# Bicubic Splines in Problems of Modeling of Multidimensional Signals

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***Abstract*— The paper is devoted to problem of spline modeling of multidimensional signals. A new method of nodes location for curves and surfaces computer construction in multidimensional spaces by means of B-splines is presented. The criteria are which links a square-mean error caused by high frequency spline distortions and approximation intervals is determined and necessary theorem is proved. In this method use a theory of entire multidimensional spectra and may be extended for the spaces of three, four and more variables.**

***Index Terms*— Signal, Basis, Approximation, Multidimensional, Bicubic splines.**

## I. INTRODUCTION

**MODERN** methods of construction of mathematical models and progress in the field of technologies of computer systems and complexes are a basis that physicists and engineers could investigate difficult multidimensional processes and objects of the real world with high degree of efficiency.

Splines as a class of piecewise functions owing to universality of algorithms of processing of readout, good differential and extreme properties, high convergence of estimations approximation, simplicity of calculations of forms and parameters, weak influence of errors of a rounding off find more and more wide application at creation of equipment rooms and software of the analysis and restoration of one-dimensional and multidimensional signals, expanding frameworks of traditional approaches [1,2,3].

## II. MULTIDIMENSIONAL B-SPLINES

Now modern computers allow to receive without vital issues full enough piecewise - polynomial descriptions of functions f (x, y) in three-dimensional space.

The multidimensional spline is an example functions of several variables when the combination of functions is set, each of which depends on one independent variable. Multidimensional basic splines which not only can be presented in the form of the sums of products of functions of one variable in this case can be allocated, but give transformation of Fure in the form of accurate analytical expression on the basis of elementary functions of frequency arguments[4].

Multidimensional polynomial B-splines of equal degrees m on each argument are defined in a kind tensor products of one-dimensional B-splines [3,4,5].

$$B_m(x,y,...,u) = B_m(x)\otimes B_m(y)\otimes ...\otimes B_m(u) \quad (1)$$

Thereof record multidimensional polynomial S-spline takes a form:

$$S_m(x_1,x_2,...,x_n = \sum_{i=-m}^{n_1+m}\sum_{k=-m}^{n_2+m}...\sum_{l=-m}^{n_n+m} b_{ikl}B_{i,m}(x_1)B_{k,m}(x_2)..B_{l,m}(x_n) \quad (2)$$

Where - $b_{ik...l}$ coefficients of multidimensional approximation.

In particular, for two-dimensional spline $S_m$ (x, y) degrees m the formula takes place:

$$S_m(x,y) = \sum_{i=-m}^{n_1+m}\sum_{k=-m}^{n_2+m} b_{ik}B_{m,i}(x)B_{m,k}(y) \quad (3)$$

The formula (3) is in the form of the double sums of multiple products where factors are coefficients and one-dimensional B-splines. Here range of definition of nonzero values of a two-dimensional basic spline.

$$B(x,y) = B_m(x)\otimes B_m(y) \quad (4)$$

Represents a rectangle $[x_i, x_{i+1}; y_k, y_{k+1}]$ received from net splitting of a following kind:

$$\Delta_x : x_0 < x_1 < x_2 < ... < x_{n1-1} < x_{n1};$$
$$\Delta_y : y_0 < y_1 < y_2 < ... < y_{n2-1} < y_{n2}. \quad (5)$$

Fig. 1.Bicubic B-spline of third degree $B_{3,3}$ (x, y) is resulted.

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Manuscript received July 21, 2011; revised August 2, 2011; accepted August 10, 2011.

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The Series-parallel algorithm of calculation of values of multidimensional splines can be realized if to enter designations for the coefficients depending on one of arguments [4]. In particular, for a case of two independent arguments it is had:

$$C_{ik}(y) = \sum_k b_{ik} B_k(y), \quad (6)$$

and expression for a spline will register in a kind:

$$S_m(x, y) = \sum_k c_i(y) B_k(x), \quad (7)$$

The $S_m$ (x, y) can be calculated through pair products in two stages.

