Link-Level Performance of Cooperative Multi-Hop Relaying Networks with MDS Codes

Katsumi Sakakibara, Daichi Ito, and Jumpei Taketsugu

Abstract: **We evaluate the link-level performance of cooperative multi-hop relaying networks with an maximum distance separable (MDS) code. The effect of the code on the link-level performance at the destination is investigated in terms of the outage probability and the spectral efficiency. Assuming a simple topology, we construct an absorbing Markov chain. Numerical results indicate that significant improvement can be achieved by incorporating an MDS code. MDS codes successfully facilitate recovery of the message block at a relaying node due to powerful error-correcting capability, so that it can reduce the outage probability. Furthermore, we evaluate the average number of hops where the message block can be delivered.**

Index Terms: **Absorbing Markov chain, cooperative multi-hop relaying networks, maximum distance separable (MDS) codes, outage probability.**

I. INTRODUCTION

Multi-hop networks are attracting researcher's attention for the use of wireless ad-hoc and sensor networks in the recent years [1]–[7]. In multi-hop networks, a source and a destination nodes can be connected through two or more routes depending on which nodes should relay the message. In such a case, it has been reported that collaborative transmission among nodes can significantly improve performance [2]–[9]. In [2] and [3], cooperative relaying techniques are discussed in conjunction with space-time coding [10] at the physical level. Particularly, Miyano *et al.* presented that the multi-route diversity gain can be obtained in terms of the packet error rate in a two-hop networks [2]. Koike *et al.* demonstrated performance improvement for inter-vehicle networks [3]. In [5], the gain obtained from convolutional codes is discussed in cooperative multi-hop networks at the physical level. In [6] and [7], performance of cooperative relay networks is discussed at the link level. Relayed packets are identical to the original message transmitted at the source node. Thus, the receiving node can recover the message when no channel errors occur in one or more received packets. From the viewpoint of error-correcting codes at the link level, erasure decoding of repetition codes for received blocks has been at the base of performance enhancement of collaborative transmission [6], [7]. The use of fountain codes [11] is proposed for collaborative two-hop relay networks [8], [9]. In the protocols proposed in [8] and [9], feedback channels from the receiver to the transmitter or intra-channels connecting all the relay nodes are inevitable in order to control packet transmissions at a node. Note that in [6]–[9], channels between nodes are modeled by erasure channel at packet level, so that a received packet with one or more channel errors is treated as packet erasure. Therefore, there will be still possibilities for further improvement by incorporating the error-correcting capability of the code employed. The authors have proposed the use of an invertible code [12], [13] to double-route cooperative multi-hop networks and evaluated the performance improvement in terms of the outage probability [14].

In this paper, we extend the results in [14] in order to be applicable to cooperative multi-hop relaying networks with two or more routes and analyze the link level performance in terms of the outage probability and the spectral efficiency. More precisely, an invertible code used in [14] is replaced by an maximum distance separable (MDS) code, since an invertible code can be applicable only to double-route networks. Then, the link-level performance is analyzed by means of arguments of an absorbing Markov chain. It should be emphasized that the protocols in this paper and in [14] require no feedback channels nor intra-channels among relaying nodes in contrast with ones in [8] and [9].

The rest of the present paper is organized as follows: Section II briefly reviews useful properties of MDS codes and describe the system model considered. In Section III, the link-level performance is derived after constructing an absorbing Markov chain. Numerical results are presented in Section IV. Finally, Section V concludes the present paper with further study.

II. SYSTEM MODEL

A. MDS Codes

Denote a linear block code of length n and dimension k by an $[n,k]$ code. An $[n,k]$ code is MDS if its minimum distance is $n-k+1$. A class of MDS codes, including Reed-Solomon codes, is known to be fruitful in tractable properties [15]. Among them, the following two theorems are used afterward:

Theorem 1: *Any combination of codeword coordinates in an MDS code may be used as message coordinates.*

For an $[n,k]$ MDS code, it follows from Theorem 1 that a receiver can recover the message, if it receives at least k code symbols with no errors.

