 10x10 rectangular terrain. Assuming that all nodes have RSSI function. Anchor nodes are randomly selected. The system-wide parameters which directly affect the localization errors are described as follows:

Anchor Node number(AN): The number of anchor nodes.

Total Node number(TN): The number of all sensor nodes in the scene including anchor nodes and unknown nodes.

Anchor Percentage(AP): The number of anchor nodes divided by the total number of nodes. The value can be calculated by the two parameters described using the formula: AP= AN /TN. This parameter is changed by the variation of anchor node number(AN) in our simulation.

Communication Radius(R): The farthest distance a node radio reaches. Assuming that all nodes have the same communication radius R in our simulation.

Degree Of Irregularity(DOI):DOI is defined as the maximum radio range variation per unit degree change in direction of radio pattern irregularity.

In our evaluation, the metric for comparison is localization estimation error. All estimation errors are related errors, which means the errors are normalized to unite of R. Simulations are implemented 10 times or more for average.

6.2 Simulation Results And Analysis

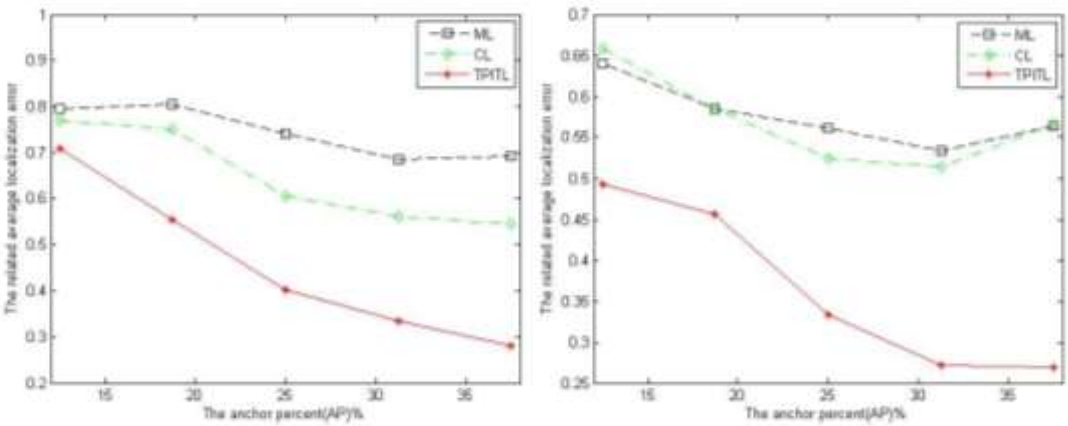
The simulation results from different system configurations are explored. We have conducted a variety of experiments to cover a wide range of conditions including varying

1)Anchor Percentage (AP)2)Communication Radius R 3)The number of total node(TN) 4)DOI. The localization errors before and after refinement are compared. Finally the algorithm complexity is analyzed.

6.2.1 Localization Accuracy When Varying AP

In this experiment, we study the effect of anchor percentage on the location errors. **Fig. 8** shows the results with TN=80 and DOI=0. **Fig. 8(a)** demonstrates that the estimation errors reduce as AP increases for all the three algorithms. Our proposed **TPITL algorithm** significantly outperforms the other two. However, TPITL is more sensitive to the addition of AP. Because for more anchors, more anchor triangles can be combined. Thus the optimized effective anchor triangle is used for trilateration in TPITL to improve the localization accuracy.

As the communication radius R increases from **Fig. 8(a)** R=4 to **Fig. 8(b)** R=5, the errors **of** the three algorithms reduce at the same anchor percentage. But the errors of ML and CL increase **in contrast** when the anchor percentage is above 30%. It means that too more anchors may introduce more estimation errors. As for TPITL, when AP is above 30%, the estimation **errors do** not increase with the increase of AP any more. So increasing AP is not an effective method for improving the accuracy of location when AP is already above an optimized value.

