Selection Based Cooperative Beamforming and Power Allocation for Relay Networks

Yi Liu and Weiqing Nie

Abstract: **Cooperative beamforming has previously been proven to be an efficient way to improve the cooperative diversity. This method generally requires all relay nodes to participate in beamforming, which can be seen as "all participate" cooperative beamforming. However, not all relay nodes have constructive impacts on the end-to-end bit error rate (BER) performance. Based on this observation, we propose a new cooperative scheme which only selects those "appropriate" relay nodes to perform cooperative beamforming. Such relay nodes can be simply determined with mean channel gains. Therefore, the selection complexity is significantly reduced as global instantaneous channel state information is not required. This scheme guarantees that energy is only allocated to the "appropriate" relay nodes, and hence provides superior diversity. We also prove that power allocation among source and selected relay nodes is a convex problem, and can be resolved with lower computational complexity. Simulation results demonstrate that our scheme achieves an essential improvement in terms of BER performance for both optimal and limited feedback scenarios, as well as high energy-efficiency for the energy-constrained networks.**

Index Terms: **Amplify-and-forward (AF), cooperative beamforming, cooperative communication, relay selection.**

I. INTRODUCTION

Cooperative communication, in which single-antenna users share their antennas cooperatively to create a virtual multiple-input multiple-output (MIMO) system [1], has attracted a lot of attentions. The spatial diversity resulted from such virtual MIMO system leads to much higher data rate and more reliable services over a larger coverage. Fundamental principles and performance results of cooperative system have been discussed in [2]–[4]. Since then, this idea has been extensively studied and several milestones in this area have been achieved.

In the relay-based cooperative communication system, cooperative beamforming performed by all relay nodes has been widely discussed to improve the performance of multiple-relay scenario. Most of these schemes weight their inputs according to the channel state information (CSI) feedback [5], [6] or a prior information available [7]. All of the relay nodes participate in beamforming, and is also known as the "all participate" cooperative beamforming (AP-BF). Unfortunately, the fading characteristics of each cooperative link (source-relay-destination link) are always different, i.e., some cooperative links are under deep fading, whereas others are in good conditions. Consequently, not all the relay nodes have constructive impacts on the end-to-end bit error rate (BER) performance. Motivated by this observation, we further study cooperative beamforming in order to improve the transmission reliability. A natural solution is to perform cooperative beamforming only among the "appropriate" relay nodes.

Previous researches in the literature have shown that a relay selection method outperforms the repetition-based scheduling. For amplify-and-forward (AF) and decode-and-forward (DF) protocols, single relay selection has been addressed in [8]–[10]. However, considering that only one relay cooperates with source-destination pair, such selection approaches cannot be directly applied to cooperative beamforming. In [11] and [12], a few very important relays are selected under the assumption that global instantaneous CSI are known at the destination, while this will incur tremendous information exchange. In particularly, according to authors' knowledge, few discussions about selection scheme have been made for cooperative beamforming. An opportunistic cooperative beamforming, in [13], selects a subset of relay nodes to conduct beamforming, whereas only DF protocol is taken into consideration. Furthermore, in these schemes, each node (source and relay nodes) has their own independent power constraints, while the sum power over source and all relay nodes is constrained jointly in our paper. The joint sum power constraint is motivated by: (i) Transmission power is a significant design consideration for the energy-constrained cooperative networks (i.e., all nodes are powered by batteries which can supply only a finite amount of energy); (ii) the central controller may limit sum transmission power to eliminate the interference with other transmissions in the network. Bearing these two problems (selection for cooperative beamforming and sum power constraint) in mind, we propose a simple relay selection scheme to determine potential relay nodes that participate in beamforming for AF relaying. In our scheme, relay selection only requires mean channel gains, and hence avoids monitoring global instantaneous CSI. Moreover, the "appropriate" relay nodes are determined before the source transmission and the selected relays remain activated as long as the fading conditions do not significantly change, in an average sense. This characteristic guarantees that only the CSI of the selected relay nodes are needed among those relays, and thus dramatically decreases the unnecessary payload of exchanging CSI under large size of network. This special cooperative beamforming can be seen as a selection cooperative beamforming (S-BF). Furthermore, under the proposed scheme, power allocation (PA) across

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source and selected relay nodes is proven to be a standard convex problem, and could further improve the outage performance with low computational complexity. Not only is S-BF scheme able to make full use of the transmission power, but also effectively exploits the cooperative diversity in the "appropriate" cooperative links.

While the cooperative beamforming is motivated by maximum ratio transmission for a transmitter configured with multiple antennas, it is an additional challenging task to obtain and exploit CSI in a distributed manner for cooperative beamforming. Although the optimal cooperative beamforming provides a performance upper bound, it is impractical for system implementation. Consequently, the beamforming with limited feedback can be a good and practical substitution of optimal cooperative beamforming. We compare the performance gap between the limited feedback case and the CSI available case when using our S-BF scheme. Simulation results demonstrate that this gap is acceptable for practical implementation. More importantly, extensive simulation results demonstrate that our cooperative scheme achieves better BER performance in addition to significant energy savings.

The rest of paper is organized as follows. In Section II, the system model is presented. In Section III, a selection cooperative beamforming scheme is proposed. The detailed procedure for practical implementation is described in Section IV. Simulation results of the cooperative scheme are provided in Section V. Finally, we conclude this paper in Section VI.