Theorem 2: *Punctured MDS codes are also MDS.*

Suppose an $[Ln,n]$ MDS code $\mathcal{C}$ over a certain finite field. Let $\boldsymbol{G}$ be a generator matrix of $\mathcal{C}$. Apparently, $\boldsymbol{G}$ is an $n\times Ln$

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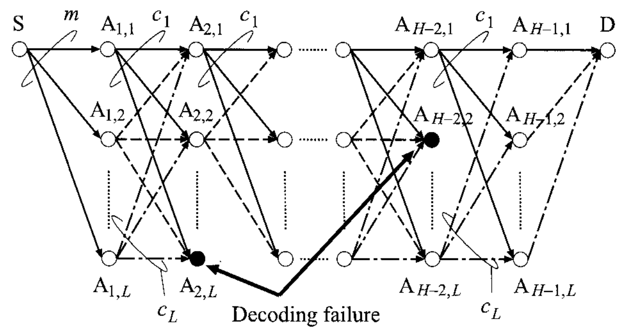

Fig. 1. System models for cooperative multi-hop relaying networks.

matrix. Let

$$G = [\underbrace{G_1}_{n} \mid \underbrace{G_2}_{n} \mid \cdots \mid \underbrace{G_L}_{n}] \quad (1)$$

be the partition of G into L blocks of identical size. It is clear that each G_ℓ is a square matrix of order n for $\ell = 1, 2, \cdots, L$. Similarly, a codeword of $\mathcal{C}$ can also be partitioned into L codeword blocks c_ℓ of length n;

$$c = mG = [\underbrace{c_1}_{n} \mid \underbrace{c_2}_{n} \mid \cdots \mid \underbrace{c_L}_{n}] \quad (2)$$

where m is a message block of length n and $c_\ell = mG_\ell$ for $\ell = 1, 2, \cdots, L$. Then, from Theorem 1, a receiver can retrieve the n-symbol message m from any received c_ℓ for $\ell = 1, 2, \cdots, L$, if no channel errors are included. On the other hand, assume that a node receives j out of L blocks in a transmitted codeword c and that all the received j codeword blocks are erroneous. In such a case, aggregation of the j blocks provides a codeword of a punctured $[jn, n]$ MDS code of $\mathcal{C}$, whose minimum distance is $(j-1)n+1$ from Theorem 2. Then, it follows that the message m can be recovered if the total number of errors occurred in the j blocks is less than or equal to

$$t_j = \left\lfloor \frac{(j-1)n}{2} \right\rfloor \quad (3)$$

where $\lfloor x \rfloor$ is the maximum integer not greater than x.

B. Cooperative Multi-Hop Relaying Network

Consider a cooperative H-hop relaying network, where nodes are orderly aligned on a two-dimensional plane as shown in Fig. 1. The hth hop stage consists of L relaying nodes, $A_{h,1}$, $A_{h,2}, \cdots, A_{h,L}$, for $h = 1, 2, \cdots, H-1$. A relaying node $A_{h,\ell}$ can receive blocks from relaying nodes in the previous hop stage, $A_{h-1,1}$, $A_{h-1,2}, \cdots, A_{h-1,L}$ for $h = 2, 3, \cdots, H-1$. Note that a topology shown in Fig. 1 is equivalent to that in [2]. However, no discussions beyond two-hop networks are addressed in [2]. Also, our topology can be viewed as a generalization of that in [3], [6]–[9].

Source node S transmits the message m of n-symbol length to L first relaying nodes $A_{1,1}, A_{1,2}, \cdots, A_{1,L}$. Each first relaying node $A_{1,\ell}$ can retrieve m, if it has received m with no symbol errors. Then, a successful relaying node $A_{1,\ell}$ encodes m by $\mathcal{C}$ and transmits the ℓth codeword block c_ℓ to L second relaying nodes $A_{2,1}, A_{2,2}, \cdots, A_{2,L}$. Let j be the number of the first relaying nodes ($j = 0, 1, \cdots, L$) which have received m from the source node with no errors. It implies that each of the second relaying nodes $A_{2,\ell}$ receives j blocks, if no transmitted blocks are lost on the channels. Then, the node $A_{2,\ell}$ can recover the message m if it receives one or more codeword blocks among j received ones with no errors (Theorem 1) or if the total number of symbol errors occurred in the j received blocks is not greater than t_j (Theorem 2). Otherwise, a second relaying node $A_{2,\ell}$ can not recover the message. In this sense, it is possible that all the L second relaying nodes can recover the message m by decoding of $\mathcal{C}$, even when all the received blocks from the first relaying nodes suffer from symbol errors. This particular property may enhance the performance of cooperative multi-hop relaying networks. The identical procedure proceeds hop-by-hop. In Fig. 1, two relaying nodes $A_{2,L}$ and $A_{H-2,2}$ fail to recover the message m, so that they do not transmit a codeword block further.

