Power Allocation in Heterogeneous Networks: Limited Spectrum-Sensing Ability and Combined Protection

Yuehuai Ma, Youyun Xu, and Dongmei Zhang

Abstract: **In this paper, we investigate the problem of power allocation in a heterogeneous network that is composed of a pair of cognitive users (CUs) and an infrastructure-based primary network. Since CUs have only limited effective spectrum-sensing ability and primary users (PUs) are not active all the time in all locations and licensed bands, we set up a new multi-area model to characterize the heterogeneous network. A novel combined interference-avoidance policy corresponding to different PU-appearance situations is introduced to protect the primary network from unacceptable disturbance and to increase the spectrum secondary-reuse efficiency. We use dual decomposition to transform the original power allocation problem into a two-layer optimization problem. We propose a low-complexity joint power-optimizing method to maximize the transmission rate between CUs, taking into account both the individual power-transmission constraints and the combined interference power constraint of the PUs. Numerical results show that for various values of the system parameters, the proposed joint optimization method with combined PU protection is significantly better than the opportunistic spectrum access mode and other heuristic approaches.**

Index Terms: **Cognitive radio, combined protection, heterogeneous network, power allocation, spectrum sensing.**

I. INTRODUCTION

Cognitive radio has been recently suggested as a solution for more efficient spectrum utilization through the principles of spectrum sensing and dynamic spectrum access [1]. In heterogeneous networks that are composed of cognitive systems and primary networks, how to appropriately control or allocate the power of the cognitive users (CUs) to obtain high spectrum-reuse efficiency, under the condition that the primary users (PUs) are not unduly interfered with, is definitely one of the important research issues [2].

At present, there are generally two PU protection models (modes) for operating dynamic spectrum access. The first is spectrum sharing (SS) [2], [3], in which CUs can transmit data even when PUs are present, provided the interference with the PUs caused by the secondary transmission is below a predefined threshold. The second is opportunistic spectrum access (OSA) [4], [5], i.e., CUs are allowed to use a certain band only when it is not occupied by any PUs (this is a spectrum hole). Power allocation under both these models has attracted much research interest. Kang *et al.* [6] research the optimal power allocation with different channel-fading statistics to achieve ergodic capacity and outage capacity in SS mode. Le *et al.* [7] present a framework to operate admission control and power allocation. In the literature, the SS model is also called the spectrum underlay [8]. Using the OSA model, Zhao *et al.* [9] propose a decentralized cognitive MAC protocol considering both the channel-availability decision process and the power allocation process. See [10]–[12] (for SS) and [13]–[15] (for OSA) for a broader overview of the state-of-the-art of cognitive power allocation. However, both OSA and SS have weaknesses. OSA requires the spectrum-sensing module of the cognitive system to perfectly sense PUs in the entire band of interest. This is difficult to achieve over the entire service area of the primary network, because of the different scale fading of the wireless channels and the limited sensing ability of the radio hardware. Moreover, the policy allowing orthogonal transmission prevents the OSA mode from further increasing the spectrum-utilization efficiency. On the other hand, although the SS mode provides an interference-avoidance mechanism when CUs coexist with PUs, it does not include a power-control policy for using the spectrum holes. In practice, because of various factors such as the hidden terminal problems [16] caused by imperfect sensing, or energy leakage on adjacent channels [17], the transmission power of CUs is constrained by not only the individual maximum power budgets but also other interference limitations.

In this paper, we focus on the power allocation for the rate maximization between CUs, considering the practical conditions that CUs have a limited effective sensing area, and PUs appear with some probability in service areas as well as licensed bands. We set up a multi-area model to characterize the heterogeneous network. We also develop a novel combined interference-avoidance policy corresponding to different PU-appearance situations to protect the primary network from unacceptable disturbance. By using dual decomposition, we transform the original power allocation problem into a two-layer optimization problem. We propose a low-complexity joint power-optimizing method taking into account both the individual power-transmission constraints of the CUs and the combined interference power constraint of the PUs. We note that in [18], the authors use a similar but considerably simpler protection idea to define the "power mask" of each subchannel. However, the power mask does not consider interference channel fading influences from CUs to PUs and is applied for totally different sensing models. To the best of our knowledge, we are the first to study power allocation for heterogeneous networks considering both the limited spectrum-sensing area and combined protec-

---

Manuscript received July 06, 2011.

