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Performance Analysis of a Vector DLL Based GPS Receiver

Deok Won Lim[1†], Heon Ho Choi[2], Sang Jeong Lee[2], Moon Beom Heo[1]
[1]Satellite Navigation Team, Korea Aerospace Research Institute, Daejeon 305-333, Korea
[2]Department of Electronics Engineering, Chungnam National University, Daejeon 305-764, Korea

ABSTRACT

For a Global Positioning System (GPS) receiver, it is known that a Vector Delay Locked Loop (DLL) in which the code signals of each satellite are tracked in parallel by using navigation results shows better performance in the aspect of the tracking accuracy and the robustness than that of a Scalar DLL. However, the quantitative analysis and the logical grounds for that performance enhancement of the Vector DLL are not sufficient. This paper, therefore, proposes the structure of the GPS receiver with the Vector DLL and analyzes the performance of it. The tracking and the positioning accuracy of the Vector DLL are theoretically analyzed and confirmed by simulation results. From the simulation results, it can be seen that the tracking and positioning accuracy has been improved about 30% in case that the receiver is static and the positioning is conducted for every Pre-detection Integration Time (PIT) while C/N0 is 45 dB-Hz.

Keywords: GPS, signal tracking, delay locked loop, Vector DLL

1. INTRODUCTION

A Vector Delay Locked Loop (DLL) technique which utilizes the navigation result and controls all Numerically Controlled Oscillator (NCOs) in parallel was firstly proposed by Spilker (Parkinson & Spilker 1996). And he observed that jamming immunity of a Vector DLL is superior to that of a Scalar DLL, and a GPS receiver with a Vector DLL can maintain operation although some of satellites are not being seen temporarily and it is more robust for dynamic stress than that with a Scalar DLL (Spilker 1995).

On the back of a research of Spilker, there are many researches for a Vector DLL until these days. Some of those are researches for GPS/INS integrated systems (Gustafson et al. 2000, Abbott & Lillo 2003, Landis et al. 2006, Petovello & Lachapelle 2006) and others are researches for weak signals environments (Pany et al. 2005, Benson 2007, Lashley & Bevly 2007). Most of above researches, however, did not analyze the performance of a Vector DLL with their structure, but give us experimental results. For example, Petovello just showed a position result, Pany showed a position result and the number of observations although a position result is not a related index with the performance of a tracking loop. Lashley showed the code phase error which is a related index with tracking performance, but there is no analysis of relation between the structure and the code phase error.

This paper, therefore, proposes the structure of the GPS receiver with the Vector DLL and theoretically analyzes the tracking and the positioning accuracy based on the performance index. And the tracking and the positioning accuracy of the receiver with the Vector DLL are confirmed by simulation results.

2. PERFORMANCE ANALYSIS OF A SCALAR DLL BASED GPS RECEIVER

2.1 Structure of a Scalar DLL Based GPS Receiver

A functional block diagram of a Scalar DLL based GPS receiver which consists of the signal tracking block and the navigation block is shown in Fig. 1. As shown in Fig. 1, the

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†Corresponding Author
E-mail: dwlim@kari.re.kr
Tel: +82-42-870-3978 Fax: +82-42-860-2789

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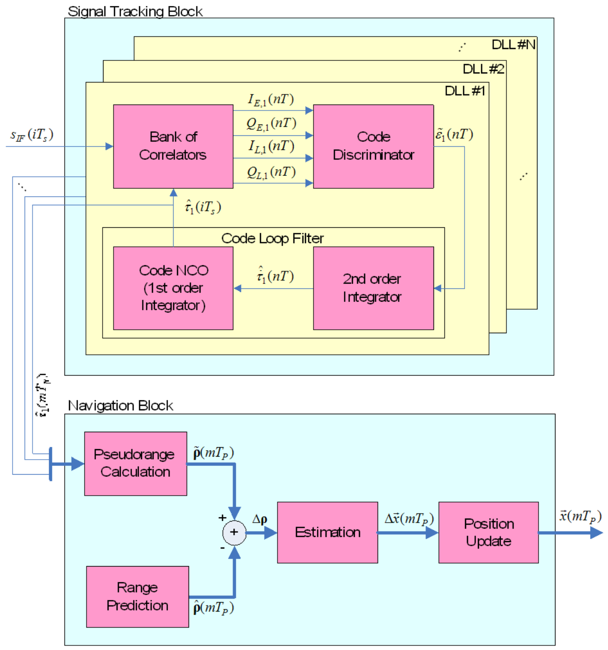

Fig. 1. Structure of a Scalar DLL based GPS receiver.