III. ANALYSIS

A. Assumptions

We model each channel between neighboring nodes by a random error channel of the symbol error rate ε, which is identical and independent among channels in order to focus on the effect of an MDS code on the link level performance. It is also assumed that no blocks are lost on the channels [6], [7] and that no retransmission mechanisms are employed, that is, no feedback channels are required in contrast with [8] and [9].

B. Probability of Successful Decoding

Denote by r_j the probability that a node succeeds in recovering the message m upon receiving j blocks for $j = 1, 2, \cdots, L$. For $j = 1$, no errors are allowable for the message to be recovered. Thus,

$$r_1 = (1-\varepsilon)^n. \quad (4)$$

When two or more blocks are delivered at a node, we can take advantage of the error-correcting capability of $\mathcal{C}$ employed. From Theorem 1 a node can recover the message m from one or more clean blocks (blocks with no errors) among j received ones, whose probability is $1-(1-r_1)^j$. Next, let i_u be the number of symbol errors occurred in the uth received block for $u = 1, 2, \cdots, j$. Then, $i_u \in \mathbb{N}$, where $\mathbb{N}$ is a set of positive integers, since all the j received blocks are assumed to include symbol errors. Define a j-tuple of positive integers by $I_j = (i_1, i_2, \cdots, i_j) \in \mathbb{N}^j$ for $j = 2, 3, \cdots, L$, where $\mathbb{N}^j$ is a set of direct product of $\mathbb{N}$ of order j. We also define the norm of I_j by $\|I_j\| = i_1 + i_2 + \cdots + i_j$. For given j ($j = 2, 3, \cdots, L$), we then denote by Γ_j a subset of $\{I_j\}$ whose norm is less than or equal to t_j;

$$\Gamma_j = \{I_j \mid I_j \in \mathbb{N}^j, \|I_j\| \le t_j\}. \quad (5)$$

Note that Γ_j represents the error distribution among j received blocks which is correctable by a punctured $[jn, n]$ MDS code of $\mathcal{C}$. Hence, the probability that a node can recover the message

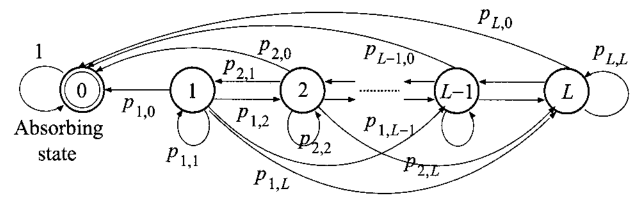

Fig. 2. Markovian model with respect to the number of successful nodes at the hop stage S_h.

$\boldsymbol{m}$ in the case that all the j received blocks include errors is

$$\delta_j = \sum_{\boldsymbol{I}_j \in \Gamma_j} \prod_{u=1}^{j} \binom{n}{i_u} \varepsilon^{i_u} (1-\varepsilon)^{n-i_u}. \tag{6}$$

In consequence, r_j is given by

$$r_j = 1 - (1-r_1)^j + \delta_j \tag{7}$$

for $j = 2, 3, \cdots, L$. Here, we neglect the probability of decoder error [15] due to its considerably small value.[1]

C. Markovian Model

The decoding procedure at a relaying node in the hth hop stage is dependent on the number of received blocks. It follows from the assumption that it is equal to the number of relaying nodes in the previous hop stage which succeed in recovering the message $\boldsymbol{m}$. Let S_h be the number of successful relaying nodes in the hth hop stage for $h = 1, 2, \cdots$. Apparently, $S_h \in \Omega = \{0, 1, \cdots, L\}$. Then, we can construct a Markovian model with respect to S_h, as shown in Fig. 2. The state S_h evolves in the state space Ω in a hop-by-hop manner. However, once the number of successful relaying nodes decreases to zero, the message $\boldsymbol{m}$ can not be delivered to the next hop stage, since no relaying nodes can recover the message. It implies that it is impossible for S_h to escape from state 0. Thus, the Markovian model in Fig. 2 is an absorbing Markov chain with one absorbing state, state 0.