II. SYSTEM MODEL

We consider a half-duplex dual-hop relay wireless network as shown in Fig. 1, where the direct path between the source $\mathcal{S}$ and destination $\mathcal{D}$ cannot be neglected. In the system model, $\mathcal{S}$ communicates with $\mathcal{D}$ under the assistance of K_R relay $\mathcal{R}$ nodes. In order to achieve low implementation complexity of the system, it is assumed that all the nodes employ a single antenna. The channels from source to relays, source to destination and relays to destination are assumed to be frequency non-selective and Rayleigh block-fading, i.e., fading coefficients remain constant within each block and are independent between different blocks. We further consider a sum power constraint over source and all the relay nodes

$$\mathcal{P}_s + \mathcal{P}_r = \mathcal{P}_{sum} \tag{1}$$

where $\mathcal{P}_s$ is the transmission power of source, $\mathcal{P}_r$ is the aggregate relay transmission power allocated to the set $\mathcal{S}_R = \{1, 2, \cdots, K_R\}$ of K_R relay nodes, and $\mathcal{P}_{sum}$ is the end-to-end (i.e., source-relay-destination) transmission power. Notice that the source and the aggregate relay power constraints are

$$\mathcal{P}_s = \zeta\mathcal{P}_{sum} \quad \text{and} \quad \mathcal{P}_r = (1-\zeta)\mathcal{P}_{sum} \tag{2}$$

where $\zeta \in (0,1)$ and $(1-\zeta) \in (1,0)$ denote the fractions of sum power $\mathcal{P}_{sum}$ allocated to the source transmission and the overall relay transmission, respectively. If the ith relay fails to participate in relaying, its transmission power is set to zero.

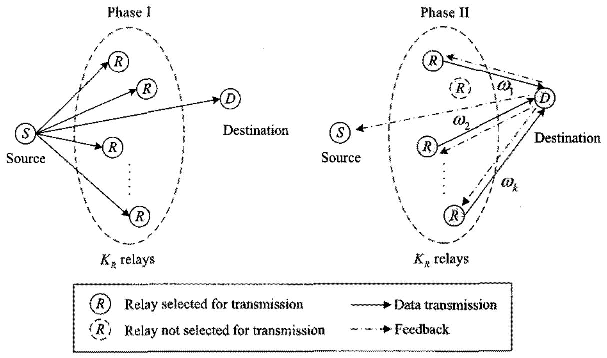

Fig. 1. System model used in this paper. Direct path between source and destination cannot be neglected.

The transmission from $\mathcal{S}$ to $\mathcal{D}$ can be split into two phases. In the first phase, source broadcasts s with power $\mathcal{P}_s$ to the relay and destination nodes. Hence, the input-output relations of $\mathcal{S} \rightarrow \mathcal{R}_i$ and $\mathcal{S} \rightarrow \mathcal{D}$ links are given as

$$y_{s,i} = \sqrt{\mathcal{P}_s} h_{s,i} s + n_i, \tag{3}$$

$$y_{d,1} = \sqrt{\mathcal{P}_s} h_{s,d} s + n_{d,1} \tag{4}$$

where s is the user data with unit energy. $h_{s,i} \sim \mathcal{CN}(0, \Omega_{s,i})$ and $h_{s,d} \sim \mathcal{CN}(0, \Omega_{s,d})$ denote the channel coefficients corresponding to the source to ith relay and source to destination links, respectively. n_i and $n_{d,1}$ represent the complex additive white Gaussian noise (AWGN) at the relay and destination nodes, separately.

In the second phase, relay node first scales the received signal uniformly. Then, it is amplified and forwarded to the destination. Therefore, the equivalent signal to be transmitted at the ith relay node can be written as[1]

$$x_i = \frac{y_{s,i}}{\sqrt{E\{|y_{s,i}|^2\}}} = \frac{\sqrt{\mathcal{P}_s} h_{s,i} s + n_i}{\sqrt{\mathcal{P}_s |h_{s,i}|^2 + N_0}} \tag{5}$$

where $E\{\cdot\}$ denotes the statistic average. All the K_R relay nodes transmit x_i, which is weighted by a complex beamforming weight w_i, simultaneously to the destination. Thus, the received signal at the destination after amplification by relay nodes is

$$\begin{aligned} y_{r,d} &= \sum_{i=1}^{K_R} \sqrt{\mathcal{P}_r} h_{i,d} w_i x_i + n_{d,2} \\ &= \sum_{i=1}^{K_R} \frac{\sqrt{\mathcal{P}_s}\sqrt{\mathcal{P}_r} h_{s,i} h_{i,d} w_i s}{\sqrt{\mathcal{P}_s |h_{s,i}|^2 + N_0}} + \sum_{i=1}^{K_R} \frac{\sqrt{\mathcal{P}_r} h_{i,d} w_i n_i}{\sqrt{\mathcal{P}_s |h_{s,i}|^2 + N_0}} + n_{d,2} \end{aligned} \tag{6}$$

where $h_{i,d} \sim \mathcal{CN}(0, \Omega_{i,d})$ is the channel coefficient from the ith relay to destination and $n_{d,2}$ stands for the AWGN at the destination during the second phase. Notice that beamforming schemes require phase synchronization across relays, an issue of practical importance.

With the prerequisite knowledge of all channel gains and noise distributions at the destination, the received signal to noise

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[1] In the rest of paper, it is assumed that the complex AWGN at each node has a power spectral density of N_0.

ratio (SNR) for AP-BF scheme can be expressed as

$$\gamma_d^{AP} = \gamma_{s,d} + \gamma_{r,d} = \frac{\mathcal{P}_s|h_{s,d}|^2}{N_0} + \frac{\left|\sum_{i=1}^{K_R} h_{sig}^i w_i\right|^2}{(1 + \sum_{i=1}^{K_R} |h_n^i|^2 |w_i|^2) N_0} \tag{7}$$

where h_{sig}^i and h_n^i represent

$$h_{sig}^i = \frac{\sqrt{\mathcal{P}_s}\sqrt{\mathcal{P}_r} h_{s,i} h_{i,d}}{\sqrt{\mathcal{P}_s |h_{s,i}|^2 + N_0}} \tag{8}$$

and

$$h_n^i = \frac{\sqrt{\mathcal{P}_r} h_{i,d}}{\sqrt{\mathcal{P}_s |h_{s,i}|^2 + N_0}}, \tag{9}$$

respectively.