DLL is only illustrated in the signal tracking block and it consists of the bank of correlators, a code discriminator and a code loop filter including a code NCO. In the navigation block, the pseudoranges are calculated by using the code phase measurement of each DLL, and the range residuals are calculated for updating the position.

2.2 Tracking Accuracy of a Scalar DLL

The accuracy of a Scalar DLL is analyzed for the structure shown at Fig. 1. Firstly, the correlator's outputs which mean the correlation power represented in Watt are modeled as Eqs. (1-4) (Van Dierendonck et al. 1992):

$$I_{E,l}(nT)=\sqrt{2C/N_0T}\,R(\varepsilon_l(nT)-d/2)\cos(\phi_l(nT))+w_{IE}(nT) \tag{1}$$

$$Q_{E,l}(nT)=\sqrt{2C/N_0T}\,R(\varepsilon_l(nT)-d/2)\sin(\phi_l(nT))+w_{QE}(nT) \tag{2}$$

$$I_{L,l}(nT)=\sqrt{2C/N_0T}\,R(\varepsilon_l(nT)+d/2)\cos(\phi_l(nT))+w_{IL}(nT) \tag{3}$$

$$Q_{L,l}(nT)=\sqrt{2C/N_0T}\,R(\varepsilon_l(nT)+d/2)\sin(\phi_l(nT))+w_{QL}(nT) \tag{4}$$

where C/N_0 is the carrier power to noise density ratio in the linear scale, T is the Pre-detection Integration Time (PIT), $R(\cdot)$ is the auto-correlation function of the GPS L1 C/A code, $\varepsilon_l(nT)$ is the true code phase difference between the input and replica, d is the chip space between early to late, $\phi_l(nT)$ is the carrier phase difference between the input and replica, and w_{IE}, w_{QE}, w_{IL}, w_{QL} are the noises whose means are zero, and variances are one.

In case of using the early minus late power discriminator, the variance of the output of l-th discriminator is given by (Van Dierendonck et al. 1992)

$$Var[\tilde{\varepsilon}_l(n)]\Big|_{\varepsilon_l(n)=0}=4d(2-d)\left[2+(2-d)C/N_0T\right], \tag{5}$$

and output of l-th discriminator for the first order feedback loop is given by (Van Dierendonck et al. 1992)

$$\sigma_l^2\equiv Var[\tilde{\varepsilon}_l(n)]\Big|_{\varepsilon_l(n)=0,closed}=\frac{B_nd}{2C/N_0}\left[1+\frac{2}{(2-d)C/N_0T}\right], \tag{6}$$

where B_n is the loop noise bandwidth. From Eq. (6), it can be found that the higher C/N_0 and lower B_n give better tracking performance.

The standard deviation of the code tracking error by the dynamics of the satellite and the receiver is given by (Ward et al. 2006)

$$R_e=\frac{d^nR/d^nt}{\omega_0^n}, \tag{7}$$

where R is the range between a satellite and a receiver, n is the loop filter order, and ω_0 is the loop filter natural radian frequency.

The total code-tracking error for the third-order loop including the dynamic stress error (R_e) can be represented as (Ward et al. 2006)

$$\sigma_{SDLL}=\sigma_l+\frac{1}{3}R_e=\sqrt{\frac{B_nd}{2C/N_0}\left[1+\frac{2}{(2-d)C/N_0T}\right]}+\frac{1}{3}\frac{d^3R/dt^3}{\omega_0^3}, \tag{8}$$

where d^3R/dt^3 is the Line-of-sight jerk dynamics. The

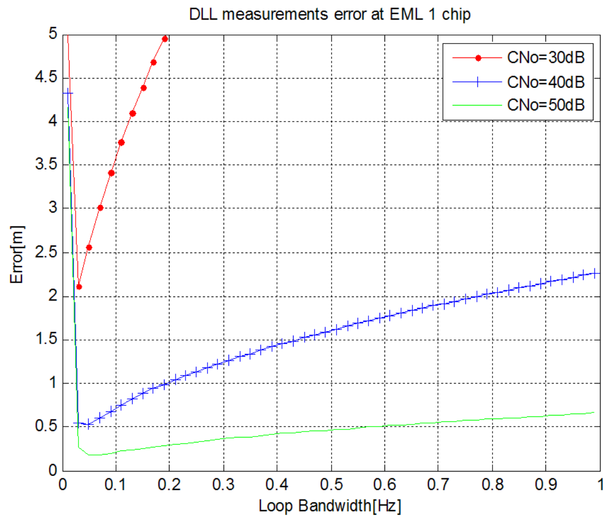

Fig. 2. Code Tracking Error according to Loop Noise Bandwidth.