This work has been supported by the National Fundamental Research Program of China under Grant 2009CB3020402, the National Hi-Tech Research and Development Program of China under Grant 2009AA01Z249, and the National Natural Science Foundation of China under Grant 61072043.

The authors are with the Nanjing Institute of Communications Engineering, PLA University of Science and Technology, Nanjing, China, email: {myuehuai, zhangdm72}@163.com, yyxu@vip.sina.com.

Y. Xu is also affiliated with the Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai, China.

1229-2370/11/$10.00 © 2011 KICS

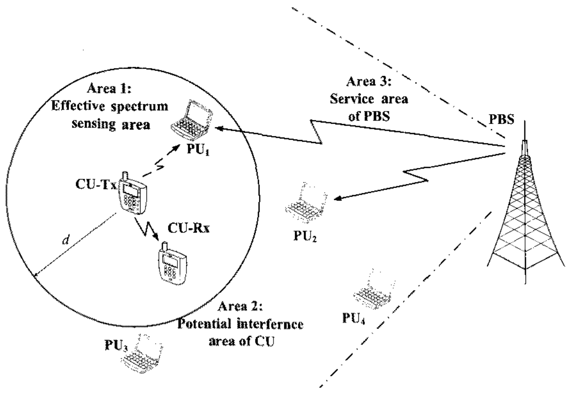


Fig. 1. Multi-area-based system model of heterogeneous network.

tion.

The rest of the paper is organized as follows. Section II presents the system model and the combined PU-protection principle. The joint optimizing power allocation method is discussed in Section III. Section IV gives numerical examples and a performance analysis. Finally, Section V provides concluding remarks.

II. SYSTEM MODEL AND COMBINED PROTECTION SCHEME

A. System Model

The system model is depicted in Fig. 1. This scenario is compatible with the IEEE 802.22 wireless regional area network (WRAN) standard [19]. The infrastructure of the primary network operating in the very high frequency (VHF)/ultra high frequency (UHF) bands consists of the primary base station (PBS) and PUs that are TV users or wireless microphones. The cognitive system consists of a pair of CUs that pursue point-to-point data transmissions as decided by higher-level network protocols. We assume that the currently transmitting CUs are cognitive transceivers (CU-transmitter (Tx) and CU-receiver (Rx)). The total bandwidth B of the network is divided into N independent subbands[1]. According to the sensing ability and interference range of the CUs [20], the network can be divided into three areas. As can be seen from Fig. 1, area 1 is the effective spectrum-sensing area in which the CUs have perfect ability to sense the subchannel occupancy results of the PUs. We assume that area 1 is a circle centered at CU-Tx with a radius of d. We further assume that the distance between the CU transceivers is much smaller than the radius. Area 2 is defined to be the CU's potential interference area, outside of which the interference with the PUs can be considered negligible because of the attenuation of unlicensed signals. Finally, area 3 is the service area of the PBS; the coverage range is not influenced by the cognitive system but depends on the fixed primary-network infrastructure[2].

In this paper, we adopt the symbol structure of [21], which is also compatible with the IEEE 802.22 standard, and assume that the heterogeneous network has perfect synchronization. Each PU occupies at most one subband with a probability of Q_d in every symbol to communicate with the PBS. The occupancy status may change only at the beginning of each symbol.

B. Combined PU-Protection Scheme

In the spectrum-sensing phase, if the CUs sense that there is a PU in a certain subband, say n, in area 1, then the CU-transmission power p_n on that subchannel must satisfy

$$p_n g_n \leq P_{th}^{(1)} \tag{1}$$

where g_n is the channel power gain from CU-Tx to the PU, and $P_{th}^{(1)}$ is the maximum tolerable interference-power threshold of the PU. We assume that the CU can obtain knowledge of g_n and $P_{th}^{(1)}$ [7], [11], and the thresholds for all PUs on every subchannel are equal. On the other hand, since the CUs do not know if there are any PUs in area 2, when the CUs do not sense a PU on subchannel n in area 1, p_n must satisfy

$$p_n d^{-\alpha} \leq P_{th}^{(1)} \tag{2}$$

i.e., the radiation power on the edge of area 1 should be below the PUs' interference-power threshold, where α is the path-attenuation factor.

