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Comparison Between DCM and Quaternion Transformation in Lever Arm Compensation of Reference System for Flight Performance Evaluation of DGPS/INS

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ABSTRACT

The flight performance evaluation of navigation system is very significant because the reliability of navigation data directly affect the safety of aircraft. Especially, the high-level navigation system such as DGPS/INS, need more precise flight performance evaluation method. The performance analysis is evaluated by comparing between the navigation system in aircraft and reference trajectory which is more precise than navigation system in aircraft. In order to verify DGPS/INS performance of m-level, the GPS receiver, which is capable post-processed Carrier-phase Differential GPS(CDGPS) method of cm-level, have to be used as reference system. The DGPS/INS is estimated the Center of Gravity (CG) point of aircraft to offer precise performance while the reference system is output the position of GPS antenna which is mounted on the outside of aircraft. Therefore, in order to more precise performance evaluation, it needs to compensate the lever arm and coordinates transformation. This paper use quaternion and Direct Cosine Matrix(DCM) methods as coordinate transformation matrix in lever arm compensation of CDGPS reference trajectory. And it compares NED errors of DCM and quaternion transformation in lever arm of reference trajectory via DGPS/INS result.

Keywords: aircraft, DGPS/INS, lever arm, quaternion, direct-cosine matrix

1. INTRODUCTION

The flight test is an important final test for the aircraft development to confirm the safety and reliability of the aircraft (Lee et al. 2009). It is also important to evaluate the performance of the navigation system through the flight test because it directly affects the safety of the aircraft. In recent years, a number of studies have been conducted to provide a reliable and precise navigation accuracy and thus a precise navigation performance evaluation method is necessary to verify the high-level navigation systems such as the Differential-GPS (DGPS)/INS integrated navigation system.

The flight navigation performance is evaluated by comparing the navigation system of the aircraft to the reference trajectory which has a higher navigation accuracy. Hence, to evaluate the performance of the DGPS/INS integrated navigation system (Farrell 2000, Redmill et al. 2001) which applies the DGPS method for several m-level position accuracy, the GPS receiver that is capable of the Carrier-phase Differential GPS (CDGPS) post-processing method (Park 2001) for precise several cm-level position accuracy needs to be used for generating the reference trajectory.

For providing the precise cm-level navigation data, the DGPS/INS, that is installed in medium and large aircraft, outputs the position of aircraft's center of gravity by compensating the lever arm error (Hong et al. 2006) which is one of the major error factors. Therefore, the CDGPS reference trajectory which outputs the position of the GPS antenna also needs to compensate the distance between the GPS antenna and the position of aircraft's center of gravity, the lever arm. In this case, the coordinate transformation

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process is necessary to match the coordinates because the lever arm coordinate is the body coordinate system while the CDGPS reference trajectory is the geodetic coordinate system. In this paper, the CDGPS reference trajectory after the lever arm compensation was used to verify the flight navigation performance of the DGPS/INS navigation system precisely, and for the coordinate transformation of the lever arm compensation, the widely used methods, quaternion and Direct Cosine Matrix (DCM) were utilized (Shin et al. 2006). And through the flight test, the NED position error was compared between the DGPS/INS integrated navigation results and the reference trajectory after the lever arm compensation with the application of each coordinate transformation method. In Section 2, the coordinate transformation method and lever arm compensation procedure are described, and in Section 3, the flight navigation performance evaluation procedure is explained. In Section 4, the test results are presented, and Section 5 is the conclusion.

2. LEVER ARM COMPENSATION

2.1. Coordinate Transformation Method

The quaternion method and DCM method are the typical coordinate transformation methods. The DCM carrys out the coordinate system transformation in the form of a matrix using 9 parameters, and the quaternion performs the coordinate transformation in the form of a differential equation using 4 parameters (Shin et al. 2006, Cai et al. 2011, Titterton & Weston 2004). When the DCM which has the order of Z-Y-X is used for transforming the navigation coordinate system into the body coordinate system, the coordinate transformation matrix C_N^B is as follows, where $\cos\psi$ is c_ψ and $\sin\psi$ is s_ψ (Cai et al. 2011, Titterton & Weston 2004).

$$\begin{aligned} C_N^B &= C_\varphi \cdot C_\theta \cdot C_\psi \\ &= \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \\ &= \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ -c_\varphi s_\psi + s_\varphi s_\theta c_\psi & c_\varphi c_\psi + s_\varphi s_\theta s_\psi & s_\varphi c_\theta \\ s_\varphi s_\psi + c_\varphi s_\theta c_\psi & -s_\varphi c_\psi + c_\varphi s_\theta s_\psi & c_\varphi c_\theta \end{bmatrix} \end{aligned} \quad (1)$$

where,

$$C_\psi = \begin{bmatrix} c_\psi & s_\psi & 0 \\ -s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_\theta = \begin{bmatrix} c_\theta & 0 & -s_\theta \\ 0 & 1 & 0 \\ s_\theta & 0 & c_\theta \end{bmatrix}$$

$$C_\varphi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\varphi & s_\varphi \\ 0 & -s_\varphi & c_\varphi \end{bmatrix}$$