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reason why the thermal noise term is represented for the first-order loop in (6) is that the thermal noise error does not much depend on the loop order and the loop order is mainly sensitive to the same order of dynamics.

By using Eq. (8), the code tracking errors according to loop noise bandwidth and C/N_0 are given in Fig. 2.

3. PERFORMANCE ANALYSIS OF A VECTOR DLL BASED GPS RECEIVER

3.1 Structure of a Vector DLL Based GPS Receiver

A functional block diagram of a Vector DLL based GPS receiver is shown in Fig 3, and it has both of a Scalar DLL and a Vector DLL. Because the Vector DLL requires the initial positioning results based on the Scalar DLL to control the code NCO. Therefore, the receiver starts with a Scalar DLL, and then the receiver switches the tracking loop to a Vector DLL after finding receiver's position.

Comparing the Fig. 3 with Fig. 1, it can be seen that the 'Pseudorange Calculation Block' has been modified and the 'Phase Error Calculation Block' is added. In the 'Pseudorange Calculation Block', firstly, the pseudorange of l-th satellite can be calculated by

$$\rho_l(mT_P) = \rho_l(mT_P - T_P) + \Delta\rho_l(mT_P) = \rho_l(mT_P - T_P) + CT_C \cdot \sum_{k=n-T_P/T+1}^{n} \tilde{\varepsilon}_l(kT), \tag{9}$$

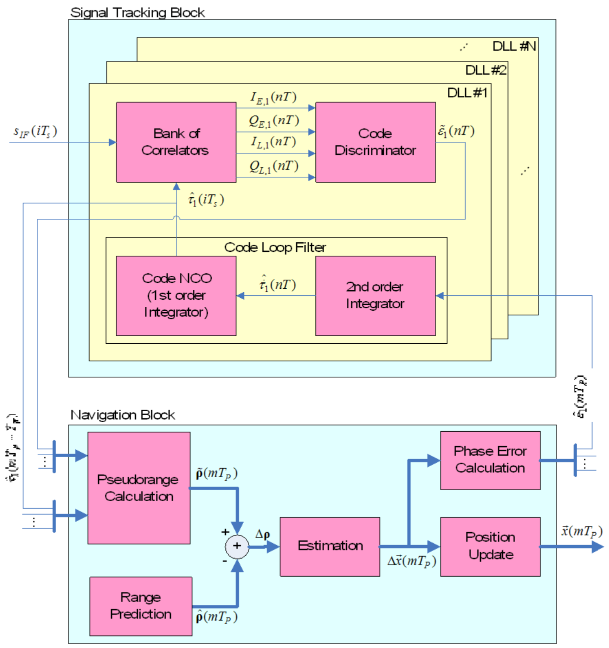

Fig. 3. Structure of a Vector DLL based GPS receiver.

where T_P is the interval of positioning, C is the speed of light, T_C is the chip duration of the C/A code and $\tilde{\varepsilon}_l(kT)$ is the output of l-th discriminator at kT. In the 'Phase Error Calculation Block', secondly, the code phase error is calculated as

$$\begin{bmatrix} \tilde{\varepsilon}_1(mT_P) \\ \tilde{\varepsilon}_2(mT_P) \\ \vdots \\ \tilde{\varepsilon}_N(mT_P) \end{bmatrix} = (\frac{1}{CT_C}) \cdot H \cdot \Delta\vec{x}(mT_P), \tag{10}$$

where N is the number of tracking signals, H is the line of sight vector.

3.2 Tracking Accuracy of a Vector DLL

In a vector DLL, the tracking error can be given by

$$\boldsymbol{\sigma}_{VDLL}^2 = \left(\frac{1}{CT_C}\right)^2 H \operatorname{cov}(d\vec{x}) H^T, \tag{11}$$

because the code phase error is calculated by the position increments as shown in Fig. 3. And the position error in Eq. (11) can by calculated by (Ward et al. 2006)

$$\operatorname{cov}(d\vec{x}) = (H^T H)^{-1} H^T \operatorname{cov}(d\boldsymbol{\rho}) H (H^T H)^{-1}. \tag{12}$$