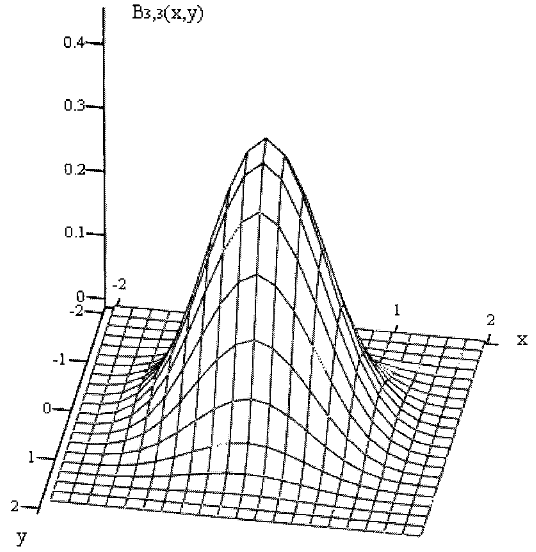


Fig. 1. A two-dimensional basic B-spline of the third degree.

For bicubic B-spline the formula (2) looks like a spline:

$$S_{3,3}(x, y) = \sum\sum b_{ij} B_{3,i}(x) B_{3,j}(y), \quad (8)$$

Local formulas of calculation of coefficients can be extended and to cases of multidimensional approximation. For example, 3-dot formulas for bicubic B-spline on uniform grids Δx and Δy can be received on the basis of formulas for one-dimensional splines [6, 7]:

$$\begin{aligned} a_{ij} &= (-f_{i-1j} + 8f_{ij} - f_{i+1j})/6, \\ & i = 1,2,\ldots,n_1 - 1; \\ b_{ij} &= (-a_{i,j-1} + 8a_{ij} - a_{i,j+1})/6, \\ & j = 1,2,\ldots,n_2 - 1; \end{aligned} \quad (9)$$

For an example we take approximation of function of two variables bicubic B-spline, i.e. m = 3. Expression (3) will be transformed to a kind:

$$\begin{aligned} f(x,y) &\equiv S_{3,3}(x,y) = \\ &= b_{i-1,j-1}B_{i-1}(x)B_{j-1}(y) + b_{i-1,j+0}B_{i-1}(x)B_{j+0}(y) + \\ &+ b_{i-1,j+1}B_{i-1}(x)B_{j+1}(y) + b_{i-1,j+2}B_{i-1}(x)B_{j+2}(y) + \\ &+ b_{i+0,j-1}B_{i+0}(x)B_{j-1}(y) + b_{i+0,j+0}B_{i+0}(x)B_{j+0}(y) + \\ &+ b_{i+0,j+1}B_{i+0}(x)B_{j+1}(y) + b_{i+0,j+2}B_{i+0}(x)B_{j+2}(y) + \\ &+ b_{i+1,j-1}B_{i+1}(x)B_{j-1}(y) + b_{i+1,j+0}B_{i+1}(x)B_{j+0}(y) + \\ &+ b_{i+1,j+1}B_{i+1}(x)B_{j+1}(y) + b_{i+1,j+2}B_{i+1}(x)B_{j+2}(y) + \\ &+ b_{i+2,j-1}B_{i+2}(x)B_{j-1}(y) + b_{i+2,j+0}B_{i+2}(x)B_{j+0}(y) + \\ &+ b_{i+2,j+1}B_{i+2}(x)B_{j+1}(y) + b_{i+2,j+2}B_{i+2}(x)B_{j+2}(y), \end{aligned} \quad (10)$$

i.e. the spline is calculated in the form of the sum from 16 members, each of which represents product of three factors generated at combinations of coefficients and one-dimensional B-splines on four sites adjoining to each other of each of arguments (flowing with numbers i and j, previous in relation to them and on two the subsequent).

