A CORDIC-Jacobi Based Spectrum Sensing Algorithm For Cognitive Radio

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Abstract

Reliable spectrum sensing algorithm is a fundamental component in cognitive radio. In this paper, a non-cooperative spectrum sensing algorithm which needs only one cognitive radio node named CORDIC (Coordinate Rotation Digital Computer) Jacobi based method is proposed. The algorithm computes the eigenvalues of the sampled covariance of received signal mainly by shift and additional operations, which is suitable for hardware implementation. Based the latest random matrix theory (RMT) about the distribution of the limiting maximum and minimum eigenvalue ratio, the relationship between the probability of false alarm and the decision threshold is derived. Simulations and discussions show the method is effective. Real captured digital television (DTV) signals and Universal Software Radio Peripheral (USRP) are also employed to evaluate the performance of the algorithm, which prove the proposed algorithm can be applied in practical spectrum sensing applications.

***Keywords**: Wireless communication, cognitive radio, spectrum sensing, eigenvalue based method, CORDIC, USRP*

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1. Introduction

Since the fast increase in wireless communication, the scarcity of electromagnetic radio spectrum has become a serious problem which has limited the development of new services in wireless applications. So a new way to solve the problem is urgently needed. Then, the Cognitive Radio (CR) was proposed, it provides a way of dynamic spectrum access for wireless users which would greatly improve the efficiency of spectrum utilization when compared with fixed spectrum allocation methods [1][2]. The CR can exploit the underutilized spectrum in an opportunistic manner, it makes the CR user can use the spectrum allocated to primary users when they are not active [3][4][5]. In this case, for the CR users, they are required to sense the spectrum frequently to find possible spectrum opportunities can be used for communication. In addition, when the primary user is become active from silence suddenly, the CR users have to detect the appearance of the primary user's signal as soon as possible with a high probability to avoid harm interference to the primary user. It is obviously that reliable spectrum sensing algorithm is a fundamental component which is worth to be studied in cognitive radio.

In this paper a CORDIC (Coordinate Rotation Digital Computer) Jacobi method based spectrum sensing algorithm for single CR user or antenna is presented. The ratio of the covariance matrix's maximum and minimum eigenvalues is also employed as the decision statistic. A new CORDIC Jacobi method (CJM) which is suitable for practical implementation by hardware is proposed to obtain the eigenvalues of the covariance matrix. According to the conclusions made in [6][7][8]**[9]** about the distributions of largest and smallest eigenvalues of a random matrix, the decision threshold and probability of false alarm of the algorithm are derived. The proposed method needs only one receiver and can be applied to various signal detection applications without prior knowledge of the signal. Compared with other eigenvalues decomposition method, the method proposed for eigenvalues computing needs mainly shift and addition operations which can be implemented easily on hardware, such as Filed Programmable Gate Array. Digital modulation signals, captured digital television (DTV) signals are used to verify the performance of the method. At the end, the spectrum sensing method proposed is tested by using GNU Radio and the Universal Software Radio Peripheral (USRP).

The rest of the paper is organized as follow: in section 2, we review the previous work in spectrum sensing in cognitive radio. Section 3, signal model and some assumptions are introduced. The eigenvalue based spectrum sensing algorithm is provided in section 4. CORDIC based Jacobi method to compute the eigenvalues of the sampled covariance matrix is presented in section 5. The computation complexity of proposed method is analyzed in section 6. Threshold and probability of false alarm of proposed method is derived in section 7. Simulations and experiments are showed in section 8 which proves the validity of the method. The whole paper is concluded in section 9.

2. Related Work

Many algorithms have been proposed for spectrum sensing, most of them are summarized well in [10], including the energy detection [11][12][13], the spectrum covariance [14][15] and the matched filtering [16]. Each method mentioned above has different requirements or assumptions, also has different disadvantages. For the energy detection, its performance much relies on the accurate estimation of the noise power otherwise it would lead to a high probability of false alarm because of SNR wall phenomena [17]. A priori known spectral features of the primary signal are needed in the spectrum covariance method, and the matched filtering method performs spectrum sensing by coherent demodulation [18] or pilot detection [19] of the primary user's signal, waveforms and channels of the primary user must be known as prior information for the CR users. A kind of spectrum sensing algorithm based on time covariance of the received signal was proposed recently [3][4][20][21][22], which needs no prior information of the primary signal. In [3][4][23], the eigenvalues of sampled covariance based technique for spectrum sensing is presented. Based on the statistical covariance of signals received by different CR users or antennas, the maximum and minimum eigenvalues are derived from the covariance matrix, and then ratio of them is employed as the test statistic to detect the presence or absence of the primary user. The technique requires a cooperative detection setting, which is accomplished by multiple antennas or cooperation among different CR users.

3. Signal Model And Assumptions

If $r(t)$ is the signal received by the cognitive radio node (CR), there are two hypotheses of the received signal

$$H_0: \ r(t) = w(t) \tag{1}$$

$$H_1: \ r(t) = s(t) + w(t) \tag{2}$$

where $s(t)$ is the possible primary user's signal received by the CR receiver and $w(t)$ is noise. The two hypotheses: (1) H_0 denotes the absence of the primary user's signal; (2) H_1 means the primary user's signal exists in received signal. And noise $w(t)$ is assumed to be with zero mean and variance σ_w^2. In addition, noise is not correlated, which indicates $E(w(t)) = 0$, $E(w^2(t)) = \sigma_w^2$ and $E(w(t)w(t+\tau)) = 0, \ \tau \neq 0$. If the CR wants to use the spectrum which is licensed to the primary user with central frequency f_c and bandwidth W, it must detect the presence or absence of the primary user's signal at first to avoid causing harmful interference to the primary user's normal activities. The signal is received by the CR and sampled with a sampling frequency f_s (so the sampling interval is T_s), and then sensing algorithm is applied to determine if the primary user's signal exists or not. Here the sample of $r(t)$ is denoted as $r(t) = r(nT_s) = r(n)$, so the two hypotheses can be rewritten as