In the case of the quaternion, the coordinate transformation matrix is as follows (Titterton et al. 2005).

$$C_N^B = \begin{bmatrix} e_x^2 + e_0^2 - e_y^2 - e_z^2 & 2(e_x e_y + e_z e_0) & 2(e_x e_z - e_y e_0) \\ 2(e_x e_y - e_z e_0) & e_y^2 + e_0^2 - e_x^2 - e_z^2 & 2(e_y e_z + e_x e_0) \\ 2(e_x e_z + e_y e_0) & 2(e_y e_z - e_x e_0) & e_z^2 + e_0^2 - e_x^2 - e_y^2 \end{bmatrix} (2)$$

where,

e_0 = scalar component of Euler-Rodrigues quaternion

e_x = x component of Euler-Rodrigues quaternion

e_y = y component of Euler-Rodrigues quaternion

e_z = z component of Euler-Rodrigues quaternion

2.2. Lever Arm Compensation

Based on the geodetic coordinate system, the Eq. (3) shows the relative distance error between the position result from the DGPS/INS integrated navigation system and the position result estimated from the CDGPS reference trajectory. And based on the body coordinate system, the Eq. (4) shows the lever arm which is the distance between the position of GPS antenna and the Center of Gravity (CG) Point of the aircraft. As the Eqs. (3) and (4) have different coordinate systems, the coordinates need to be matched before the lever arm compensation.

$$\hat{e}_G = (\hat{P}_{REF} - \hat{P}_{EGI})_G \quad (3)$$

$$L_B = (P_{CG} - P_{GPS\ Ant})_B \quad (4)$$

where,

$$\hat{P}_{EGI} = (P_{CG} + e_{EGI})_G$$

$$\hat{P}_{REF} = (P_{GPS\ Ant} + e_{REF})_G$$

$(\ )_G$ is the geodetic coordinate system, $(\ )_B$ is body coordinate system, and $(\ )_N$ represents the navigation coordinate system. $\hat{P}_{EGI}$ and $\hat{P}_{REF}$ are the output positions of the DGPS/INS integrated navigation system and CDGPS reference trajectory, respectively. And PCG is the position of aircraft's center of gravity and P_{GPSAnt} is the position of GPS antenna. e_{EGI} and e_{REF} are the actual errors of the DGPS/INS and CDGPS reference trajectory, respectively.

Fig. 1 shows the procedure of the lever arm compensation. First, the lever arm data of Eq. (4) are transformed from the body coordinate system to the navigation coordinate system as in Eq. (5) using the inverse matrix of the

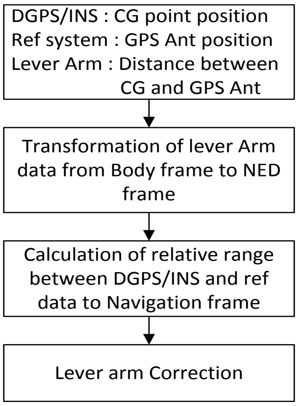


Fig. 1. Procedure of lever arm compensation.

coordinate transformation matrix Eqs. (1) or (2) (Cai et al. 2011, Titterton & Weston 2004). In this case, the roll, pitch, and yaw information from the outputs of the DGPS/INS integrated navigation system are used as the attitude information of the aircraft which are the parameters necessary for the coordinate transformation.

$$L_N = \left(C_N^B\right)^{-1} L_B = C_B^N\, L_B \tag{5}$$

And the relative distance error estimated from Eq. (3) is calculated into the navigation coordinate system as in Eq. (6). The data necessary for transforming the geodetic coordinate system into the navigation coordinate system are the radius of curvature in the prime vertical (R_N), radius of curvature in the meridian (R_M), and the variables for the WGS 84, semi-major axis (R) and flattering (f) of the Earth (Cai et al. 2011, National Imagery and Mapping Agency 1984).

$$d\hat{e}_N = \begin{bmatrix} dN \\ dE \\ dD \end{bmatrix} = \begin{bmatrix} \dfrac{d\mu}{\mathrm{atan}\left(\frac{1}{R_M}\right)} \\ \dfrac{dl}{\mathrm{atan}\left(\frac{1}{R_N \cos\mu_0}\right)} \\ -dh \end{bmatrix} \tag{6}$$

where,

$$R_N = \frac{R}{\sqrt{1-(2f-f^2)\sin^2\mu_0}}$$
$$R_M = R_N \frac{1-(2f-f^2)}{\sqrt{1-(2f-f^2)\sin^2\mu_0}}$$
$d\mu = \mu - \mu_0$: latitude difference
$dl = l - l_0$: longitude difference
$dh = h - h_0$: height difference
$1/f = 298.257223563$
$R = 6378137$ m