Given the different occupancy realizations of the PUs on every subchannel, we can divide the subchannels into two sets. Specifically, let Ω_1 be the channel set whose elements represent the subchannels that have not sensed any PUs in area 1, and Ω_2 be the set whose elements are the subchannels that have sensed PUs. Thus, $\Omega_1 \cap \Omega_2 = \emptyset$ and $\Omega_1 \cup \Omega_2 = \{1, \cdots, N\}$. Let h_n be the channel power gain between CU-Tx and CU-Rx, and let the noise at CU-Rx be additive white Gaussian noise (AWGN) with zero mean and variance σ^2. Moreover, if a PU in area 1 uses subchannel n, the interference with the CUs can be viewed as white noise with zero mean and variance σ^2. It is assumed that the interference from the PUs outside area 1 is negligible (or can be integrated into the AWGN). Hence, the maximum transmission rate corresponding to the two PU occupancy statuses on subchannel n is

$$r_n = \begin{cases} \log_2\left(1 + \frac{p_n h_n}{\Gamma(\sigma^2 + \sigma_p^2)}\right), & n \in \Omega_1 \\ \log_2\left(1 + \frac{p_n h_n}{\Gamma \sigma^2}\right), & n \in \Omega_2 \end{cases} \tag{3}$$

where Γ is the signal-to-noise ratio (SNR) gap [22]. With the MQAM modulation, Γ has a simple relationship with bit error rate (BER): $\Gamma = -\ln(5\text{BER})/1.5$.

III. POWER ALLOCATION BASED ON JOINT OPTIMIZATION

Once the necessary channel state information (CSI) and PU occupancy information has been acquired, we focus on how to appropriately allocate power to different subchannels. The goal is to maximize the sum-rate between the CUs, while satisfying not only their individual power-transmission budgets but also the combined interference power protection threshold. Let

---

[1] Throughout this paper, we use the concept of subband and subchannel interchangeably.

[2] We note that in the network model area 2 contains area 1, and area 3 contains area 2.

the power allocation vector of the CUs on all subchannels be $\mathbf{P} = [p_1, p_2, \cdots, p_N]$. The problem can be expressed mathematically as

Problem 1:

$$\mathbf{P}^* = \arg\max_{\{p_n|p_n\geq 0,\; n\in\Omega_1\cup\Omega_2\}} \left\{ \sum_{n\in\Omega_1} \log_2\left(1+\frac{p_n h_n}{\Gamma(\sigma^2+\sigma_p^2)}\right) + \sum_{n\in\Omega_2} \log_2\left(1+\frac{p_n h_n}{\Gamma\sigma^2}\right)\right\} \tag{4}$$

$$\begin{aligned} \text{subject to:} \quad & \text{C1}: \quad p_n g_n \leq P_{th}^{(1)}, \quad \text{if } n\in\Omega_1 \\ & \text{C2}: \quad p_n \leq P_{th}^{(2)}, \quad \text{if } n\in\Omega_2 \\ & \text{C3}: \quad \sum_{n=1}^{N} p_n \leq P_{\max} \end{aligned}$$

where $P_{th}^{(2)} = d^\alpha P_{th}^{(1)}$ and $P_{\max}$ is the maximum power transmission of the CUs.

It can be proved that problem 1 is convex with respect to the power p_n of each subchannel. However, it is not easy to derive its closed form because when a subchannel belongs to different channel sets it has different maximum rates and constraints. This also makes it difficult to derive the Lagrangian function with respect to p_n using the Karush-Kuhn-Tucker (KKT) conditions. However, by observing the characteristics of problem 1 and using dual decomposition theory [23], we can transform problem 1 into an equivalent form:

Problem 2:

$$\mathbf{P}^* = \max_{P_{sum}^{(1)}, P_{sum}^{(2)}} \left\{ \max_{\{p_n|p_n\geq 0,\; n\in\Omega_1\}} \sum_{n\in\Omega_1} \log_2\left(1+\frac{p_n h_n}{\Gamma(\sigma^2+\sigma_p^2)}\right) + \max_{\{p_n|p_n\geq 0,\; n\in\Omega_2\}} \sum_{n\in\Omega_2} \log_2\left(1+\frac{p_n h_n}{\Gamma\sigma^2}\right)\right\} \tag{5}$$

$$\begin{aligned} \text{subject to:} \quad & \text{C1, C2, and} \\ & \text{C4}: \quad \sum_{n\in\Omega_1} p_n \leq P_{sum}^{(1)} \\ & \text{C5}: \quad \sum_{n\in\Omega_2} p_n \leq P_{sum}^{(2)} \\ & \text{C6}: \quad P_{sum}^{(1)} + P_{sum}^{(2)} \leq P_{\max} \end{aligned}$$

where $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$ represent the sum-power constraints of channel sets Ω_1 and Ω_2, respectively. We can easily prove that problems 1 and 2 have identical solutions, but we omit the details for brevity. It can be seen in problem 2 that although constraints are added, the PU-appearance situations are totally separated by the introduction of new variables. Furthermore, we see that the optimizations of $P_{sum}^{(1)}$, $P_{sum}^{(2)}$, and $\{p_n|n\in\Omega_1\}$, $\{p_n|n\in\Omega_2\}$ are independent of each other. Therefore, similarly to [24], we can treat this as a two-layer optimization problem. We first optimize the inner layer, which involves the optimization of $\{p_n|n\in\Omega_1\}$ and $\{p_n|n\in\Omega_2\}$ when $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$ are fixed. We then optimize the outer layer, which involves $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$, to get the optimal power allocation. Notice that this hierarchy is similar to that of the classic dual algorithm in convex optimization theory [25]. We will show that the detailed technique is different.

Let $\mathbf{P}_1 = \{p_n|\forall n\in\Omega_1\}$ and $\mathbf{P}_2 = \{p_n|\forall n\in\Omega_2\}$. When the values of $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$ are given, the power allocation in each subchannel set is equivalent to the following two subproblems:

Subproblem 1:

$$\mathbf{P}_1^* = \arg\max_{\{p_n|p_n\geq 0,\; n\in\Omega_1\}} \sum_{n\in\Omega_1} \log_2\left(1+\frac{p_n h_n}{\Gamma(\sigma^2+\sigma_p^2)}\right) \tag{6}$$

$$\text{subject to:} \qquad \text{C1 and C4}$$

Subproblem 2:

$$\mathbf{P}_2^* = \arg\max_{\{p_n|p_n\geq 0,\; n\in\Omega_2\}} \sum_{n\in\Omega_2} \log_2\left(1+\frac{p_n h_n}{\Gamma\sigma^2}\right) \tag{7}$$

$$\text{subject to:} \qquad \text{C2 and C5.}$$

Subproblem 1 is a convex problem in $\{p_n|n\in\Omega_1\}$. It can be solved by the interior point method or by using a subgradient method to update the Lagrangian coefficients to minimize the dual. However, these approaches are computationally complex, and the interior point method cannot obtain the closed-form solution. To solve subproblem 1 more efficiently, we first present the optimal solution theorem and then propose a low-complexity cognitive water-filling algorithm (CW-FA) to find the optimal solution quickly.

Theorem 1: Let $\Omega_1^A = \{n|p_n g_n < P_{th}^{(1)},\ n\in\Omega_1\}$ and $\Omega_1^B = \{n|p_n g_n \geq P_{th}^{(1)},\ n\in\Omega_1\}$. The sum-rate of subproblem 1 is maximized for the power allocation:

$$p_n = \begin{cases} \left(\Delta_1 - \frac{\Gamma(\sigma^2+\sigma_p^2)}{h_n}\right)^+, & \forall\, n\in\Omega_1^A \\ \frac{P_{th}^{(1)}}{g_n}, & \forall\, n\in\Omega_1^B \end{cases} \tag{8}$$

where $(x)^+ \triangleq \max\{x, 0\}$, and Δ_1 satisfies

$$\sum_{n\in\Omega_1^A}\left(\Delta_1 - \frac{\Gamma(\sigma^2+\sigma_p^2)}{h_n}\right)^+ = P_{sum}^{(1)} - \sum_{n\in\Omega_1^B}\frac{P_{th}^{(1)}}{g_n}. \tag{9}$$

Proof: See the Appendix.