$$H_0: \ r(n) = w(n) \tag{3}$$

$$H_1: \ r(n) = s(n) + w(n) \tag{4}$$

For H_1, the signal $s(n)$ received by the CR may be different from the signal $s_0(n)$ transmitted by the primary user. It depends on the channel between the primary user and the CR receiver. In AWGN (Additive Gaussian White Noise) channel, it is obviously that $s(n) = s_0(n)$. Different from [3][4], there is only one receiver or antenna participating the spectrum sensing, the received data samples are defined as follow

$$\mathbf{r} = \left[r(1) \ \ r(2) \ \ \cdots \ \ r(N) \right] \tag{5}$$

$$\mathbf{s} = \left[s(1) \;\; s(2) \;\; \cdots \;\; s(N) \right] \tag{6}$$

$$\mathbf{w} = \left[w(1) \;\; w(2) \;\; \cdots \;\; w(N) \right] \tag{7}$$

where $\mathbf{r}$ is a $1 \times N$ vector. For practical applications. In this paper, narrow band modulated signal and captured DTV signals are the candidates of primary user's signal for spectrum sensing.

4. Eigenvalue Based Spectrum Sensing

The autocorrelation of the received samples can be computed by the equation followed

$$\Re(l) = \frac{1}{N} \sum_{n=0}^{N-1} r(n) r(n-l) \qquad l = 0, 1, \cdots, \alpha - 1 \tag{8}$$

where α is a positive integer named the smooth factor. So the statistical covariance matrix of the received signals can be defined as

$$\mathbf{R}_r = E\left[\mathbf{r}\mathbf{r}^H \right]_{\alpha \times \alpha} = \begin{bmatrix} \Re(0) & \cdots & \Re(\alpha-1) \\ \vdots & \ddots & \vdots \\ \Re(\alpha-1) & \cdots & \Re(0) \end{bmatrix}_{\alpha \times \alpha} \tag{9}$$

Obviously, it is a toeplitz and symmetric covariance matrix. With the same method, the covariance matrix of the signal and noise also can be obtained

$$\mathbf{R}_s = E\left[\mathbf{s}\mathbf{s}^H \right] \tag{10}$$

$$\mathbf{R}_w = E\left[\mathbf{w}\mathbf{w}^H \right] \tag{11}$$

According to the signal model and assumptions of this paper, we can easily obtain that

$$\mathbf{R}_r = \mathbf{R}_s + \sigma_w^2 \mathbf{I} \tag{12}$$

with $\mathbf{I}$ is the identity matrix of order N. If primary user's signal doesn't exist, $\mathbf{R}_s = 0$, so the off-diagonal elements of $\mathbf{R}_r$ are all zeros, then the eigenvalues of the matrix $\mathbf{R}_r$ are all σ_w^2. If the primary user's signal existing in the received samples, $\mathbf{R}_s$ is not a diagonal matrix and the off-diagonal elements of $\mathbf{R}_r$ should be non-zeros, that indicates the eigenvalues of $\mathbf{R}_r$ are no longer all σ_w^2. Hence, we can detect the primary user's signal presented or not by utilizing the eigenvalues of the covariance matrix of the received signals.

Hence the maximum and minimum eigenvalues of the covariance matrix is used to detection of the primary user's signal. Compared with the maximum-minimum eigenvalue (MME) method proposed in [3], in proposed method, modified MME method (mMME), only one CR node participates in detection instead of cooperatively detecting by using more than one CR nodes, consequently the derivations and expressions of the decision threshold must be modified although the steps of the two algorithms are the same. The following are the steps of the algorithm:

- *Step 1*. Compute the sample covariance matrix $\mathbf{R}_r$;
- *Step 2*. The maximum and minimum eigenvalue of the covariance matrix are obtained, which can be denoted as $\lambda_{\max}$ and $\lambda_{\min}$.
- *Step 3*. Decision making: if $\lambda_{\max} / \lambda_{\min} > \gamma$, we say that the primary user's signal exists

(H_1), otherwise, the signal doesn't exist (H_0). And γ is the decision threshold.

From the description of the steps, we can see that the key of this algorithm is eigenvalue decomposition of the covariance matrix and the selection of the decision threshold. They will be further discussed in the following section.

Energy detection is also a classical method for spectrum sensing [24]. Let T_{egy} be the energy of the received signal, it can be obtained by the following equation

$$T_{egy} = \sum_{i=1}^{N} |r(i)|^2 \tag{13}$$

The decision rule of the energy detection is

$$\begin{cases} H_1 : T_{egy} \geq \gamma_{egy} \\ H_0 : T_{egy} < \gamma_{egy} \end{cases} \tag{14}$$

where γ_{egy} is the decision threshold of energy detection, it can set based on probability of false alarm P_{fa_egy} :

$$P_{fa\_egy} = P(T_{egy} > \gamma_{egy} | H_0) = P\left(\frac{\frac{T_{egy}}{\sigma_w^2} - N}{\sqrt{2N}} > \frac{\frac{\gamma_{egy}}{\sigma_w^2} - N}{\sqrt{2N}} \right) \approx Q\left(\frac{\frac{\gamma_{egy}}{\sigma_w^2} - N}{\sqrt{2N}} \right) \tag{15}$$

From the equation we can see that the energy detection needs accurate knowledge of the noise power σ_w^2, but in practical applications, the estimated noise power may be different from the actual one. If the noise power estimated by the CR node is $\hat{\sigma}_w^2 = \mu\sigma_w^2$, the noise uncertainty factor (in dB) can be defined as

$$\Omega = \max\{10\log_{10}\mu\} \tag{16}$$

Which indicates the μ (in dB) is randomly distributed at a range of $[-\Omega, \ \Omega]$.