III. SELECTION BEAMFORMING FOR AF COOPERATIVE NETWORKS

In this section, we first give the expression of optimal beamforming vector for AF cooperative networks and then develop a simple relay selection scheme to find the "appropriate" relay subset to conduct cooperative beamforming. Based on this selection beamforming for AF (S-BF-AF) scheme, a PA method is presented to further improve the end-to-end BER performance and energy efficiency.

A. Cooperative Beamforming with Unlimited Feedback

Given cooperative networks with multiple relays, the adoption of cooperative beamforming is efficient to improve the error performance and energy efficiency of the transmission from relays to destination. In the optimal cooperative beamforming, the relays, with the knowledge of acquired CSI, weight their transmission signals with w_i so that they can be added up coherently at the destination. The optimal cooperative beamforming vector is developed under the assumption of knowing full CSI at each relay node. Although it is impractical for system implementation, the optimal cooperative beamforming serves as a reference comparison to other beamforming schemes with limited feedback.

The first term of equivalent received SNR in (7) for AF networks depends on $\mathcal{S} \to \mathcal{D}$ link. Obviously, the SNR of cooperative channel (i.e., $\gamma_{r,d}$, the second term in (7)) influences the beamformer design. For notation brevity, $\gamma_{r,d}$ can be rewritten as

$$\gamma_{r,d} = \frac{\mathbf{w}^\dagger \mathbf{h}_{sig} \mathbf{h}_{sig}^\dagger \mathbf{w}}{\mathbf{w}^\dagger (\mathbf{I} + \mathbf{h}_n \mathbf{h}_n^\dagger) \mathbf{w}} \tag{10}$$

where $\mathbf{w}$ and $\mathbf{h}_{sig}^\dagger$ are column vectors, whose entries are w_i and h_{sig}^i, respectively. $\mathbf{h}_n = \mathrm{diag}\{h_n^1, \cdots, h_n^i, \cdots, h_n^{K_R}\}$ is a diagonal matrix. $(\cdot)^\dagger$ denotes the Hermitian or conjugate transpose.

The design of beamformer is equivalent to maximize the $\gamma_{r,d}$ over $\mathbf{w}$ under the constraint of $\|\mathbf{w}\|_F = 1$. It has been shown, in [14, chap. 5.6], that the optimal beamforming vector with unlimited feedback can be simplified as

$$\mathbf{w}^* = \frac{(\mathbf{I} + \mathbf{h}_n \mathbf{h}_n^\dagger)^{-1} \mathbf{h}_{sig}}{\left\|(\mathbf{I} + \mathbf{h}_n \mathbf{h}_n^\dagger)^{-1} \mathbf{h}_{sig}\right\|_F} \tag{11}$$

where $\|\cdot\|_F$ represents the Frobenius norm.

B. Relay Selection Scheme

Different source-relay-destination cooperative pairs have different fading characteristics in multiple-relay networks (i.e., some pairs have constructive impacts on end-to-end BER performance, while others have destructive ones). For a sum power constrained network, the energy should only be allocated to the relay nodes that are constructive to improve the end-to-end BER performance as well as energy efficiency. For the optimal selection scheme, the "best" relay subset (Ψ_m^*) can be formulated as

$$\Psi_m^* = \underset{m \in \{1,2,\cdots,K_R\}}{\arg\min} \, [\mathrm{BER}(\Psi_m)] \tag{12}$$

where Ψ_m is the subset of all the relay nodes in any size. Notice that (12) means the "best" relay subset with m relays should has the minimum end-to-end average BER probability over all the relay combinations. Such subset can be obtained through $2^{K_R} - 1$ searches, but the computation cost exponentially increases with the increasing of K_R.

A simple relay selection scheme is developed to determine which relay nodes perform cooperative beamforming in the second phase. Initially, all the relay nodes are included in the potential relay subset. Then, the ones, which cannot satisfy the threshold will be excluded step by step. Let x denotes a relay node in the relay subset Ψ_{m+1} and cannot satisfy the proposed selection threshold. Consequently, the BER relationship between Ψ_{m+1} and Ψ_m (excluding xth relay) is

$$\frac{P_e^{m+1}}{P_e^m} > 1 \tag{13}$$

where P_e^m denotes the end-to-end average BER probability from source to destination with m relays. Given the relay set Ψ_m, average BER probability for the cooperative beamforming has been developed [15]

$$P_e^m \approx \frac{(2m+1)! N_0^{m+1}}{m!(m+1)!(2c)^{m+1}} \frac{1}{\mathcal{P}_s \Omega_{s,d}} \prod_{i=1}^{m} \left(\frac{1}{\mathcal{P}_s \Omega_{s,i}} + \frac{1}{\mathcal{P}_r \Omega_{i,d}} \right) \tag{14}$$

where c is the coefficient corresponding to modulation type (i.e., $c = 2$ for binary phase shift keying (BPSK) modulation). It has been proven that (14) has a better approximation at high SNRs. Therefore, the left part of (13) can be rewritten as

$$\frac{P_e^{m+1}}{P_e^m} = \prod_{\substack{i \in \Psi_{m+1} \\ i \neq x}} \frac{\frac{1}{\mathcal{P}_s \Omega_{s,i}} + \frac{1}{\mathcal{P}_r \Omega_{i,d}}}{\frac{1}{\mathcal{P}_s \Omega_{s,i}} + \frac{1}{\mathcal{P}_r \Omega_{i,d}}} \cdot \frac{N_0(2m+3)}{c(m+2)} \left(\frac{1}{\mathcal{P}_s \Omega_{s,x}} + \frac{1}{\mathcal{P}_r \Omega_{x,d}} \right). \tag{15}$$