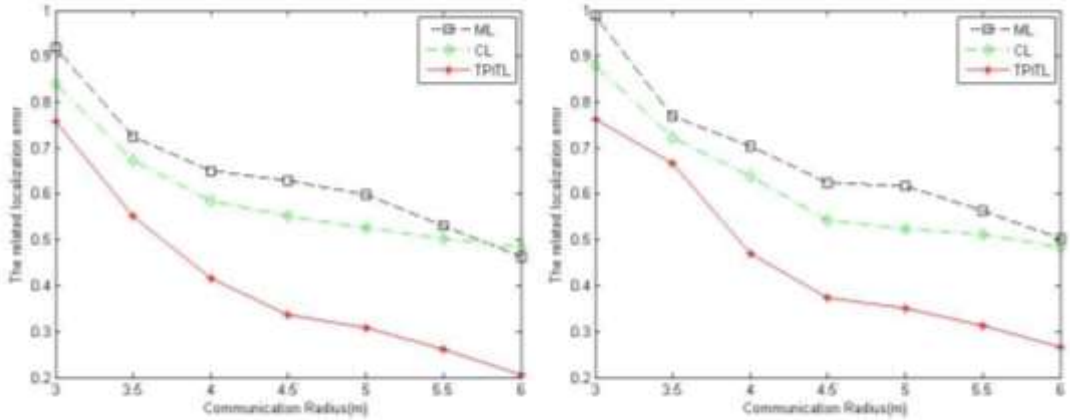


(a)R=4, TN=80 , DOI=0 (b)R=5, TN=80, DOI=0

Fig. 8. Localization error varying AP

6.2.2 Localization Accuracy When Varying R

Fig. 9 explores the effect of communication radius R on the localization errors. The overall estimation errors decrease because more neighbor anchors are added to locate the unknown node as R increases. This can provide more position information for the algorithms. If R is too small, for example below 3.5, the location errors may reach as high as over 50%. As in **Fig. 9(b)**, the value of DOI setting 0.1, the accuracy of TPITL reduces compared with **Fig. 9(a)**, the DOI setting 0. This happens because of the irregular radio patterns of the transmitting model. But the errors do not increase so significantly for CL.



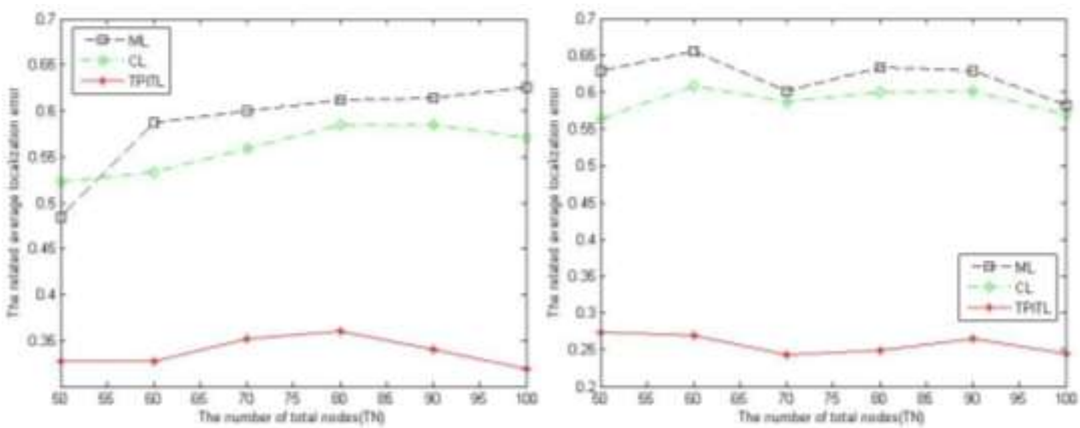
(a)AP=25%, NT=80, DOI=0 (b)AP=25%, NT=80, DOI=0. 1

Fig. 9. Localization error varying R

6.2.3 Localization Accuracy When Varying TN

In this experiment, we analyze the effect of node number on the location accuracy. The results are displayed in **Fig.10**. For all the three algorithms, the location accuracy do not change distinctly and regularly with the increase of the total node. This is due to the fact that the variation of the node number mainly effects the node density of the scene. The value of node density does not impact evidently on the location accuracy of the three algorithms. But in all, TPITL is the most precise positioning algorithm among the three.

In **Fig. 10(b)**, there are 10 more anchors than that in **Fig. 10(a)**. As is analyzed in 6.2.1, with the increase of the AP above 30, the accuracy of ML and CL may decrease. The implication of this conclusion is also investigated in this experiment. We can see from **Fig. 10** the errors of ML and CL in **Fig. 10 (b)** are larger than that in **Fig. 10 (a)**. But the accuracy of TPITL is improved.