Let $p_{j,i}$ represent a transition probability from state j to state i. From the assumption that no blocks are lost on the channels, all the relaying nodes in the hth hop stage receive j blocks if $S_{h-1} = j$. The identical and independent assumption on the channel errors provides us

$$p_{j,i} = \Pr[S_h = i \mid S_{h-1} = j] = \binom{L}{i} r_j^i (1-r_j)^{L-i} \tag{8}$$

for $j \in \Omega - \{0\}$, $i \in \Omega$ and $h = 2, 3, \cdots$, since every relaying node decodes the j received blocks in an independent manner. Clearly,

$$p_{0,i} = \begin{cases} 1, & \text{for } i = 0 \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

First, consider the L relaying nodes in the first hop stage. Each of these nodes receives only the message $\boldsymbol{m}$ transmitted by the source node S. Thus, they can obtain no benefit from the MDS code $\mathcal{C}$ and the probability of successful reception at each first relaying node is r_1. The probability distribution of S_1 is then given by the binomial distribution;

$$\begin{aligned} \boldsymbol{\pi}_1 &= [\, \Pr[S_1 = 0], \Pr[S_1 = 1], \cdots, \Pr[S_1 = L] \,] \\ &= \left[(1-r_1)^L, \binom{L}{1} r_1 (1-r_1)^{L-1}, \cdots, r_1^L \right]. \end{aligned} \tag{10}$$

Next, the probability distribution of S_h for $h = 2, 3, \cdots$ can be obtained in a recursive manner:

$$\begin{aligned} \boldsymbol{\pi}_h &= [\, \Pr[S_h = 0], \Pr[S_h = 1], \cdots, \Pr[S_h = L] \,] \\ &= \boldsymbol{\pi}_{h-1} \begin{bmatrix} 1 & 0 & \cdots & 0 \\ p_{1,0} & p_{1,1} & \cdots & p_{1,L} \\ \vdots & \vdots & & \vdots \\ p_{L,0} & p_{L,1} & \cdots & p_{L,L} \end{bmatrix} \\ &= \boldsymbol{\pi}_{h-1} \left[\begin{array}{c|c} 1 & 0 \ \cdots \ 0 \\ \hline p_{1,0} & \\ \vdots & \boldsymbol{Q} \\ p_{L,0} & \end{array} \right] \\ &= \boldsymbol{\pi}_{h-1} \boldsymbol{P} = \cdots = \boldsymbol{\pi}_1 \boldsymbol{P}^{h-1} \end{aligned} \tag{11}$$

where $\boldsymbol{P}$ is the transition matrix and

$$\boldsymbol{Q} = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,L} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,L} \\ \vdots & \vdots & & \vdots \\ p_{L,1} & p_{L,2} & \cdots & p_{L,L} \end{bmatrix} \tag{12}$$

is a submatrix which represents transition probabilities among transient states.

D. Outage Probability

The destination node D, which is located at the Hth hop stage, receives S_{H-1} blocks. The outage probability, that is, the probability that the destination node D can not recover the message $\boldsymbol{m}$, is evaluated as

$$\begin{aligned} \eta_H &= \sum_{\ell=0}^{L} (1 - r_\ell) \Pr[S_{H-1} = \ell] \\ &= \boldsymbol{\pi}_{H-1} [1, 1-r_1, 1-r_2, \cdots, 1-r_L]^T \\ &= \boldsymbol{\pi}_1 \boldsymbol{P}^{H-2} [1, 1-r_1, 1-r_2, \cdots, 1-r_L]^T \end{aligned} \tag{13}$$

where a superscript T is transpose of a matrix. Note that, if $S_{H-1} = 0$, the destination node can never recover the message.

E. Spectral Efficiency

In general, it is well known that the use of error-correcting codes expands the required bandwidth due to transmission of redundant symbols. Also, in cooperative relaying networks shown in Fig. 1, it is required for each relaying node except $A_{1,1}, \cdots, A_{1,L}$ to receive at most L blocks. This results in the bandwidth expansion of degree L regardless of the use of MDS codes. Hence, we define the spectral efficiency as

$$\xi_H = \frac{1 - \eta_H}{L} \tag{14}$$

[1]For example, according to [17], the probability of decoder error is at most 1.12×10^{-47} for the $[128, 64]$ MDS code over GF(2^8) ($L = 2$), 4.22×10^{-103} for the $[192, 64]$ MDS code over GF(2^8) ($L = 3$), and 8.14×10^{-160} for the $[256, 64]$ MDS code over GF(2^8) ($L = 4$).

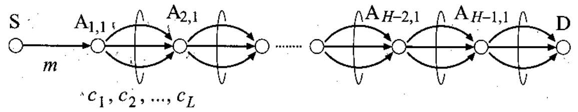

Fig. 3. System models for single-route multi-hop relaying networks with MDS codes.