From Eq. (9), the pseudorange errors of N satellites can be represented as

$$\operatorname{cov}(d\boldsymbol{\rho}) = \begin{pmatrix} \operatorname{var}(\Delta\rho_1) & 0 & \cdots & 0 \\ 0 & \operatorname{var}(\Delta\rho_2) & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \operatorname{var}(\Delta\rho_N) \end{pmatrix}, \tag{13}$$

and the variance of the delta pseudorange ($\Delta\rho_l$) can be calculated as

$$\operatorname{var}(\Delta\rho_l) = \left({}^{C}\!/_{T_C}\right)^2 \cdot \operatorname{var}\left(\sum_{k=n-T_P/T+1}^{n} \tilde{\varepsilon}_l(kT)\right) = \left({}^{T_P}\!/_{T}\right)(CT_C)^2 \operatorname{var}(\tilde{\varepsilon}_l(kT)) = \left({}^{T_P}\!/_{T}\right)(CT_C)^2 \boldsymbol{\sigma}_{SDLL}^2, \tag{14}$$

by using the statistical properties of the variance operation. Finally, the variance of a Vector DLL is given as

$$\boldsymbol{\sigma}_{VDLL}^2 = \left({}^{T_P}\!/_{T}\right) H (H^T H)^{-1} H^T \boldsymbol{\sigma}_{SDLL}^2 H (H^T H)^{-1} H^T, \quad T_P \geq T. \tag{15}$$

Through this result, it can be found that the tracking accuracy of a Vector DLL depends on PIT and interval of positioning and satellite's geometry. In case that $T_P = T$, it can be expected that the tracking error of the Vector DLL is smaller than that of the Scalar DLL because the components of H is naturally smaller than 1.

4. SIMULATION RESULTS

4.1 Simulation Environment

The goal of the simulation was to validate the theoretical analysis and to compare the accuracy of the Vector DLL with those of Scalar DLL. In a point of the accuracy, the tracking and positioning accuracy were compared for static case.

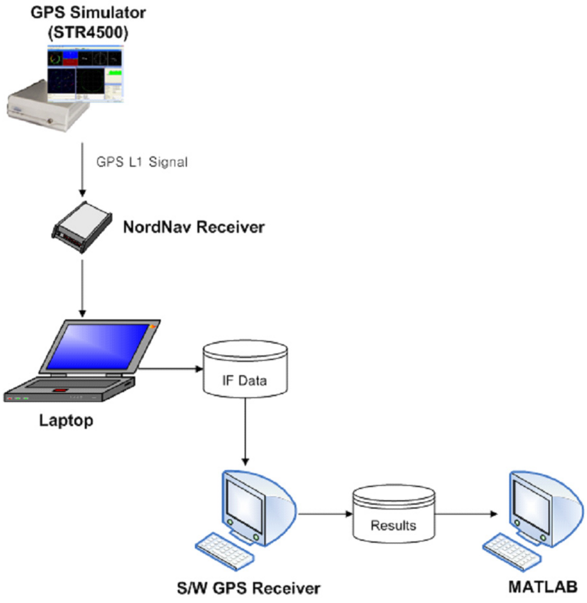

Fig. 4. Simulation Environment.

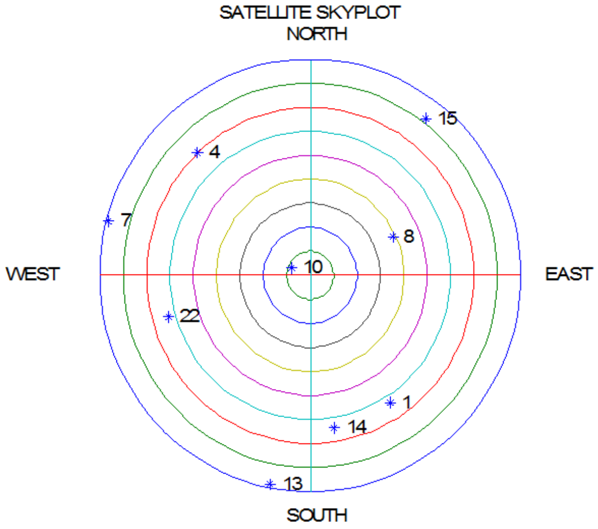

Fig. 5. Sky plot for the visible satellites.

An environment for the simulation is shown in Fig. 4. As shown in Fig. 4, the GPS RF signal has been generated by the Spirent simulator and IF data have been collected for 120 seconds by the RF front-end of the Nordnav receiver. And then they are processed in a software GPS receiver with the tracking loop of a Scalar DLL and a Vector DLL and the positioning algorithm of Least Squares. Finally, the processed results of the receiver are presented by the MATLAB.