5. Eigenvalues Computation

As mentioned in the last section, the most crucial point of mMME algorithm is eigenvalue computation. However, to apply MME method to spectrum sensing in practical application, we must obtain the eigenvalues of the covariance matrix in reality instead of computer simulations, which requires mMME method being carried out in hardware. As a solution, a new CORDIC based Jacobi method (CJM), which can be implemented on hardware, is proposed. It is well known that the Jacobi method can be expressed as

$$\begin{cases} \mathbf{A}_{k+1} = \mathbf{U}_k(p,q,\theta)\mathbf{A}_k\mathbf{U}_k^T(p,q,\theta) \quad k = 0,1,\cdots \\ \mathbf{A}_0 = \mathbf{R}_r \end{cases} \tag{17}$$

When $k \to \infty$, the matrix $\mathbf{A}_k$ converges to a diagonal matrix $\mathbf{A}_\infty$ in terms of a rigid rotation. For each rotation, two elements of $\mathbf{A}_k$, a_{pq}^k and a_{qp}^k are settled to be zero. After the norm of off-diagonal elements of $\mathbf{A}_k$ equal to zero, the rotation is stopped, and the diagonal elements of $\mathbf{A}_k$ is considered to be the eigenvalues of the covariance matrix. The matrices $\mathbf{U}_k(p,q,\theta)$ are rotational matrices which annihilate off-diagonal elements, it can be written as

$$\mathbf{U}_k(p,q,\theta)=\begin{bmatrix} 1 & 0 & \cdots & \overset{p}{0} & \overset{q}{0} & 0 \\ 0 & 1 & 0 & \cos\theta & \sin\theta & \vdots_p \\ \vdots & 0 & \ddots & -\sin\theta & \cos\theta & \vdots_q \\ & & & \ddots & 0 & \\ \vdots & \vdots & & & \ddots & 0 \\ 0 & 0 & \cdots & & 0 & 1 \end{bmatrix}_{N\times N} \tag{18}$$

where $p<q$ and $p,q=1,2,\cdots$, θ is the rotation angle. Because after each rotation, a_{pq}^{k+1} and a_{qp}^{k+1} must be zero, that indicates

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} a_{pp}^k & a_{pq}^k \\ a_{qp}^k & a_{qq}^k \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^T=\begin{bmatrix} a_{pp}^{k+1} & 0 \\ 0 & a_{qq}^{k+1} \end{bmatrix} \tag{19}$$

From last equation we can obtain the relationship between the rotation angle θ and the elements of the matrix can be denoted as

$$\tan 2\theta=\frac{2a_{pq}^k}{a_{pp}^k-a_{qq}^k} \tag{20}$$

so we can derive the iterative rules for the elements of covariance matrix as follow:

$$a_{pp}^{k+1}=\cos^2\theta\left(a_{pp}^k+2\tan\theta a_{qp}^k+\tan^2\theta a_{qq}^k\right) \tag{21}$$

$$a_{qp}^{k+1}=\cos^2\theta\left(-\tan\theta a_{pp}^k+(1-\tan^2\theta)a_{qp}^k+\tan\theta a_{qq}^k\right) \tag{22}$$

$$a_{qq}^{k+1}=\cos^2\theta\left(\tan^2\theta a_{pp}^k-2\tan\theta a_{qp}^k+a_{qq}^k\right) \tag{23}$$

$$a_{pj}^{k+1}=\cos\theta\left(a_{pj}^k+\tan\theta a_{jq}^k\right) \tag{24}$$

$$a_{iq}^{k+1}=\cos\theta\left(a_{iq}^k-\tan\theta a_{pi}^k\right), i,j\neq q、p \tag{25}$$

From the iterative rules of Jacobi method, in order to compute the eigenvalues of the matrix, the rotation angle must be computed at first, then multiplication and other operations such as divisions, are needed here. That's time consuming and difficulty to be implemented in hardware. So the CORDIC algorithm is applied to avoid rotation angle computation and complicated operations.

The CORDIC algorithm was invented by J.Volder in 1959[25], which is an iterative algorithm that only shift and add operations are needed. For the Jacobi method, the CORDIC module can be used to compute the rotation angle, such as in [26], but for one rotation, at least two CORDIC modules are needed. Here one CORDIC module with rotation mode is employed to eigenvalue computation. In the CORDIC iterative process, the rotation with angle θ is divided into series of consecutive micro-rotations with angle $\pm\arctan 2^{-m}$ and requires only shift and addition operations, $m=0,1\cdots,b$, where b is the times of micro-rotation during one rotation with angle θ . The iterative process can be denoted as follow

$$\begin{cases} x_{m+1} = K_b\left(x_m - y_m \cdot d_m \cdot 2^{-m}\right) \\ y_{m+1} = K_b\left(y_m + x_m \cdot d_m \cdot 2^{-m}\right) \\ z_{m+1} = z_m - d_m \cdot \arctan(2^{-m}) \end{cases} \tag{26}$$

when $z_m \to 0$, one CORDIC iterative process is completed. Where $d_m = sign(z_m) = \pm 1$, which decides the rotation direction. Compared with Eq.(26) and Eq.(21-25), we approximates $2^{-m} = \tan\theta$, $\cos\theta = K_b$, and $\mathbf{R}_r^m(p,q) = \mathbf{R}_r^m(q,p)$, $\mathbf{R}_r^m(j,p) = \mathbf{R}_r^m(p,j)$, $\mathbf{R}_r^m(i,q) = \mathbf{R}_r^m(q,i)$, $d = sign(\mathbf{R}_r^m(p,q))$, $i, j \neq q$、p . The new CORDIC based Jacobi algorithm proposed for eigenvalue computation can be derived as follow:

- *Step 1.* Find the maximum absolute value of the off-diagonal elements of covariance matrix $\mathbf{R}_r^m(p,q)$, where p and q are the row and column, respectively.
- *Step 2.* One CORDIC iterative process is applied. For the elements of row p (q) and column q (p), the iterative process is showed in **Table 1**.
- *Step 3.* Repeat step 1 and 2, until the absolute values of all the off-diagonal elements of the $\mathbf{R}_r^m$ are smaller than e . In this condition, the diagonal elements of the $\mathbf{R}_r^m$ are considered to be the eigenvalues of the sample covariance matrix $\mathbf{R}_r$.

where e is a parameter defined by the users, its value is close to zero, which indicates the diagonalization of $\mathbf{R}_r^m$ for a finite m, and $K_b = \sqrt{1+2^{-2m}}$ is the scalling factor, we noticed that if $m > 5$, $K_b \approx 0.6073$. So the value of K_b can be pre-computed off-line, and only 6 values of it need to be stored in the memory of the hardware.