Next, the final NED position error can be estimated by eliminating the lever arm error of the navigation coordinate system obtained from Eq. (5), with respect to Eq. (6).

$$de_N = d\hat{e}_N + L_N, \tag{7}$$

3. FLIGHT PERFORMANCE EVALUATION PROCEDURE

3.1. Flight Test Environment and Procedure

For performance evaluation, the CDGPS reference trajectory was generated by the DL-V3 of NovAtel Inc. which is the high precision navigation equipment GPS receiver. The DL-V3 provides a precise navigation solution of 5mm when the CDGPS post-processing method is used.

Fig. 2 shows the block diagram of the equipments for the flight test. The interior of the aircraft is equipped with the DGPS/INS integrated navigation system which is the target for the performance evaluation, and the DL-V3 which is the rover station (RS) for obtaining the reference trajectory. And the control tower is euipped with the base station (BS). Using the GPS signal distributor, the GPS antenna was shared between the DGPS/INS integrated navigation equipments and the rover station DL-V3. With the use of the jig, the rover station DL-V3 was installed at the mounting angle which is equipped with the DGPS/INS. In addition, for the CDGPS post-processing, the base station DL-V3 was established which receives the signal from the GPS antenna installed at the location where the geodetic survey was previously carried out.

While checking the aircraft prior to the flight test, the inspection and alignment of the integrated navigation

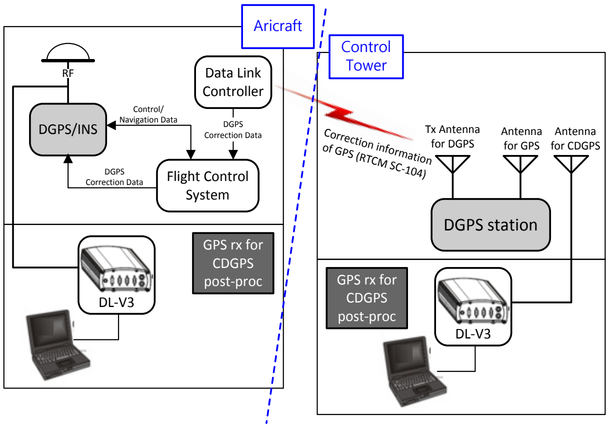


Fig. 2. Block diagram of flight test equipments.

system were performed. The alignment was carried out using the accelerometer and gyroscope, and after the alignment, the flight test was performed at the navigation mode. During the flight test, the raw GPS measurements information for the reference trajectory was received on the ground by operating the base station DL-V3 at the location where the geodetic survey was previously carried out.

3.2. Navigation Performance Evaluation Procedure

Fig. 3 shows the performance evaluation procedure for the DGPS/INS integrated navigation system following the flight test. Utilizing the commercial CDGPS post-processing software of NovAtel Inc., the CDGPS reference trajectory of the GPS antenna position was acquired by processing the raw GPS measurements obtained from the rover station DL-V3 installed in the aircraft and the base station DL-V3 installed at the control tower. For the acquired CDGPS reference trajectory, the NED position error was calculated relative to the DGPS/INS integrated navigation system. For this calculated NED position error, the lever arm compensation was performed as explained in section 2 using the attitude data obtained from the DGPS/INS, and in this way the final NED position error was estimated.

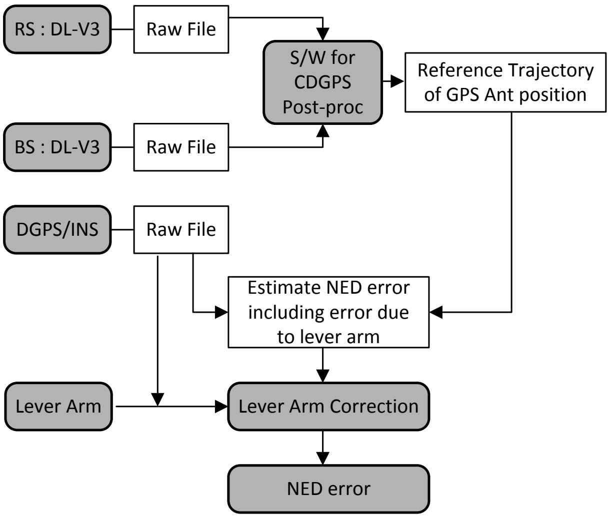


Fig. 3. Procedure of flight performance evaluation.