The satellites allocation for this simulation is given in Fig. 5. As shown in Fig. 5, there are 9 satellites and PDOP is 1.64. And the receiver design parameters are also given in Table 1.

4.2 Simulation Results

As shown in Fig. 6, the tracking accuracy is compared by using Eqs. (8) and (15), and then the horizontal positioning error when the C/N_0 is 45 dB-Hz and $T_P = T$ is compared as shown in Fig. 7. From these results in Fig. 6, it can be check that the tracking error of the Vector DLL is smaller

Table 1. Design parameters of the receiver.

Parameters	Value
C/N0	45 [dB-Hz]
Early to Late chip space	1 [chip]
DLL Loop noise bandwidth	1 [Hz]
DLL Order	3rd
FLL Loop noise bandwidth	5 [Hz]
FLL order	2nd
Pre-detection integration time	1 [ms]

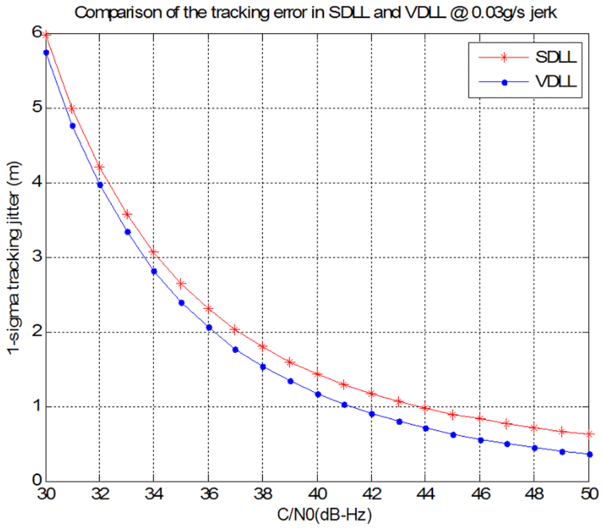

Fig. 6. Comparison of the positioning accuracy in static case.

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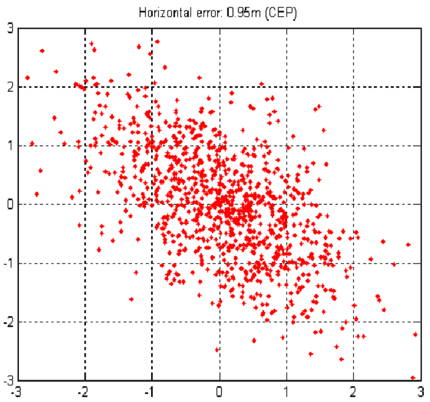
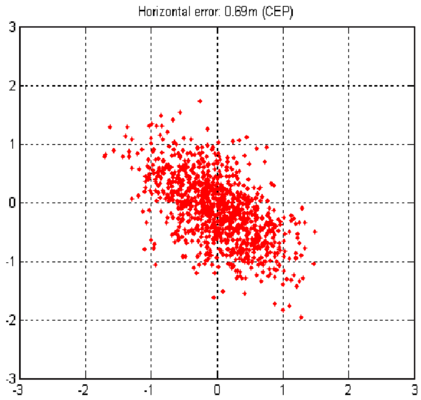


(a) GPS receiver with the Scalar DLL



(b) GPS receiver with the Vector DLL

Fig. 7. Comparison of the positioning accuracy in static case.

than that of the Scalar DLL for all C/N0 when a receiver is static in which the jerk is 0.03 g/s by the motion of the satellite. From Fig. 7, it can be also checked that the positioning error of the GPS receiver with the Scalar DLL and Vector DLL are given as 0.95 m (CEP) and 0.69 m (CEP), respectively. By these results, it can be concluded that the tracking and positioning performance of the receiver with the Vector DLL are improved about 30% than those of the receiver with the Scalar DLL when the receiver is static and PDOP is 1.64.

Through this paper, eventually, it is confirmed that the performance of the GPS receiver with the Vector DLL is superior to the receiver with the Scalar DLL for specific environments, and performance expectation can be roughly done for a given environment.

5. CONCLUSIONS

In this paper, performance of the GPS receiver with a Vector DLL has been analyzed in a point of the tracking accuracy, and it also compared with that of the receiver with the Scalar DLL in a point of the positioning accuracy for the static case. From the results, it is confirmed that the tracking and positioning accuracy of the Vector DLL has been improved about 30% when a receiver is static and $T_P = T$ while C/N_0 is 45 dB-Hz and the PDOP is 1.64. Based on these observations, eventually, the performance expectation can be roughly done for a given environment.

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