Table 1. One CORDIC Jacobi iterative process

While $\left|\mathbf{R}_r^m(p,q)\right| < e$

$\mathbf{R}_r^{m+1}(p,p) = K_b^2\left(\mathbf{R}_r^m(p,p) + 2^{-m+1} \cdot d \cdot \mathbf{R}_r^m(q,p) + 2^{-2m}\mathbf{R}_r^m(q,q)\right)$

$\mathbf{R}_r^{m+1}(q,p) = K_b^2\left(-2^{-m}\mathbf{R}_r^m(p,p) + d \cdot (1-2^{-2m})\mathbf{R}_r^m(q,p) + 2^{-m}\mathbf{R}_r^m(q,q)\right)$

$\mathbf{R}_r^{m+1}(q,q) = K_b^2\left(2^{-2m}\mathbf{R}_r^m(p,p) - 2^{-m+1} \cdot d \cdot \mathbf{R}_r^m(q,p) + \mathbf{R}_r^m(q,q)\right)$

$\mathbf{R}_r^{m+1}(p,q) = \mathbf{R}_r^{m+1}(q,p)$

for $i = 1:\alpha(i \neq p,q)$

$\mathbf{R}_r^{m+1}(p,i) = K_b\left(\mathbf{R}_r^m(q,i) + d \cdot 2^{-m}\mathbf{R}_r^m(i,q)\right)$

$\mathbf{R}_r^{m+1}(i,q) = K_b\left(\mathbf{R}_r^m(i,q) - d \cdot 2^{-m}\mathbf{R}_r^m(p,i)\right)$

$\mathbf{R}_r^{m+1}(i,p) = \mathbf{R}_r^{m+1}(p,i)$

$\mathbf{R}_r^{m+1}(i,q) = \mathbf{R}_r^{m+1}(q,i)$

end

end

Suppose that $\lambda_1, \lambda_2 \cdots \lambda_\alpha$ are the eigenvalues of the covariance matrix $\mathbf{R}_r$, and $\tilde{\lambda}_1, \tilde{\lambda}_2 \cdots \tilde{\lambda}_\alpha$ are the eigenvalues computed by CJM. Because the existence of e, when the CORDIC iterative process stopped the matrix $\mathbf{R}_r^m$ can be approximated as follow

$$\mathbf{R}_r^m = \begin{bmatrix} \tilde{\lambda}_1 & e & \cdots & e \\ e & \tilde{\lambda}_2 & e & \vdots \\ \vdots & e & \ddots & e \\ e & \cdots & e & \tilde{\lambda}_\alpha \end{bmatrix}_{\alpha\times\alpha} \tag{27}$$

It can be easily proved when e is small enough, $\lim_{\substack{e\to 0 \\ i=1\cdots\alpha}} \tilde{\lambda}_i = \lambda_i$. In order to evaluate the convergence performance of CORDIC Jacobi method, two metrics named average eignvalue computation errors and maximum eigenvalue computation errors are defined respectively. The average eignvalue computation error at the end of the j th iteration is defined as follow:

$$err_j = \frac{1}{\alpha}\sum_{i=1}^{\alpha}\left|\frac{\lambda_i - \tilde{\lambda}_i}{\lambda_i}\right| = \frac{1}{\alpha}\sum_{i=1}^{\alpha} err_{ji} \tag{28}$$

Where α is the number of eigenvalues, λ_i is the i th eigenvalue of the covariance matrix computed by Matlab eig function and $\tilde{\lambda}_i$ is the i th corresponding diagonal element of the matrix during the procedure of iteration. The maximum eigenvalue computation error at the end of the j th iteration can be presented as:

$$Merr_j = \max\left\{err_{j1}, err_{j2}, \cdots, err_{j\alpha}\right\} \tag{29}$$

As shown in **Fig.1**, with the increase of iterations, both of the errors are converging to 0. And the iteration times is determined by the computation accuracy, the higher accuracy, the more times of iteration are needed. If e is small enough, the computation error would converge to 0. when $e = 10^{-3}$, 1576 Jacobi iterations are needed and the average computation error is 1.242×10^{-7}, but when $e = 0.5$, only 76 Jacobi iterations are executed while the average computation error is 0.0193. In practical applications, there is a tradeoff between computation accuracy and complexity.