As a result of application bicubic basic B-splines for restoration of functional dependences and experimental data of two variables mentioned below figure (3) are received.

As an example, consider the approximation using bicubic B-spline analytically given function:

$$F = \sin(\sqrt{x^2 + y^2} + eps) / \sqrt{x^2 + y^2} + eps,$$

where eps=2.2204e-016

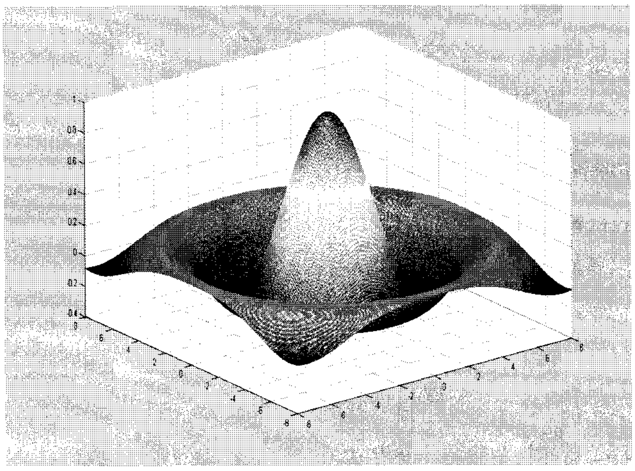

Fig. 2. The results of analytical function $F = \sin(\sqrt{x^2 + y^2} + eps) / \sqrt{x^2 + y^2} + eps$

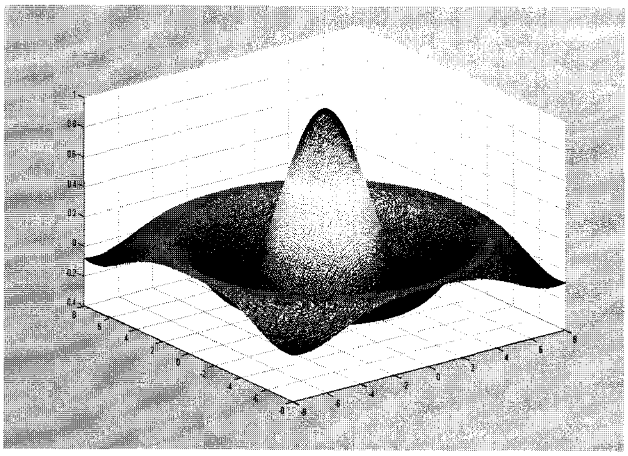

Fig. 3. The results of restoration of analytical function using bicubic B-spline.

## III. MULTIPROCESSING COMPUTING STRUCTURES ON A BASIS BICUBIC B-SPLINES FOR CALCULATION OF TWO-DIMENSIONAL DEPENDENCES

Last years hardware methods of construction of computing structures for functions evaluation of two and more variable used piecewise-plane and piecewise–quadratic approximation, being guided by the minimum number of adders and multipliers [8, 9] that didn't provide both high speed, and accuracy of transformation.

With the advent of multiprocessing VLSI (Very Large Scale Integration) there was a possibility of fast functions evaluation of many variables with application of mathematical apparatus of basic splines and table–algorithmic methods. As composed at calculation the B-spline – the sums of one-dimensional area, equally $m + 1$, on an example bicubic splines it is possible to show number how to construct the economic computing structure containing limited number of processors. The system one-dimensional a spline – functions is entered:

$$C_i(y) = \sum_{j=-1}^{n_2+1} b_{ij} B_j(y)$$
$$i = -1, 0, 1, \dots, n+1, \quad (11)$$

Then the two-dimensional spline is calculated:

$$S_3(x, y) = \sum_{i=-1}^{n_1+1} C_i(y) B_i(y), \quad (12)$$

That is the sequence of operations on calculation a spline – functions is carried out. Each of transformations of type (11) and (12) can be realized in the parallel form in the presence $m + 1$ of processors-multipliers.