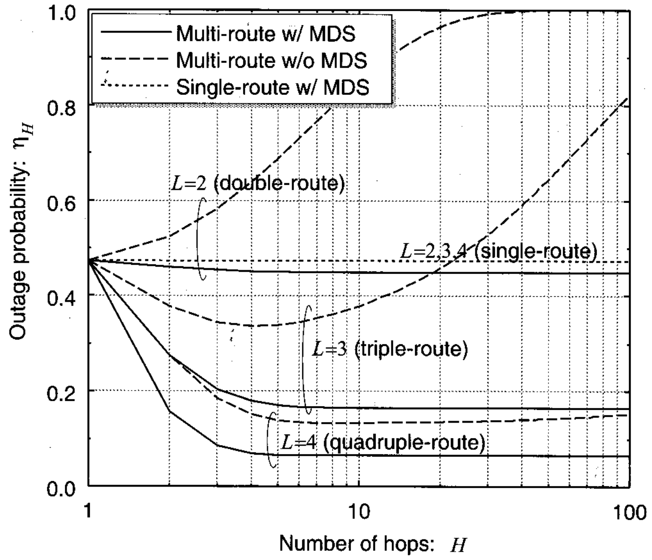

Fig. 4. Outage probability of 64-symbol message block for $\varepsilon = 10^{-2}$.

for L-route cooperative multi-hop relaying networks, since the end-to-end throughput can be defined as $1 - \eta_H$.

IV. NUMERICAL EXAMPLES AND DISCUSSIONS

We numerically examine the obtained expressions for a $[64L, 64]$ MDS code $\mathcal{C}$ ($n = 64$). Three types of cooperative multi-hop networks are considered, that is, double-, triple-, and quadruple-route multi-hop networks ($L = 2, 3, 4$). The channel symbol error rate is assumed to be $\varepsilon = 10^{-2}$. It should be emphasized that the above analysis can also provide the link-level performance for cooperative multi-hop networks without MDS codes, when we impose $t_\ell = 0$ instead of (3), which results in $\delta_\ell = 0$ from its definition (6).

For comparison, we consider a single-route multi-hop relaying network shown in Fig. 3, where the message block $\boldsymbol{m}$ is transmitted on the first hop and all the L codeword blocks, $\boldsymbol{c}_1, \boldsymbol{c}_2, \cdots, \boldsymbol{c}_L$, are relayed afterward. The outage probability can be evaluated by

$$\eta_H = \begin{cases} 1 - r_1, & \text{for } H = 1 \\ \eta_{H-1} + (1 - \eta_{H-1})(1 - r_L), & \text{for } H \geq 2 \end{cases} \tag{15}$$

in a recursive manner, since a relaying node receives L codeword blocks as long as the previous node can recover the message block $\boldsymbol{m}$.

A. Outage Probability

The outage probability η_H of three types of the cooperative multi-hop networks is presented in Fig. 4. In Fig. 4, the outage probability of multi-route cooperative multi-hop networks with and without the MDS code $\mathcal{C}$ are presented by solid lines and dashed lines, respectively, and that of single-route multi-hop networks is depicted by dotted line. It is clearly observed that the use of an MDS code significantly reduces the outage probability. For example, in the case of a double-route multi-hop network without an MDS code, the outage probability converges into unity for approximately thirty hops. On the contrary, the outage probability is still less than 0.5 for 100 hops if an MDS code can be collaboratively incorporated. Comparing to the case of no MDS code, an MDS code affords performance improvement by δ_ℓ due to its error-correcting capability t_ℓ. The calculated values of the probability of correct decoding r_ℓ upon ℓ-block reception are given in Table 1. From Table 1, we can find that the message $\boldsymbol{m}$ can be retrieved with probability approximately one, if an MDS code is employed and if two or more blocks are received. The outage probability of single-route multi-hop networks is a slight increasing function with respect to H due to the same reason of $r_\ell \approx 1.0$ for $\ell = 2, 3, 4$, so that it is almost independent of L. As a result, triple- and quadruple-route cooperative multi-hop relaying networks outperform single-route multi-hop networks for $H \leq 20$ (triple-route) in terms of the outage probability, even if no MDS codes are incorporated in cooperative multi-hop networks.