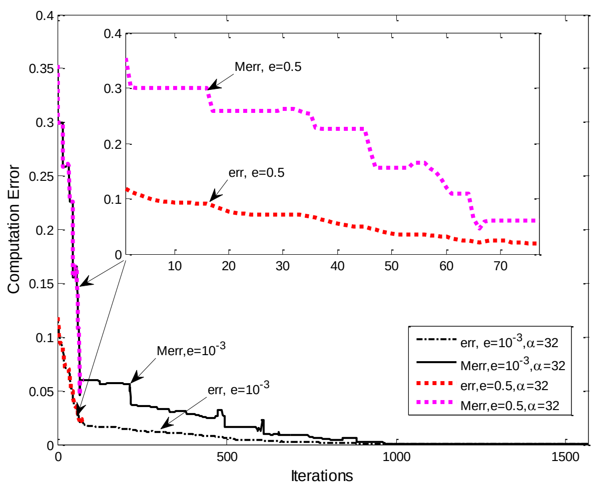

Fig. 1. The convergence procedure of proposed method

6. Computational complexity

The complexity of mMME is mainly consisted by three parts: computation of the statistical covariance matrix (as in equation (8) and (9)), operation of sorting the absolute value of the off-diagonal elements of covariance matrix and the eigenvalus computation. For the first part, since the covariance matrix is symmetrical, the main computational complexity is to compute the autocorralation of received samples. Hence $\alpha(N+1)$ multiplications and $\alpha(N-1)$ additions are needed. Taking advantages of the symmetric property of the covariance matrix, the second part is comparisions of the absolute value of the upper triangular elements (the diagonal elements are not included), so $0.5\alpha(\alpha-1)$ comparisions are executed in each Jacobi iteration. In practical implementation, if the word length is l, hence the number of CORDIC rotations in each Jacobi iteration is l at most, as shown in Table 1, after l times of right shifts, both of the $-2^{-l}\mathbf{R}_r^l(p,p)$ and $2^{-l}\mathbf{R}_r^l(q,q)$ equal to 0, the value of $(1-2^{-2l})$ approximates to 1, in this case, the value of $\mathbf{R}_r^m(q,p)$ never changes, one Jacobi iteration is finished. For each CORDIC rotation, $2(1+\alpha)$ shifts and $(3+2\alpha)$ additions are needed. The scaling can be executed with approximatedly $0.25l$ shifts and additions. The times of Jacobi iteration are determined by the computation accuracy, the smaller of e, the more times of iteration are needed. We assume J Jacobi iterations are needed to complete the diagonalization of the covariance matrix, therefore the complexity of the proposed method is summarized in **Table 2**.

Table 2. Computation complexity of proposed method

multiplications	additions	comparisions	shifts
$\alpha(N+1)$	$\alpha(N-1)+Jl(3.25+2\alpha)$	$0.5J\alpha(\alpha-1)$	$(2.25+2\alpha)Jl$

We assume that the word length $l=16$, $\alpha=8$. As mentioned in [27], its total computation complexity of the eigenvalue computation is

$$\{4\alpha(\alpha-1)(\alpha+2)+1\}\times\{4.25l\ shifts+5.25l\ additions\}$$

While the method proposed in this paper are $0.5J\alpha(\alpha-1)$ comparisons, $(2.25+2\alpha)Jl$ shifts and $(3.25+2\alpha)Jl$ additions, respectively. So in the same experiment settings, the method proposed in [26] needs at least 43928 clock cycles. If proposed CORDIC Jacobi method executes the same clock cycles, it is easy to see that least 71 Jacobi iterations can be carried out for eigenvalue computation. But for such a covariance matrix, only after 28 Jacobi iterations, the maximum eigenvalue computation error is below 0.5%, which is precise enough for most of the spectrum sensing application.

7. Decision Threshold And Probability Of False Alarm

If the primary user's signal doesn't exist, the sample covariance matrix $\mathbf{R}_r$ turns to be

$$\mathbf{R}_r=E\left[\mathbf{r}\mathbf{r}^H\right]_{\alpha\times\alpha}=E\left[\mathbf{w}\mathbf{w}^H\right]_{\alpha\times\alpha}=\mathbf{R}_w \quad (30)$$

according to the conclusions of [21], $\mathbf{R}_w$ can be considered as a Wishart random matrix. Many researchers, such as Soshnikov, Ruzmaikina, Feldheim[7], and Federico Penna [23] have done lots of work on studying the eigenvalue distributions of a random matrix. Based on the results they had given, when N and α are large, and $\lambda_1 \le \lambda_2 \le \cdots \le \lambda_\alpha$ are the eigenvalues of $\frac{N}{\sigma_w^2}\mathbf{R}_w$, let

$$\varepsilon = N + \alpha + 2\sqrt{N\alpha} \tag{31}$$

$$\eta = N + \alpha - 2\sqrt{N\alpha} \tag{32}$$

$$\phi = \sqrt{\varepsilon}\left(\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{\alpha}}\right)^{\frac{1}{3}} \tag{33}$$

$$\varphi = \sqrt{\eta}\left(\frac{1}{\sqrt{N}} - \frac{1}{\sqrt{\alpha}}\right)^{\frac{1}{3}} \tag{34}$$

we can obtain that $L_\alpha = \frac{\lambda_\alpha - \varepsilon}{\phi}$ and $L_1 = \frac{\lambda_1 - \eta}{\varphi}$, both of them converge in distribution to the Tracy-Widom law Tw_β [23], because the noise here is real, so $\beta = 1$ (if complex, $\beta = 2$). The Tracy-Widom law Tw_2 is defined, its cumulative density function is

$$F_{Tw_2}(x) = \exp\left\{-\int_x^{+\infty}(t-x)\chi^2(t)dt\right\} \tag{35}$$

where $\chi(\cdot)$ is the solution to the differential equation:

$$\chi^{''}(t) = t\chi(t) + 2\chi^3(t) \tag{36}$$

In [6], it had shown that when $\beta = 1$, then

$$F_{Tw_1}^2(x) = F_{Tw_2}(x)\exp\left\{-\int_x^{+\infty}\chi(t)dt\right\} \tag{37}$$

so the distribution function is defined as

$$F_{Tw_1}(x) = \exp\left\{-\frac{1}{2}\int_x^{+\infty}((t-x)\chi^2(t) + \chi(t))dt\right\} \tag{38}$$

The decision statistic T can be written as[9]:

$$T = \frac{\lambda_\alpha}{\lambda_1} = \frac{\phi L_\alpha + \varepsilon}{\varphi L_1 + \eta} \tag{39}$$