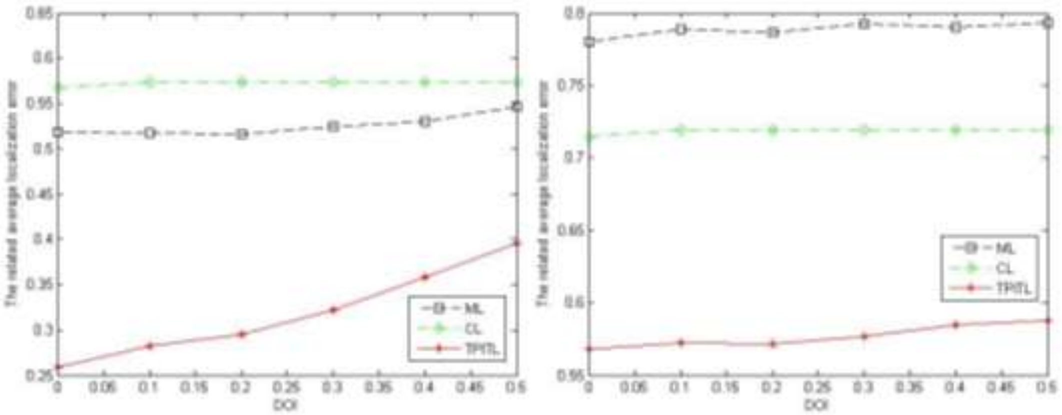


(a)R=5, AN=20, DOI=0 (b)R=5, AN=30, DOI=0

Fig. 10. Localization error varying TN

6.2.4 Localization Accuracy When Varying DOI

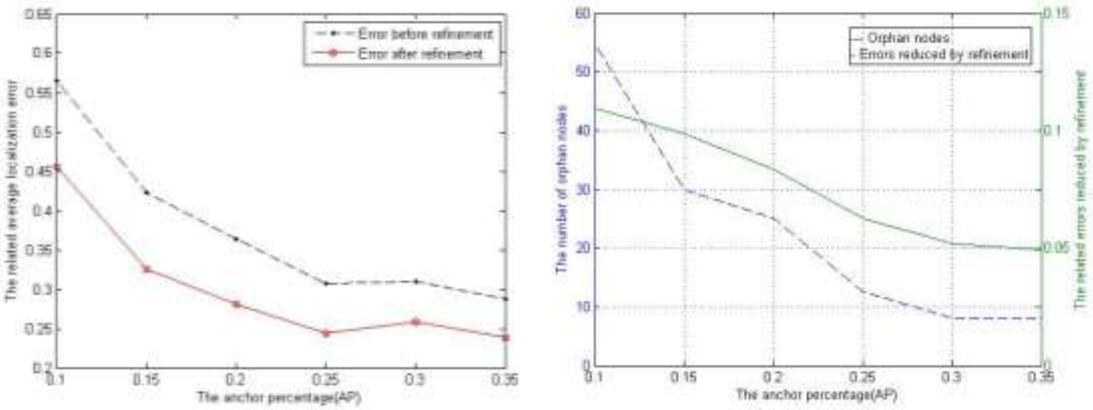
Fig. 11 explores the impact of irregular radio patterns on the location accuracy. For CL, the location errors are nearly constant when varying DOI. Because CL is a range-free algorithm. It leads to the accuracy of CL independent on the distances estimation errors. The location results of ML change slightly with the increase of DOI because the distances estimation errors mildly affect the accuracy of multilateration. But for TPITL, the location errors increase evidently with the increase of DOI in **Fig. 11(b)**, the communication radius R=4. This occurs because the distances estimation errors largely affect the PIT testing errors and the accuracy of trilateration in TPITL. While in **Fig. 11(b)**, the radius is 5, larger than that in **Fig. 11(a)**. This results in less sensitivity to DOI for TPITL.



(a)R=4, AP=20%, NT=80 (b)R=5, AP=20%, TN=80

Fig. 11. Localization error varying DOI

6.2.5 Localization Error Before And After Refinement



(a) Localization errors before and after refinement (b)Orphan nodes and errors reduced by refinement

Fig. 12. Localization error and orphan nodes before and after refinement(R=5,TN=100,DOI=0)

In **Fig. 12**, localization error before and after refinement is compared. It is obviously seen from **Fig. 12(a)** that the localization errors after refinement are lower than the errors before refinement. The errors reduced by refinement are displayed in **Fig. 12(b)**. **Fig. 12(b)** is a double y axis figure. The real line and the right y axis show the errors reduced. The broken line and the left axis show the number of orphan nodes. From the figure we can find that as

AP increases, both the number of orphan nodes and the errors reduced by refinement decrease. This is because there are more anchors and thus more unknown nodes can be located by TPITL. So the number of orphan nodes decrease and meanwhile the errors reduced by refinement decrease with less orphan nodes.