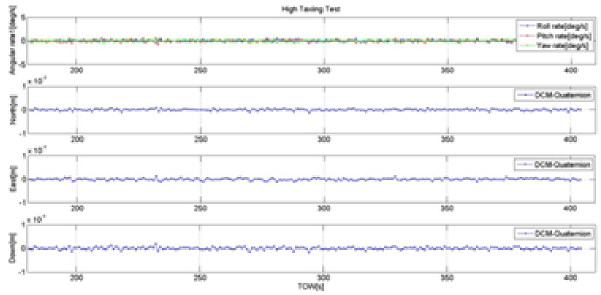
Table 1. Range of angular rate (deg/s).

	Roll rate	Pitch rate	Yaw rate
High Taxiing	0	0	0
Flight Test #1	±5	±3	±5
Flight Test #2	±8	±4	±6
Flight Test #3	±30	±12	±15

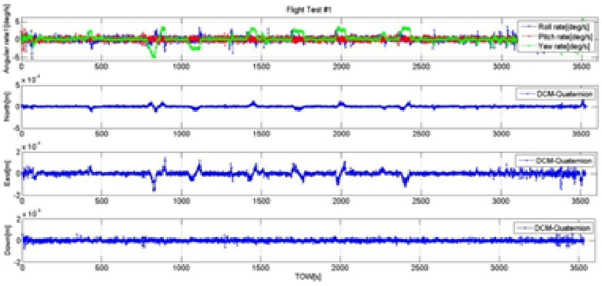
4. FLIGHT TEST RESULT

One high taxiing test and three flight tests were performed, and Table 1 summarizes the range of the angular rate for the aircraft's roll and pitch at each flight test.

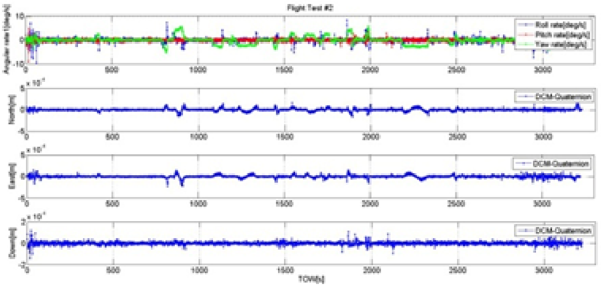
Fig. 4 shows the angular rate and the difference of the NED errors for DCM and Quaternion when the lever arm compensation of the reference trajectory is performed relative to the DGPS/INS integrated results during high



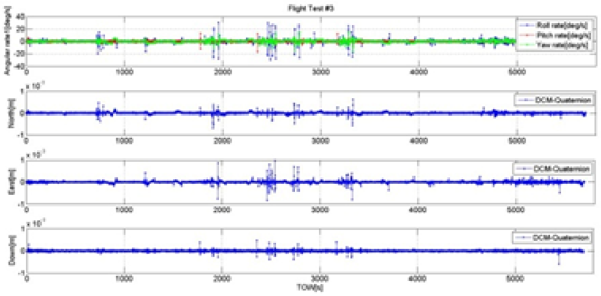
(a) High Taxiing test



(b) Flight test #1



(c) Flight test #2



(d) Flight test #3

Fig. 4. NED position error difference between quaternion and DCM.

taxiing and flight tests. Fig. 4a is the result of the high taxiing test and the variations of the aircraft's roll and pitch are close to 0 deg/s as shown in Table 1. In this case, the difference between the two methods is less than 10-5m, which is almost negligible. Figs. 4b,c, and 4d show the results of the flight test. The difference between the two methods is almost negligible similar to the result of the high taxiing test, but because of the change in the roll axis, (b) has the difference of less than 0.2 mm, (c) has the difference of less than 0.3 mm, and (d) has the difference of less than 1 mm. The difference induced by the roll axis appears distinct because the aircraft typically has more frequent change in the roll axis with higher angular rate compared to the pitch or yaw axes. If the variations in the pitch and yaw axes increase at a flying vehicle other than the aircraft, the difference induced by the pitch and yaw axes is expected to occur.

5. CONCLUSION

In this paper, the CDGPS reference trajectory after the lever arm compensation was used to verify the flight navigation performance of the DGPS/INS navigation system precisely, and for the coordinate transformation of the lever arm compensation, the widely used methods, quaternion and DCM (Shun et al. 2006) were utilized and compared for the performance difference. And through the high taxiing and flight tests, the NED error components were compared between the DCM and Quaternion which are used for lever arm compensation of reference trajectory. The results of the two methods were generally similar, but the difference between the methods occurred because of the change in the roll axis, which showed more difference at higher roll rate. In addition, as the difference was larger at higher variation in the roll axis, the difference is expected to be larger and more frequent with highr maneuverability of the flying vehicle. The error is negligible in terms of the navigation, but the difference occurred for the two methods depending on the maneuverability and therefore these effects need to be considered in accordance with the requirements for the maneuverability of the flying vehicle and applications.

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