If $f_{\lambda_\alpha}(t)$ and $f_{\lambda_1}(t)$ denote the limiting probability density function (PDF) of λ_α and λ_1, respectively, and the two PDFs can be expressed by a linear random variable transformation of the Tracy-Widom PDF, hence they can be presented as follow:

$$f_{\lambda_\alpha}(t) = \frac{1}{\phi} f_{Tw1}\left(\frac{t-\varepsilon}{\phi}\right) \tag{40}$$

$$f_{\lambda_1}(t) = -\frac{1}{\varphi} f_{Tw1}\left(\frac{t-\eta}{\varphi}\right) \tag{41}$$

Where $f_{Tw1}(\cdot)$ is the PDF of the Tracy-Widom distribution with order 1. Because $f_{\lambda_\alpha}(t)$ and $f_{\lambda_1}(t)$ are independent for $N, \alpha \to \infty$, so the PDF of T is:

$$f_{T|H_0}(t) = -\int_0^{\infty} x\frac{1}{\phi\varphi} f_{Tw1}\left(\frac{tx-\varepsilon}{\phi}\right) f_{Tw2}\left(\frac{tx-\eta}{\varphi}\right)dx \quad (t > 1) \tag{42}$$

The condition $t > 1$ is to assure $\lambda_\alpha > \lambda_1$. Let $F_{T|H_0}(t)$ is the cumulative density function of the decision statistic, it can be written as

$$F_{T|H_0}(t) = \int_1^{\infty} f_{T|H_0}(t)dt \tag{43}$$

So the probability of false alarm can be given as follow

$$P_{fa} = P(T > \gamma | H_0) = 1 - F_{T|H_0}(\gamma) \tag{44}$$

At the same time, if the false probability of detection is fixed, the decision threshold can be computed by this equation

$$\gamma = F_{T|H_0}^{-1}(1 - P_{fa}) \tag{45}$$

At the same time, if the false probability of detection is fixed, the decision threshold can be computed by this equation which means we can set the decision threshold according to the required error constraints. For the Tracy Widom distribution, since its importance in Rand Matrix Theory, this distribution has been extensively studied and tabulated, and a Matlab routing to compute is available at [28]. In practical applications, the values of $F_{T|H_0}^{-1}(\cdot)$ can be computed off-line and stored in a look-up table.

However, the eigenvalues of the sample covariance matrix are computed by CJM, the errors between the true values and the computed ones can not be avoided because the approximate rotations instead of complex accurate computations. The error would lead to a wrong sensing result especially when the SNR is very low. Parts of the error can be reduced by selecting a suitable e. Computer simulation proves that when e is small enough, the ratio $T_{CJM} = \tilde{\lambda}_{\max} / \tilde{\lambda}_{\min}$ is approximating to $T = \lambda_{\max} / \lambda_{\min}$. But to find an analytical formula to analyze the error is difficult and complex, fortunately, the following computer simulation proves that when e is small enough, the error's influence to performance of spectrum sensing can be neglected.

8. Simulation Results And Experiments

In this part, some simulation results are given to evaluate the performance of proposed method. Here digital modulated signals and captured DTV signals are employed as the primary user's signals. The probability of false alarm is set to 0.05 and times of test for each SNR are 1000 unless redefined in simulations specially.

The decision statistic

The ratio of maximum and minimum eigenvalues of the sample covariance is the decision statistic of the spectrum sensing method, so the computation of the ratio is very important. Here the eigenvalues are computed by CORDIC Jacobi method and under the hypothesis of H_0, to evaluate the accuracy of the proposed method, we fixed $e = 10^{-3}$, and the length of received 16QAM signal sample is 8000 and 32000, the smooth factor is 8 and 16, respectively. Where T is the ratio of maximum and minimum eigenvalues computed by Matlab eig function while T_{CJM} is the one computed by proposed method. **Fig.2** shows the ratio changes versus noise variance from 3.162 to 31.62. For each noise variance, the average value of 2000 simulations is considered to be the final result of the ratio. From the figure we can see that the two ratios matched very well, it proves that proposed method can obtain the ratio with a high accuracy when $e = 10^{-3}$. **Fig.3** shows the errors between the two ratios versus e, in the same way, 2000 trials are made for each e. It is obvious that the error is increased when e become larger. It is reasonable, just because a larger e indicates the off-diagonal elements of the sample covariance matrix is not close enough to zero when the iterative process stopped. We noticed that when $e < 10^{-3}$ the error between the two results can be neglected, and when $e > 10^{-1}$, the errors between them approximate to a constant value. These two simulations showed that if we can choose a suitable e for proposed eigenvalue decomposition method, a better tradeoff between computation accuracy and complexity can be achieved. If e is too small, a more accurate ratio can be obtained while the computation complexity would be increased. If e is too large, the computation complexity would be reduced but it may lead to a result with low accuracy. In this paper, e is set to 0.001 without special illustration.

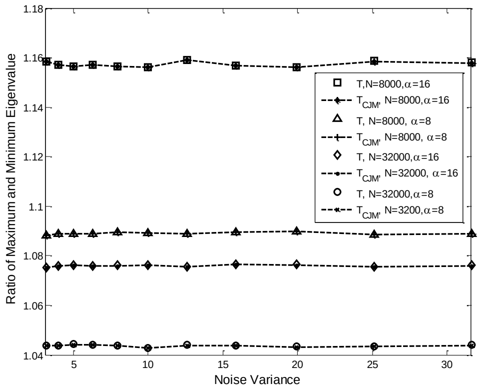

Fig. 2. The ratios computed by two methods versus noise variance (under H_0).

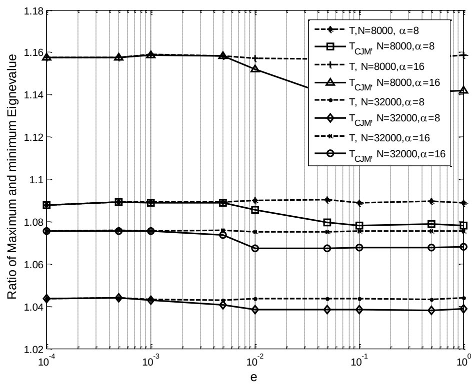

Fig.3. The ratios computed by two methods versus e (under H_0).