Table 1. Probability of correct decoding when ℓ blocks are received for $n = 64$ and $\varepsilon = 10^{-2}$.

r_ℓ	r_1	r_2	r_3	r_4
with MDS	0.5256	≈ 1.0000	≈ 1.0000	≈ 1.0000
without MDS	0.5256	0.7749	0.8932	0.9493

Among six curves for multi-route cooperative multi-hop networks in Fig. 4, only the result for $L = 2$ without the MDS code $\mathcal{C}$ is monotonously increasing with respect to an increase of the number of hops H. Other curves are decreasing for small H. In the case of no MDS codes we have $1 - r_j = (1 - r_1)^j$ from (7), since $\delta_j = 0$ from $\Gamma_j = \emptyset$ (no error-correcting capability is employed). Then, the outage probability for $H = 2$ is

$$\begin{aligned} \eta_2 &= \sum_{j=0}^{L} \binom{L}{j} r_1^j (1 - r_1)^{L-j} (1 - r_j) \\ &= (1 - r_1)^L \sum_{j=0}^{L} \binom{L}{j} r_1^j \\ &= (1 - r_1^2)^L \end{aligned} \tag{16}$$

from (10) and (13). After simple algebraic manipulations and numerical calculation, we obtain $\eta_1 > \eta_2$, if and only if

$$\varepsilon < \begin{cases} 7.49 \times 10^{-3}, & \text{for } L = 2 \\ 1.46 \times 10^{-2}, & \text{for } L = 3 \\ 1.96 \times 10^{-2}, & \text{for } L = 4. \end{cases} \tag{17}$$

Thus, in the case without the MDS code $\mathcal{C}$, we have $\eta_1 < \eta_2$ for $L = 2$ and $\eta_1 > \eta_2$ for $L = 3, 4$, since $\varepsilon = 10^{-2}$ in Fig. 4. It reveals that cooperative two-hop relaying networks are inefficient in terms of the outage probability, if the channel quality is bad and if no error-correcting codes are employed.

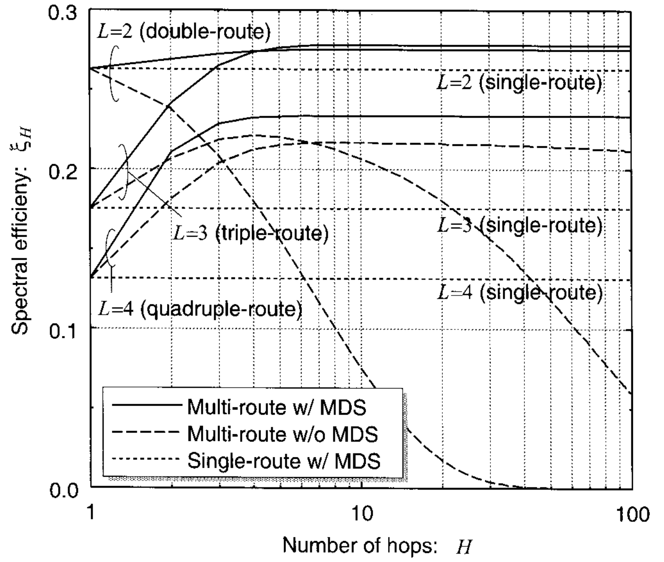

Fig. 5. Spectral efficiency of 64-symbol message block for $\varepsilon = 10^{-2}$.

B. Spectral Efficiency

The spectral efficiency ξ_H is presented in Fig. 5. From Fig. 5, the double-route cooperative multi-hop network ($L = 2$) with the MDS code $\mathcal{C}$ exhibits the highest efficiency for $H \leq 4$ and the triple-route network ($L = 3$) with the MDS code $\mathcal{C}$ slightly outperforms the double-route case for $H \geq 5$. We can reduce the outage probability if a long MDS code is employed, as shown in Fig. 4. However, it requires a large bandwidth, so that the spectral efficiency is degraded.

C. Probability Distribution in the Markov Chain

Let us investigate the evolution of the probability distribution $\boldsymbol{\pi}_h$ in the Markov chain given in Fig. 2. The probability distribution for the six multi-route cases in Fig. 4 is shown in Fig. 6 for $n = 64$ and $\varepsilon = 10^{-2}$.

From Fig. 6 we can find that in the case without the MDS code $\mathcal{C}$, $\Pr[S_h = 0]$ gradually increases and $\Pr[S_h = L]$ first increases and then turns to decrease. Even for $L = 4$, this is true. Since State 0 is the unique absorbing state for the Markov chain in Fig. 2, the probability $\Pr[S_h = 0]$ in an increasing function with respect to H. Also, the probability distribution $\boldsymbol{\pi}_h$ asymptotically approaches to

$$\lim_{h\to\infty} \Pr[S_h = \ell] = \begin{cases} 1, & \text{for } \ell = 0 \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

On the contrary, in the case with the MDS code $\mathcal{C}$, the state merges into two extremes, state 0 and state L, after several hops. As shown in Table 1, a node can recover the message $\boldsymbol{m}$ with probability almost one if it receives two or more blocks. It enhances the probability that all of L relaying nodes can recover the message. The probability of intermediate states, states 2 to $L-1$, vanishes at the same time. Note that for sufficiently large h, however, we asymptotically have $\Pr[S_h = 0] = 1$ and $\Pr[S_h = L] = 0$, that is, (18) also holds in this case.