It is obvious that

$$\prod_{\substack{i\in\Psi_{m+1}\\ i\neq x}} \frac{\frac{1}{\mathcal{P}_s\Omega_{s,i}}+\frac{1}{\mathcal{P}_r\Omega_{i,d}}}{\frac{1}{\mathcal{P}_s\Omega_{s,i}}+\frac{1}{\mathcal{P}_r\Omega_{i,d}}}=1.$$

Subject to (13), it should be

$$\frac{N_0(2m+3)}{c(m+2)}\left(\frac{1}{\mathcal{P}_s\Omega_{s,x}}+\frac{1}{\mathcal{P}_r\Omega_{x,d}}\right)>1 \tag{16}$$

and after performing some algebraic derivations in (16), we have

$$\frac{1}{\zeta\Omega_{s,x}}+\frac{1}{(1-\zeta)\Omega_{x,d}}>\frac{\mathcal{P}_{sum}c(m+2)}{N_0(2m+3)}. \tag{17}$$

In the next subsection, we will discuss how to calculate the PA coefficient ζ. It is observed that the selected relays depend on the sum power, size of relay nodes and modulation type. Before source transmission, potential relay nodes can be decided only by mean channel gains. Thus, the selection complexity is considerably reduced since the acquisition of instantaneous CSI is spared, and in the meantime, the payload for exchanging CSI is alleviated. The detailed procedure of the proposed selection cooperative beamforming scheme is introduced in Table 1. Although discussions, here, mainly focus on the selection for optimal beamforming, the results can be easily extended to a limited feedback scenario and further reduce the exchanging information.

Table 1. Structure of the S-BF for AF cooperative networks.

Step 1:	Initially, all potential relay nodes are included in the relay subset Ψ_m and set counter to $m = K_R$.
Step 2:	Calculate the selection metrics of each relay, $\theta_i = \frac{1}{\zeta\Omega_{s,i}} + \frac{1}{(1-\zeta)\Omega_{i,d}}$, and define two quantities (a) $Th_{K_R-m+1} = \frac{\mathcal{P}_{sum}c(m+2)}{N_0(2m+3)}$, (b) $\Gamma_m = \max\limits_{i\in\{1,\cdots,m\}} (\theta_i)$. If $\Gamma_m \leq Th_{K_R-m+1}$, go to **Step 5**, or else if Γ_m cannot satisfy the threshold, $\Gamma_m > Th_{K_R-m+1}$, the ith relay is excluded from Ψ_m.
Step 3:	Set size of relay nodes to $m = m - 1$. Update Γ_m and threshold Th_{K_R-m+1}.
Step 4:	When $m = 1$, go to **Step 5**. Else if $m > 1$, go to **Step 2**.
Step 5:	Finally, each selected relay calculates the beamforming vector with (11). And forward the received signal of first hop multiplied by beamforming vector simultaneously.

Obviously, only K_R searches are needed for our proposed scheme, which reduces the complexity of selection procedure dramatically. The threshold in (17) is a monotone function of sum power and relay number. With fixed sum power, the larger size of subset is chosen, the smaller value of threshold is set, which in turn means more relay nodes selected is a small probability event. On the contrary, the threshold increases with the increasing of sum power when m is fixed. This characteristic shows that smaller size subset is more suitable in poor channel conditions, while larger size subset leads to more cooperative diversity gain in good channel conditions.

C. Power Allocation Scheme

For the sum-power-constraint networks, the overall transmission power of source and selected relays is a significant requisite resource. Adopting the appropriate PA algorithm, we can further improve the end-to-end BER performance. In this section, a simple PA algorithm is proposed. This method can be used to resolve the PA for sum power constrained cooperative networks, where only part of the relay nodes are selected for cooperative beamforming. Mathematically, the PA problem can be written as

$$\begin{aligned}\zeta^* &= \arg\min_{0<\zeta<1}\ (P_e^m)\\ \text{subject to : } \mathcal{P}_s &= \zeta\mathcal{P}_{sum}\\ \mathcal{P}_r &= (1-\zeta)\mathcal{P}_{sum}\\ \mathcal{P}_s,\mathcal{P}_r &\geq 0.\end{aligned} \tag{18}$$

Considering the relay subset with m relay nodes, this optimization problem can be simplified by

$$\begin{aligned}\zeta^* &= \arg\min_{0<\zeta<1}\left(\frac{1}{\zeta\mathcal{P}_{sum}^{m+1}\Omega_{s,d}}\prod_{i=1}^{m}[\frac{1}{\zeta\Omega_{s,i}}+\frac{m}{(1-\zeta)\,\Omega_{i,d}}]\right)\\ \text{subject to : } &\mathcal{P}_s+\mathcal{P}_r=\mathcal{P}_{sum}\\ &\mathcal{P}_{sum}\geq 0,\end{aligned} \tag{19}$$

and with the relation between arithmetic and geometric inequality,

$$\begin{aligned}f(\zeta) &= \frac{1}{\zeta\mathcal{P}_{sum}^{m+1}\Omega_{s,d}}\prod_{i=1}^{m}\left[\frac{1}{\zeta\Omega_{s,i}}+\frac{m}{(1-\zeta)\,\Omega_{i,d}}\right]\\ &\leq \frac{1}{m^m\mathcal{P}_{sum}^{m+1}\Omega_{s,d}}\left\{\sum_{i=1}^{m}\left[\frac{1}{\zeta^{(1+\frac{1}{m})}\Omega_{s,i}}+\frac{m}{\zeta^{\frac{1}{m}}\,(1-\zeta)\,\Omega_{i,d}}\right]\right\}^m.\end{aligned} \tag{20}$$

The following theorem proves the fact that the right part of inequality (20) is a standard convex problem. Furthermore, the convex solution is developed.