QAM signals detection

The 16QAM signal is generated with carrier frequency 24MHz, and symbol rate 6Mbps, the base-band signal is filtered by a root squared raise cosine filter with roll off factor 0.5 before modulation. The sampling frequency of the receiver is 96MHz, which are 4 times the carrier frequency and 16 times of the symbol rate. The probabilities of detection and false alarm are showed in **Fig.4**. In this figure, the sample size is 8000, the SNR varies from -25dB to 0dB with a step 1dB. The P_{dmMME} (P_{famMME}) denotes the detection (false alarm) probability of the proposed method, while P_d (P_{fa}) denotes the detection (false alarm) probability of the method which the eigenvalues used in simulations are computed by Matlab eig function. The probability of false alarm is set to 0.05 and the decision threshold of mMME method is computed according to Eq.(45)**.** From the figure we can see that P_d (P_{fa}) and P_{dmMME} (P_{famMME}) are matched very well. Enegy detection is employed to spectrum sensing, and noise uncertainty is also introduced in the simulations. When there is no noise uncertainty, the mMME and energy detection perform similar detection results, and the simulated probability of false alarm fits well with theoretical value. Obviously, from the figure we can see that the probability of false alarm is not related to the SNR of received signal. However when we assume 2dB noise uncertainty exists in received signal, the performance of energy detection decreases heavily while there is no obvious influence on the performance of mMME method. These simulations prove the proposed mMME method overcomes the noise uncertainty problem which is the main problem of energy detection.

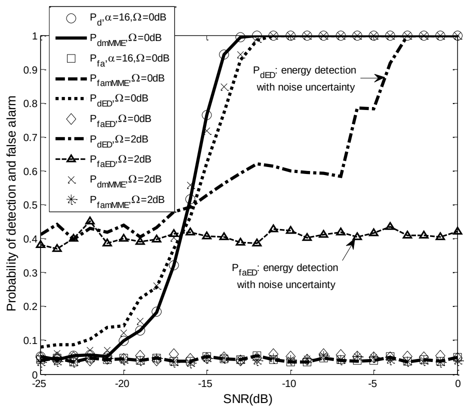

Fig. 4. Probabilities of detection and false alarm for QAM signal.

Fig.5 shows the changes of probabilities of detection and false alarm with different e when SNR is -14dB. For each e , 2000 tests are applied. In this simulation setting, we noticed that when e is smaller than 0.01, the probabilities of proposed method approximately equal to the ones which eigenvalues are computed by Matlab eig function, that is because when e is smaller than 0.01, the computation errors of the ratio computed by proposed method is small. So the performances of them are similar. The simulation also proves that proposed method can detect the presence or absence of the primary signal effectively at a very low SNR.

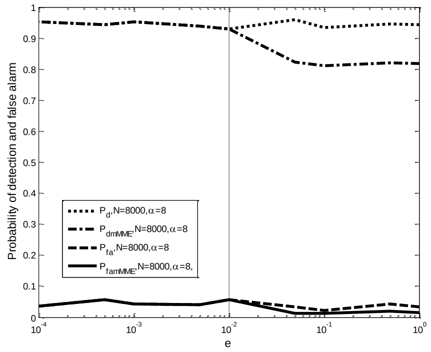

Fig.5. Probabilities of detection and false alarm with different e for QAM signal.

Fig.6 and **Fig.7** show the influence of received data samples and smooth factors to the performance of proposed method. It is obvious that if more received data samples can be obtained, it would improve the performance of detection when they are with the same probabilities of false alarm. And it indicates that the probability of false alarm of proposed method is not sensitive to the length of received data samples. From **Fig.7**, when the smppth factor is smaller than 8, the probability of detection is increased with the increase of smooth factor. But when it is larger than 8, there is only slight variation in detection performance. In a spectrum sensing application, we always hope when the smooth factor is fixed, the probability of detection can be as larger as possible while smaller for probability of false alarm. So when the sample size is 8000 and the smooth factor is bigger than 8, we can get a difference large enough between the two probabilities. Due to the computation complexity, we prefere to choose smaller smooth factor and sample size when they are with the same performance guarantee. From the result of **Fig.7**, a larger smooth factor may improve the detection performance, but it also increases the computing complexity.

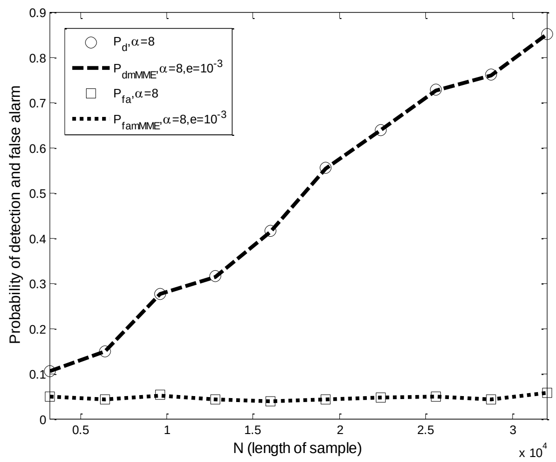

Fig. 6. Probabilities of detection and false alarm with different length of data samples for QAM signal (SNR=-18dB).

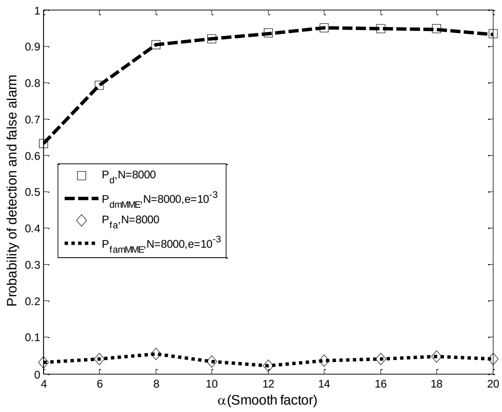

Fig. 7. Probabilities of detection and false alarm with different smooth factors for QAM signal (SNR=-14dB).