6. 3 Algorithm Complexity Analysis

Among the three algorithms, CL is the simplest one. It only needs the coordinates information of the neighbor anchors for the unknown node. And the position can be easily estimated by calculating the centroid of all the neighbor anchors. As for ML and TPITL, they require additional distances information between the unknown node and its neighbor anchors. So, the communication overhead of ML and TPITL are the same. In TIPITL, the estimation position is computed through more complex calculation. Firstly all the anchor triangles are constructed and the area of each is calculated. Then PIT testing is implemented and the smallest one is searched.

Supposing there are N unknown nodes and $M_i (i=1, 2...N)$ neighbor anchors for the ith unknown node. The communication and computation cost are listed in **Table 1**. The metric for communication cost is the packages sent between nodes. The computation cost scale is the times the addition and multiplication implemented. In the table, $A(p)$ and $M(q)$ represent p times of additions and q times of multiplications.

Table 1. Algorithm complexity

Algorithms	Communication cost	Computation cost
CL	$O(N^2)$	$A(\sum_{i=1}^{N} M_i)$
ML	$O(N^2)+\sum_{i=1}^{N} M_i$	$\sum_{i=1}^{N}\{A[6*(M_i-1)]+M[7*(M_i-1)^2)]\}$
TPITL	$O(N^2)+\sum_{i=1}^{N} M_i$	$\sum_{i=1}^{N}\{C_{Mi}^3[A(12)+M(28)]\}$

7. Conclusion And Outlook

In order to improve the location accuracy in WSNs, a novel algorithm based on trilateration named TPITL is proposed in the paper. Unlike the traditional trilateration location algorithm, which chooses the neighbor anchor nodes randomly for positioning, the TPITL algorithm chooses three special neighbor anchors for trilateration. The three special anchors compose the smallest anchor triangle among all of the anchor triangles which the unknown node locates in, and the smallest anchor triangle is named effective anchor triangle in the paper.To select the vertexes of the effective anchor triangle, we propose PITD method. The distances information which are applied in trilateration, are used in PIT testing to reduce the PIT testing errors and thus the location accuracy is improved.

Simulation results show that our proposed TPITL algorithm significantly outperforms the related algorithms CL and ML. Nevertheless, the improvement of the location accuracy is at the cost of additional communication and computation overhead. Reducing the complexity and studying the effect of distances estimation errors on location accuracy in the proposed algorithm are the future work.

Appendix A

In **Fig. 2**, supposing the real distances between O_1 and A_1, A_2, A_3 are d_{11}, d_{12}, d_{13} separately. The real distances between O_2 and A_1, A_2, A_3 are d_{21}, d_{22}, d_{23}. The estimation distances between O_1 and A_1, A_2, A_3 are d_1, d_2, d_3. The estimation distances between O_2 and A_1, A_2, A_3 are d_1', d_2', d_3'. O_1O_2 is symmetric about line A_1A_2, so

$d_{11}= d_{21}$, $d_{12}=d_{22}$,

The distances estimation error variance for O_1 and O_2 are both Er. Then

$d_1= d_{11}+ d_{11}*Er*N(0, 1)$, $d_1'= d_{21}+ d_{21}*Er*N(0, 1)$

It is easy to see that d1= d1'. There is the same result for A2, that is $d_2= d_2'$.

For anchor A_3,

$d_3= d_{13}+ d_{13}*Er*N(0, 1)$, $d_3'= d_{23}+ d_{23}*Er*N(0, 1)$.

It is obviously that $d_{23}> d_{13}$, So $d_3'> d_3$.

In **Fig. 2**, the positioning area of O_1 is the intersecting of the real circulars while the location of O_2 is showed by the intersecting of the dashed circulars. The real circular A_1 and the dashed circular A_1 are superimposed because $d_1= d_1'$. It is the same for A_2. The real circular A_3 is included in the dashed circular A_3 because $d_3'> d_3$. So the cross area of real circulars DEF is included in the cross area of the dashed circulars FGH, which means the area of DEF is smaller than that of FGH.

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