Theorem 1: Given function $g(\zeta) = \zeta^{-\frac{1}{m}}[\frac{A}{\zeta} + \frac{Bm}{(1-\zeta)}]$, where $0<\zeta<1$, $A = 1/(\sum\limits_{i=1}^{m}\Omega_{s,i})$, and $B = 1/(\sum\limits_{i=1}^{m}\Omega_{i,d})$. $g(\zeta)$ is a convex function and the convex solution of $g(\zeta)$ is

$$\zeta^* = \frac{-b+\sqrt{b^2-4ac}}{2a} \tag{21}$$

where

$$\begin{aligned}a &= (m+1)(Bm-A),\\ b &= 2A(m+1)-Bm,\\ c &= -A(m+1).\end{aligned}$$

Proof: The second-derivative of $g(\zeta)$ with respect to ζ can be given by

$$\frac{d^2g(\zeta)}{d\zeta^2} = A\left(1+\frac{1}{m}\right)\left(2+\frac{1}{m}\right)\zeta^{-\frac{1}{m}-3}+B\zeta^{-\frac{1}{m}-1}(1-\zeta)^{-3}$$

$$\cdot\left[\left(\frac{m+1}{m}+2m+2\right)\zeta^2+2\left(\frac{m+1}{m}-1\right)\zeta+\frac{m+1}{m}\right]. \tag{22}$$

Obviously, the first term of the right part of (22) is greater than zero. For notation brevity, we define the quadratic function

$$y=\left(\frac{m+1}{m}+2m+2\right)\zeta^2+2\left(\frac{m+1}{m}-1\right)\zeta+\frac{m+1}{m} \tag{23}$$

and with the relation between roots and coefficients

$$\begin{aligned}\Delta&=4\left(\frac{m+1}{m}-1\right)^2-4\left(\frac{m+1}{m}+2m+2\right)\left(\frac{m+1}{m}\right)\\&=\frac{-8m^2-20m-16}{m}\end{aligned} \tag{24}$$

therefore, $\Delta<0$. Since $\frac{m+1}{m}+2m+2>0$, it can be concluded that $y>0$. Consequently, we have $\frac{d^2g(\zeta)}{d\zeta^2}>0$. With the help of [16, chap. 3.1.4], $g(\zeta)$ is a convex function.

The derivative of $g(\zeta)$ with respective to ζ is given by

$$\begin{aligned}\frac{dg(\zeta)}{d\zeta}=&\,m\zeta^{-\frac{1}{m}}\zeta^{-2}(1-\zeta)^{-2}[(Bm-A)(m+1)\zeta^2\\&+(2Am+2A-Bm)\zeta-A(m+1)].\end{aligned} \tag{25}$$

Setting (25) to zero, we get

$$\zeta_1=\frac{-b+\sqrt{b^2-4ac}}{2a}\quad\text{or}\quad\zeta_2=\frac{-b-\sqrt{b^2-4ac}}{2a} \tag{26}$$

and with the prerequisite that $0<\zeta<1$

$$\zeta^*=\frac{-b+\sqrt{b^2-4ac}}{2a} \tag{27}$$

where a, b, and c are defined in Theorem 1. □

Under the PA coefficient ζ, relay nodes will update the metrics θ_i and threshold Th_{K_R-m+1} before each selection. The practical structure of selection AF beamforming algorithm combined with PA is illustrated in Fig. 2.

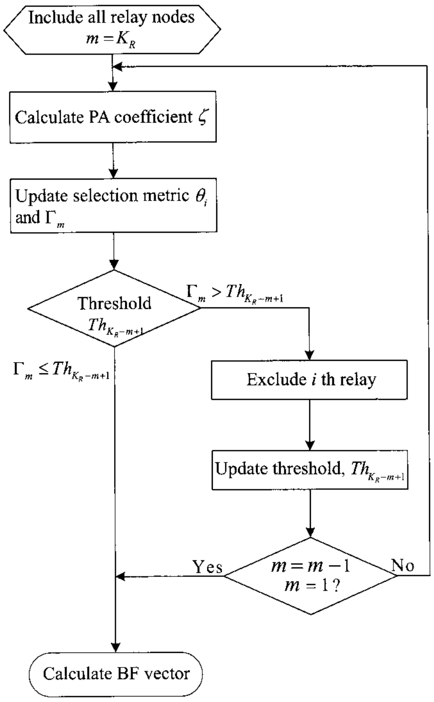


Fig. 2. The procedure of S-BF with PA for AF cooperative networks.