Fig.8 shows the probability of false alarm versus the decision threshold. The theoretical probability of the false alarm is computed according to Eq. (44). The probability of false alarm of proposed method is also simulated at a range of γ from 1 to 1.4. For each γ , 5000 tests are made. From the simulation results, the simulated results and the theoretical ones are matched well. It is significant for us to find an appropriate decision threshold when the probability of false alarm needed in real applications has been determined.

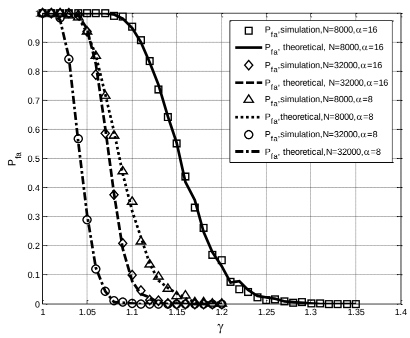

Fig. 8. Probability of false alarm versus γ .

Captured DTV signal

In this part, we use the captured real ATSC DTV signals (field measurements) to evaluate the proposed method[29]. The captured signal data used here is WAS_3_27_06022000_REF, which is provided by Dr.Y.H.Zeng[30]. The following is the test condition of the captured DTV signal: the DTV signal is received by an antenna with height of 30 feet, suburban, Washington D.C., USA. The distance between the DTV station and the receiver is 48.41 miles, and the received signal is sampled with a frequency of 10.762MHz. The multi-path channel between the receiver and the DTV station is unknown and also the SNR of signal received. In this test, white Gaussian noise is added to the signal to evaluate the performance of proposed method at different SNR levels. The decision threshold is selected according to Eq. (45). **Fig.9** shows the results of detection. From the figure we can see that if the SNR of the received signal is higher than -13dB, with 32000 data samples (2.9734ms), the method can detect the DTV signal successfully. According to the IEEE 802.22 standard [31], a cognitive radio device should finish spectrum sensing within 2 seconds.

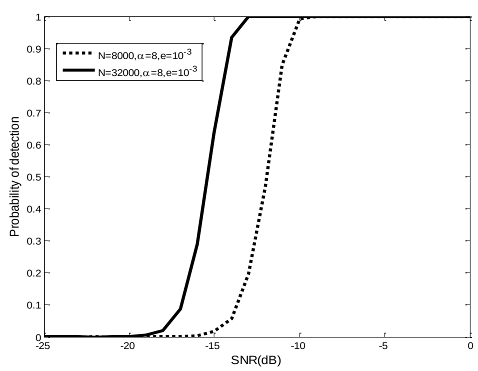

Fig.9. Probability of detection for captured DTV signals in Washington, USA.

If the cognitive radio uses a quiet period of 40ms (time consuming for signal processing is included) every 2 seconds for spectrum sensing, the proposed method can provide its user 98% of time to transmission.

Captured digital modulated signal

In order to prove the proposed method in a real test environment, the Ettus USRP hardware is utilized as an RF front-end of the CR user. The GNU Radio open-source software and matlab are used as software interface. The Agilent Vector Signal Generator (E4438C) is employed as the primary user. The primary signal is a QAM signal, with modulation level 16, carrier frequency 2.441GHz and symbol rate 1Mbps. The distance between the two antennas of the CR and primary is about 30 centimeters (see **Fig. 10**). The signal samples collected by the USRP are sent back through USB to the laptop for further processing.

Fig. 10. Experiment setting

The SNR of received signal is measured by a software defined spectrum analyzer developed by GNU Radio and USRP. The tests prove that when SNR is larger than 0dB, the proposed method always can detect the primary signal successfully. Because there is no effective way to measure the SNR of received signal smaller than 0dB, white Gaussian noise is added to the signal samples captured by the USRP to test the proposed method at different SNR levels.

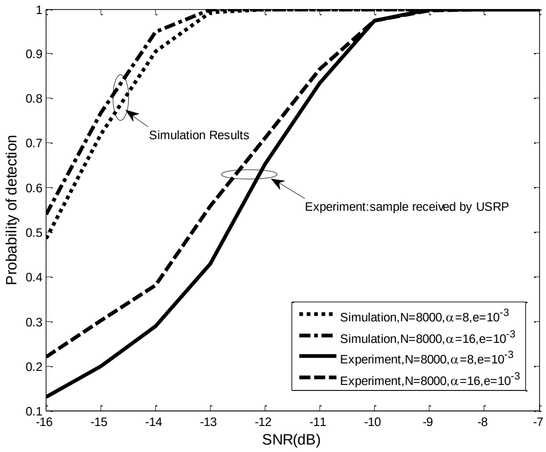

Fig. 11. Probability of detection for QAM signals which are captured by USRP.

From **Fig.11**, compared with the results of computer simulations, the experimental probability is much lower. The explanation to this phenomenon is the signal samples received by the USRP have been polluted by noise before white Gaussian noise is added, what we can do in the experiment is try our best to improve the SNR of received data samples, such as increase the transmission power of the signal generator. Hence the actual SNR of the received data sample is much lower than the SNR presented in **Fig.11**, while the data samples are pure in computer simulations. Although there is a performance decrease, we still see that the proposed method also can detect the presence of primary signal at a low SNR with a high probability of detection.

9. Conclusion

In this paper, a CORDIC-Jacobi based spectrum sensing algorithm is proposed. The algorithm can obtain the maximum and minimum eigenvalues of sampled covariance matrix of received signal mainly by shift and additional operations. Latest random matrix theories have been employed to set the decision threshold according to a given probability of false alarm. Simulations based on digital modulated QAM signals showed the method can detect the absence and presence of primary signal effectively. Real captured DTV signal and digital modulated QAM signal are applied to evaluate the performance of proposed method. All the simulations and experiments show that the method works well without using the information of signal, channel and noise.

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