Table 2. Average number of hops h_{av} to enter the absorbing state for $n = 64$ and $\varepsilon = 10^{-2}$.

No. of routes	$L = 2$	$L = 3$	$L = 4$
Multi w/ MDS	2.66×10^{18}	1.41×10^{38}	2.56×10^{58}
Multi w/o MDS	6.50	6.22×10^{1}	3.69×10^{3}
Single w/ MDS	4.22×10^{14}	5.88×10^{14}	8.23×10^{14}

D. Average Number of Hops to Enter the Absorbing State

From Fig. 4, it appears that for large H, the outage probability with the MDS code $\mathcal{C}$ converges into a certain value less than unity. However, this is not true, since the multi-hop network shown in Fig. 1 is modeled by the absorbing Markov chain in Fig. 2. It implies that for sufficiently large H, the state enters into state 0 and never leave it, so that the outage probability must reach unity for large H. In such a case, it is of importance to estimate the average number of hops for the system to enter into state 0.

Based on the argument of absorbing Markov chains [16], the fundamental matrix is defined by $\boldsymbol{I} - \boldsymbol{Q}$, where $\boldsymbol{I}$ is the identity matrix of order L and $\boldsymbol{Q}$ is given by (12). Then, the conditional average number of hops τ_ℓ for the system to be absorbed in state 0, given that it starts at state ℓ for $\ell = 1, 2, \cdots, L$, is defined by

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_L \end{bmatrix} = (\boldsymbol{I} - \boldsymbol{Q})^{-1} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}. \quad (19)$$

As a result, the average number of hops to enter the absorbing state, state 0, is

$$h_{\mathrm{av}} = 1 + \boldsymbol{\pi}_1 [0, \tau_1, \tau_2, \cdots, \tau_L]^{\mathrm{T}} \quad (20)$$

since $\boldsymbol{\pi}_1$ provides the initial distribution of the Markovian model in Fig. 2. Table 2 presents the calculated values of the average number of hops for the double-, triple-, and quadruple-route multi-hop networks with and without MDS code and for single-route multi-hop networks with MDS code to be absorbed into state 0 for $n = 64$ and $\varepsilon = 10^{-2}$.

Let us consider the double-route multi-hop networks ($L = 2$) as an example. The message can be delivered to six or seven hops on average, since $h_{\mathrm{av}} = 6.504$, and no further delivery can be accomplished if no MDS code is employed. However, incorporation of the MDS code $\mathcal{C}$ can augment h_{av} in orders of magnitude, which results in $h_{\mathrm{av}} = 2.66 \times 10^{18}$.

V. CONCLUSION

In this paper, we have considered multi-route cooperative multi-hop relaying networks and evaluated the link-level performance when an MDS code is applied. An absorbing Markov model has been constructed for the performance evaluation. The expressions of the outage probability and the spectral efficiency have been derived. Numerical results with a $[64L, 64]$ MDS code for a double-, triple-, and quadruple-route cooperative multi-hop relaying network, $L = 2, 3, 4$, have indicated