The way of substantial increase of speed of processes of functions evaluations of many variables lies in the field of use of a principle of natural parallelism and tables nonlinear B–splines. The formula of formation of values of functions of many variables is put in a basis of calculations in the form of the approximating weighed sums of multiple products of one-dimensional local basic splines of degree $m \geq 2$.

The block diagram is showen on figure 4. It consists of the register of argument (RgAr), a memory of coefficients (RAMbij), a memory of basic functions (ROMBF) and the target adder-store of accumulating adders, [10]. As all basic splines identical under the form, that, using as ROM BF block, it is possible to be limited to storing of the table of values only one basic spline. On figure 5 the basic cubic B-spline which is defined as nonzero on four sites of piecewise splitting is presented. For sample of various four values of B-splines for the purpose of summation under the formula four subsections of the ROM to be required, the part of a curve of the B-spline should be stored in each of which, set on one site.

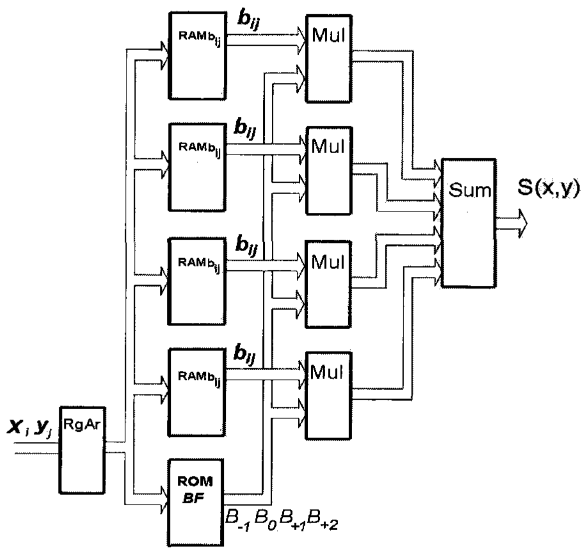


Fig. 4. Table-algorithmic computing structure for realization of approximation of functions by bicubic basic splines.

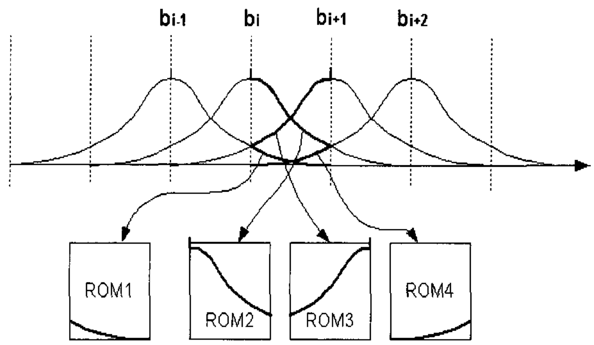


Fig. 5. The scheme of distribution of memory for corresponding sites of a basic B-spline.

## IV. CONCLUSIONS

Connection of advantages table-algorithmic methods of reproduction of functions and multidimensional local B-splines conducts to realization of in parallel-conveyor computing structures of many variables providing most high efficiency of operations, close to the peak.

The in parallel-conveyor computing structure is developed for realization two-dimensional basic a spline - approximations. She allows to save memory for storage of values of basic splines twice, at the limited number of processors.

The basic characteristics of realization table-algorithmic computing structures for processing of signals in piecewise - polynomial bases are received. Are shown that table-algorithmic computing structures on a basis bicubic B-splines function faster (in case of bicubic basic splines 3 times), than classical polynomials of the same degrees.

The cores advantage of structure is high speed, almost limiting for table-algorithmic methods.

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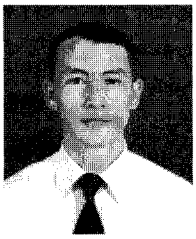


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