D. Comparison of Energy Efficiency Improvement

For brevity, we define two quantities which denote the ratios of received SNR achieved by AP-BF for AF (AP-BF-AF) to equal PA (EPA) scheme and S-BF-AF to AP-BF-AF scheme, respectively, as

$$G_{AP/Eq}=\frac{\gamma_d^{AP}}{\gamma_d^{Eq}},$$

$$G_{S/AP}=\frac{\gamma_d^{S}}{\gamma_d^{AP}}$$

where γ_d^{AP} is defined in (7). γ_d^{Eq} and γ_d^{S} represent the received SNR of EPA scheme and S-BF-AF, respectively. For the EPA and S-BF-AF schemes, the received SNR at the destination can be easily derived as

$$\gamma_d^{Eq}=\frac{\mathcal{P}_s\left|h_{s,d}\right|^2}{N_0}+\frac{\left|\sum_{i=1}^{K_R}h_{sig}^i\right|^2}{(1+\sum_{i=1}^{K_R}\left|h_n^i\right|^2)N_0} \tag{28}$$

and

$$\gamma_d^{S}=\frac{\mathcal{P}_s\left|h_{s,d}\right|^2}{N_0}+\frac{\left|\sum_{i\in\Psi_m^*}h_{sig}^i w_i\right|^2}{(1+\sum_{i\in\Psi_m^*}\left|h_n^i\right|^2|w_i|^2)N_0} \tag{29}$$

where h_{sig}^i and h_n^i are defined in (8) and (9). EPA can be seen as a special case of AP-BF-AF scheme where $w_i=\mathcal{P}_r/K_R$. Since optimal w_i can maximize the received SNR, the SNR gain of AP-BF-AF over EPA meets $G_{AP/Eq}>1$. From previous discussion, it also can be easily obtained that $G_{S/AP}>1$. Hence, the relation between γ_d^S, γ_d^{AP}, and γ_d^{Eq} is given by

$$\gamma_d^S>\gamma_d^{AP}>\gamma_d^{Eq}. \tag{30}$$

In the following section, simulation results will validate this conclusion.

IV. IMPLEMENTATION IN PRACTICAL APPLICATION

In this section, a semi-distributed method is presented for the proposed cooperative scheme. The "appropriate" relay nodes are selected to perform cooperative beamforming in a distributed manner, while the PA problem between source and "selected" relay nodes is solved by the destination in a centralized manner. We assume that the destination has mean channel gains of all the channels. In Fig. 3, the transmission frame structure of this semi-distributed scheme is depicted.

The source node and destination node send orthogonal training sequences in the first phase. The relay nodes can listen to the pilot to estimate channel variances from the source to relays. The channel variance of $\mathcal{R}\rightarrow\mathcal{D}$ is identical to that of

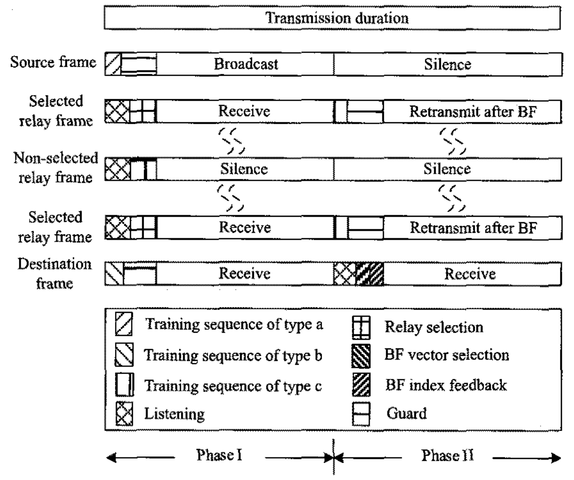


Fig. 3. Transmission frame of the proposed cooperative scheme. Training sequences of "type a", "type b", and "type c" are orthogonal. Guard is the protection interval.

the $\mathcal{D} \rightarrow \mathcal{R}$ because of the reciprocity of uplink and downlink in time-division-duplex (TDD) systems. With (17) and the selection procedure in the previous section, each relay node decides whether it is selected or not. Then, the response to communication (RTC) signal is broadcasted to notify the rest of nodes in the network about its availability. Subsequently, destination calculates the power coefficient ζ^* with (27) and broadcasts it, through which relay nodes update the selection threshold Th_{K_R-m+1}. This procedure iteratively continues until there is no relay node can be excluded. This method costs K_R searches and each relay node only needs channel variance.

It is natural that optimal beamforming with unlimited feedback needs each relay node to have full information of all the channels. However, this is hard for system implementation. In traditional MIMO systems, the optimal beamforming codebook design has been studied in [17]–[19]. For cooperative communication, [20] has already proven the efficiency of Grassmannian codebooks for quantizing the optimal beamforming vectors. In our research, each relay node shares the same codebook, which is design by Grassmannian line packing (GLP). We will not pay much attention to the design of codebook, and more considerations are given to the BER performance gap between the utilization of GLP codebook and optimal beamforming. For each transmission, destination selects the best codeword in the codebook to maximize the received SNR in (29) as

$$m^* = \arg\max_{\mathbf{w}_m \in \mathbb{C}} (\gamma_d^S) \tag{31}$$

where the codebook $\mathbb{C} = \{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_N\}$ consists of N distinct unit vectors as codeword (e.g., $\mathbf{w}_m$ is one of the codewords) and m^* represents the label of the best beamforming codeword in the codebook. Throughout the overall procedure, the destination broadcasts two real number: m^* and ζ^* with $B_1 + B_2$ feedback, where B_1 and B_2 are the amount of bits needed for broadcasting m^* and ζ^*, respectively. Selected relay nodes choose the m^*th codeword from their codebook to perform beamforming, and calculate their transmit power with ζ^*. The proposed scheme is more beneficial in slow fading channels, where mean channel gains will not change dramatically.

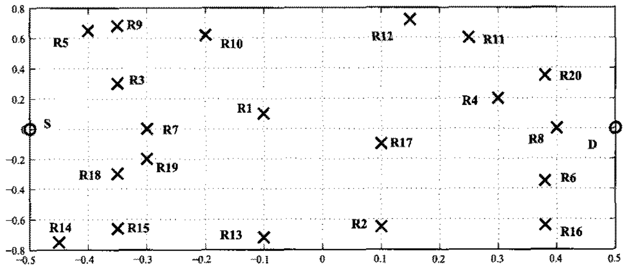


Fig. 4. Relay distribution used in the simulation where 20 relay nodes are randomly deployed.