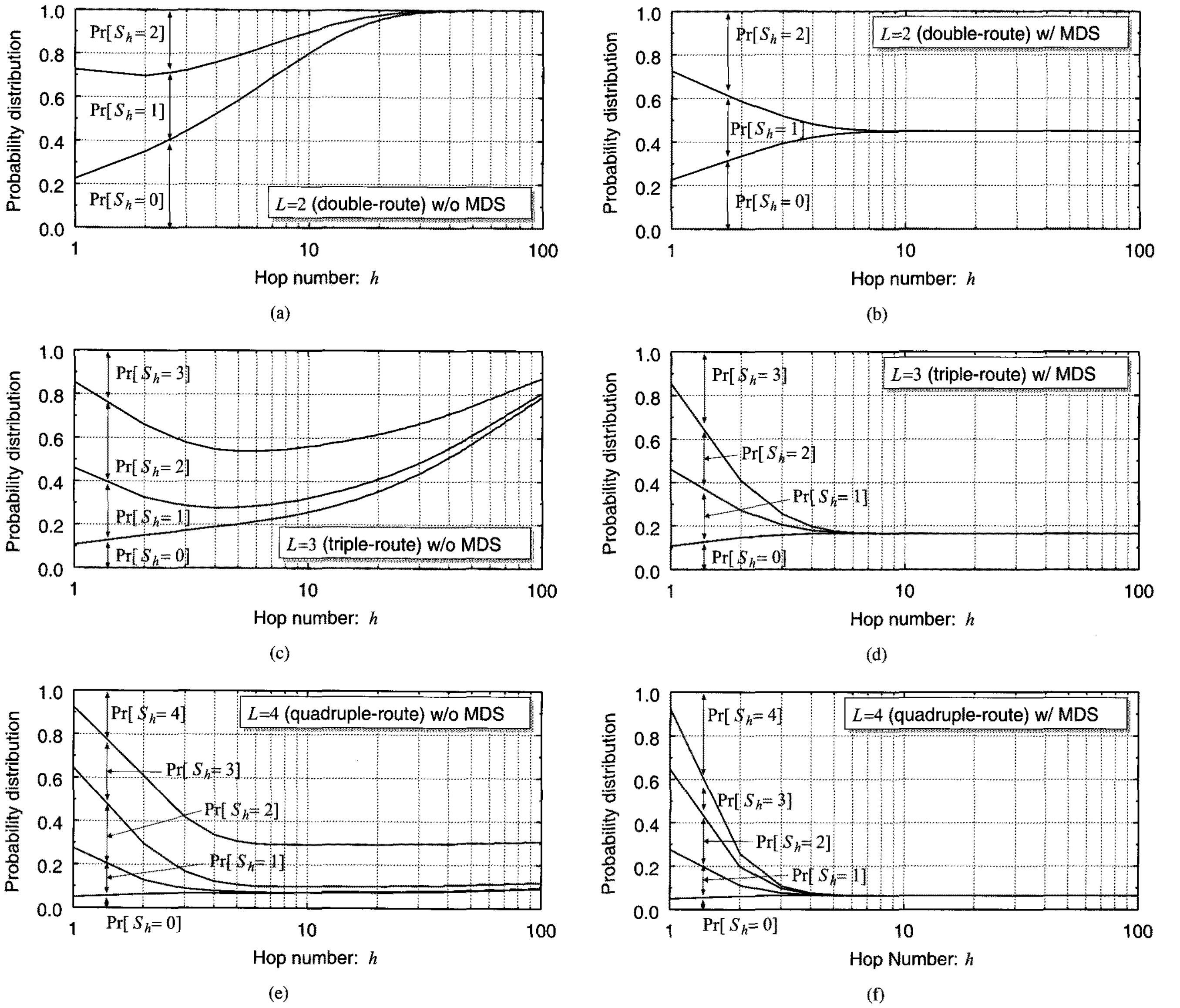

Fig. 6. Probability distribution in the Markov chain MDS code π_h for $n = 64$ and $\varepsilon = 10^{-2}$: (a) Double-route ($L = 2$) without MDS code $\mathcal{C}$, (b) double-route ($L = 2$) with MDS code $\mathcal{C}$, (c) triple-route ($L = 3$) without MDS code $\mathcal{C}$, (d) triple-route ($L = 3$) with MDS code $\mathcal{C}$, (e) quadruple-route ($L = 4$) without MDS code $\mathcal{C}$, and (f) quadruple-route ($L = 4$) with MDS code $\mathcal{C}$.

that significant improvement can be achieved by incorporating an MDS code. Due to the powerful error-correcting capability of an MDS code, an information block can be reproduced with probability approximately one when a relaying node receives blocks from two or more neighboring nodes. This results in reduction of the outage probability.

Further study includes the incorporation of retransmission mechanisms and the consideration of the physical level issues such as the channel correlation and the power consumption at a node.

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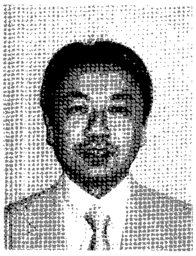

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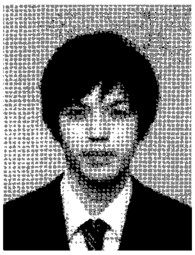

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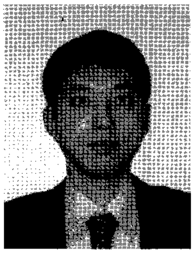

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