V. NUMERICAL AND SIMULATION RESULTS

In this section, both the numerical and simulation results are given to show the performance of proposed cooperative scheme. In the simulation, the source and destination nodes adopt single antenna. There are $K_R = 20$ relays randomly located, which also maintain one antenna each. In Fig. 4, the distribution of relay nodes is depicted. We consider a sum power constrained relay network, in which $\mathcal{P}_s + \mathcal{P}_r = \mathcal{P}_{sum}$. It is also assumed that the variance of AWGN at relay and destination nodes is $N_0 = 0$ dB. We will quantify the performance difference between the proposed scheme and other schemes.

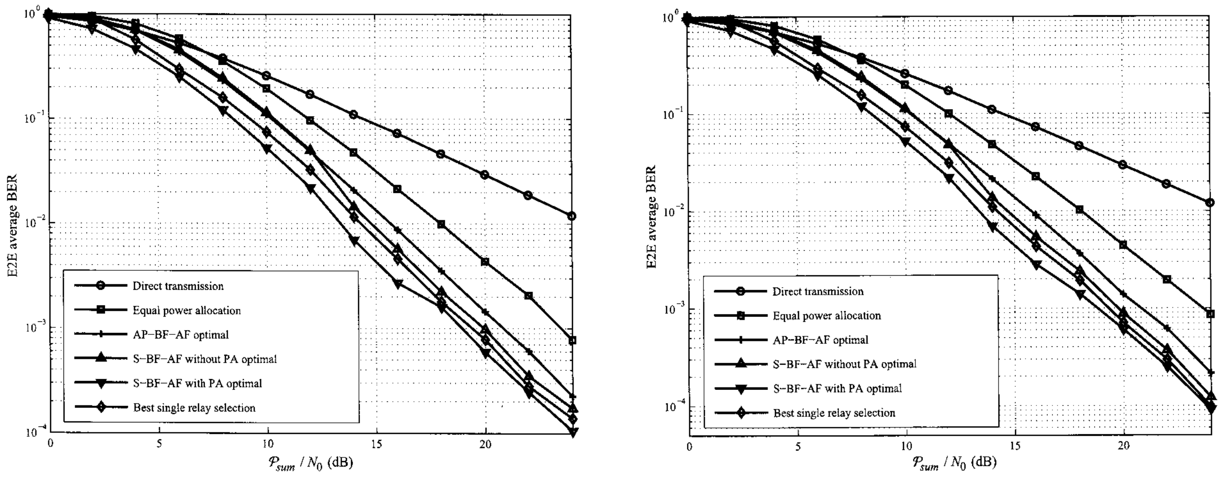
A. Comparison of Proposed Scheme with Other Schemes

In this section, we compare the proposed scheme with other cooperative schemes in the unlimited feedback and limited feedback scenarios. It is assumed that all the channels from source to destination obey complex Gaussian distributions

$$h_{t,r} \sim \mathcal{CN}\left(0, \frac{1}{(1+d)^{\alpha}}\right)$$

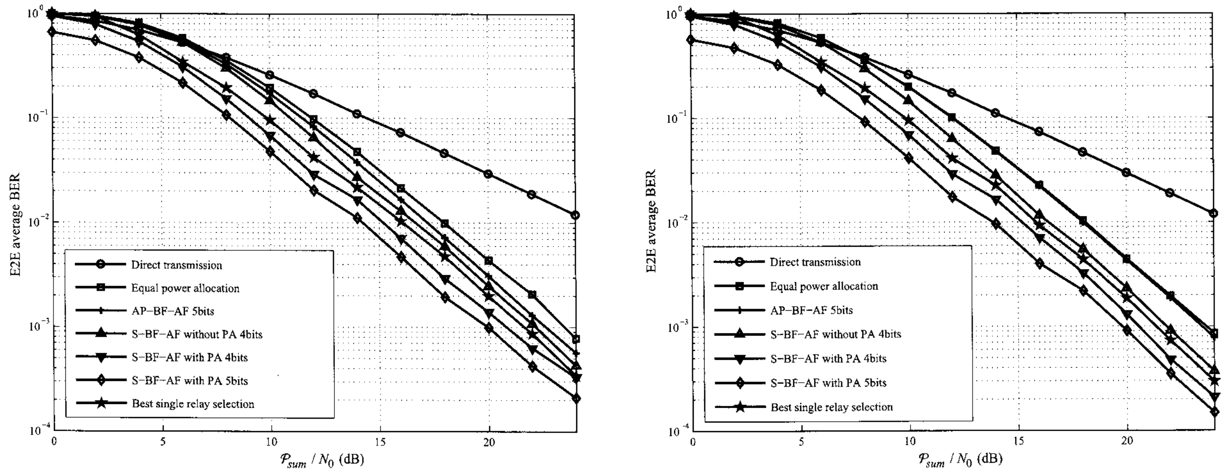
where "t" denotes the transmit node, "r" is the receive node, "d" stands for the distance between node "t" and "r", and α represents the pathloss exponent (i.e., in this paper, $\alpha = 3$).

Fig. 5 shows the average BER performance for the cooperative beamforming, in which 6 relays and 8 relays are deployed, separately. Cooperative beamforming schemes (i.e., AP-BF-AF and S-BF-AF) outperform the EPA among all the relay nodes. In the low SNR region, the performance improvement of S-BF-AF is finite for both 6 relays and 8 relays scenarios. Under this condition, only one relay is selected to cooperate with source-destination pair, and hence dramatically decreases the amount of information exchange across all the relay nodes without performance degradation. However, about 1~2 dB improvement is achieved compared with AP-BF-AF in the high SNR region. When the PA method is adopted, S-BF-AF achieves about 2~3 dB performance improvement regardless of the reign of transmission power. Although (17) is derived based on the high SNR assumption, simulation results prove that S-BF-AF efficiently improves the BER performance. From the figures, one can also see that Fig. 5(b) achieves more performance improvement than Fig. 5(a). The finding shows that the proposed scheme is more meaningful to a larger size cooperative network which subjects to the sum power constraint. Under different SNR, Table 2 demonstrates the index of selected relay nodes for PA and non-PA scenarios, where 8 relays are deployed (i.e., $R1$~$R8$



(a) (b)

Fig. 5. End-to-End average BER probability is compared with respect to $\mathcal{P}_{sum}/N_0$ for AF cooperative networks with unlimited feedback: (a) $K_R = 6$ relay nodes are randomly deployed and (b) $K_R = 8$ relay nodes are randomly deployed.



(a) (b)

Fig. 6. End-to-End average BER probability is compared with respect to $\mathcal{P}_{sum}/N_0$ for AF cooperative networks with limited feedback: (a) $K_R = 6$ relay nodes are randomly deployed and (b) $K_R = 8$ relay nodes are randomly deployed.

Table 2. The index of selected relay nodes under different SNR for PA and non-PA scenarios.

SNR/(dB)	$0 \sim 12$	$14, 16$	$18, 20$	$22, 24$
non-PA	$R1$	$R1, R7$	$R1, R4, R7$	$R1, R4, R7, R8$
PA	$R1$	$R1, R7$	$R1, R7, R8$	$R1, R4, R7, R8$

in Fig. 4). It can be seen that more potential relay nodes will be selected with the increasing of SNR. This observation tells us, under the low SNR region, the limited power should be allocated to the relay with better channel condition. Compared to the non-PA scenario, the selected relay nodes may be a little different when the proposed PA scheme is utilized. It can be also observed that best single relay selection outperforms S-BF-AF without PA, while S-BF-AF with PA has a better performance over best single relay selection scheme.

For the limited feedback scenario, all the relay nodes share the same codebook, which is designed by GLP in [21]. Fig. 6 shows that all the limited feedback scenarios outperform the EPA scheme. It also can be seen that the proposed scheme achieves better performance than the "all participate" cooperative beamforming in the limited feedback scenario. In particular, the increasing amount of feedback leads to much higher performance improvement. Comparing the BER performance between Figs. 5 and 6, our scheme with limited feedback has about 1 dB performance loss than the unlimited feedback scenario in both 6 relays and 8 relays cases. Therefore, limited feedback beamforming is an acceptable substitute to the unlimited feedback one for practical implementation. The best single relay selection scheme also has the same results with unlimited feedback scheme.

B. Improvement of Energy Efficiency over EPA

Fig. 7 compares the improvement of SNR gain over the EPA achieved by S-BF-AF and AP-BF-AF schemes. SNR gain is compared when the number of relay nodes and $\mathcal{P}_{sum}/N_0$ change simultaneously. Through the simulation, it can be observed that the number of selected relay nodes is changing with the increas-

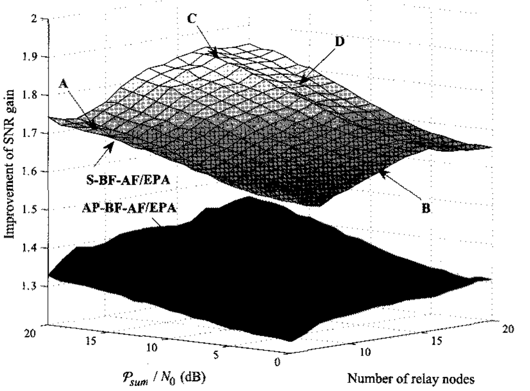


Fig. 7. Improvement of SNR gain with respect to number of relay nodes and $\mathcal{P}_{sum}/N_0$. The gains of S-BF-AF and AP-BF-AF over EPA (i.e., $G_{S/Eq}$ and $G_{AP/Eq}$) are demonstrated, respectively.

ing of sum power, and results in the variant of diversity order with different transmission power levels. As shown in Fig. 7, there are four regions (depicted by "A", "B", "C", and "D") in the figure, where the SNR gain is obviously different. In region "B", about 25% of the transmission power can be saved over AP-BF-AF scheme. The maximum power saved by S-BF-AF is about 50% in region "C". The fact shows that our scheme also outperforms "all participate" cooperative beamforming in terms of energy efficiency.

VI. CONCLUSIONS

In this paper, a cooperative beamforming scheme is presented for dual-hop multiple-relay cooperative networks under the sum power constraint. Different from the "all participate" cooperative beamforming, a relay selection algorithm is utilized to select the "appropriate" relay nodes to perform beamforming. This scheme can make full use of the channel resource. Besides, a simple PA method between source and the selected relay subset is developed to further improve the energy efficiency. For practical implementation, the beamforming with limited feedback is presented to substitute the impractical beamforming with unlimited feedback. Monte-Carlo simulations show that our proposed cooperative scheme has a better BER performance and the BER performance gap between limited and unlimited scenarios is acceptable for implementation. Therefore, we conclude that this selection cooperative beamforming conducted by only those "appropriate" relay nodes could further improve the energy efficiency with low implementation complexity.

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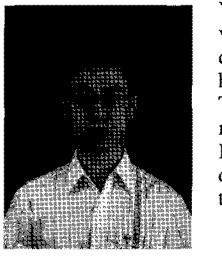

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