When $n\in\Omega_1^A$, p_n has the same form as the classical water-filling solution. From Theorem 1 and constraint C1 we see that we cannot determine whether subchannel n belongs to set Ω_1^A or Ω_1^B before the value of p_n is given. Moreover, it is difficult to calculate p_n when the subchannel division information is not fixed. One method to overcome this is to identify all the subchannel-division combinations over sets Ω_1^A and Ω_1^B, calculate the power allocation for every combination, and select the one that maximizes the sum-rate and satisfies all the power constraints. Clearly, the high complexity of exhaustive search makes it prohibitive in practice. We therefore develop an iterative CW-FA to find the optimal power allocation more efficiently. The

Table 1. CW-FA.

```
Initialization: Define $\Omega_1^A = \{n | \forall n \in \Omega_1\}$, $\Theta = \emptyset$, $P_{\text{int}} = P_{sum}^{(1)}$.
S1) Taken $P_{\text{int}}$ as sum-power constraint, implement water-filling
    based power allocation for all the subchannels $n \in \Omega_1^A$ to get
    $p_n$, i.e., $p_n = (\Delta_1 - \Gamma(\sigma^2 + \sigma_p^2)/h_n)^+$, $n \in \Omega_1^A$, where
    $\Delta_1$ satisfies: $\sum_{n \in \Omega_1^A} (\Delta_1 - \Gamma(\sigma^2 + \sigma_p^2)/h_n)^+ = P_{\text{int}}$.
S2) For every $n \in \Omega_1^A$, check whether $p_n > P_{th}^{(1)}/g_n$?
    If there exists a subchannel $n^*$ satisfies the inequality, let
    $p_{n^*} = P_{th}^{(1)}/g_{n^*}$, goto step 3), otherwise, the algorithm is end.
S3) Update: $\Omega_1^A = \Omega_1^A / \{n^*\}$, $P_{\text{int}} = P_{\text{int}} - P_{th}^{(1)}/g_{n^*}$,
    $\Theta = \Theta \cup \{P_{th}^{(1)}/g_{n^*}\}$.
S4) Check whether $\Omega_1^A = \emptyset$? or $p_n \le P_{th}^{(1)}/g_n$? $\forall n \in \Omega_1^A$.
    If either of above condition is satisfied, the algorithm is end;
    otherwise, goto step S1)
```

basic idea of the algorithm is that at each step, we perform classical water-filling for all the subchannels remaining in set Ω_1^A. If the power obtained by water-filling on some channel is beyond the corresponding interference threshold, we set the power of the subchannel equal to the threshold, remove the channel from set Ω_1^A, and update the sum-power of the set. This process is performed iteratively until the powers on all the channels satisfy the interference power constraints, or there are no more subchannels in set Ω_1^A. A detailed description is given in Table 1.

In this algorithm, $\emptyset$ represents the empty set, and the power allocation of CW-FA is $\{p_n \, | \forall n \in \Omega_1^A\} \cup \Theta$. The complexity of the classical water-filling algorithm increases linearly with the number of channels, which is $O(|\Omega_1|)$ here. From the algorithm, it can be seen that in the worst case, the number of iterations of CW-FA is $|\Omega_1| - 1$, so the complexity of CW-FA is approximately $O(|\Omega_1|^2)$. Since the complexity of the exhaustive search method is $O(|\Omega_1| 2^{|\Omega_1|})$ and that of the interior point method [25] is at least $O(|\Omega_1|^{3.5} \log_2(1/e))$, the superiority of CW-FA is obvious[3].

For subproblem 2, we have Theorem 2 similarly as follows:

Theorem 2: Let $\Omega_2^A = \{n | p_n < P_{th}^{(2)}, \; n \in \Omega_2\}$ and $\Omega_2^B = \{n | p_n \ge P_{th}^{(2)}, \; n \in \Omega_2\}$. The optimal power allocation of subproblem 2 to maximize the sum-rate of CU is

$$p_n = \begin{cases} (\Delta_2 - \Gamma\sigma^2/h_n)^+ & \forall \, n \in \Omega_2^A \\ P_{th}^{(2)} & \forall \, n \in \Omega_2^B \end{cases} \tag{10}$$

where Δ_2 satisfies $\sum_{n \in \Omega_2^A} (\Delta_2 - \Gamma\sigma^2/h_n)^+ = P_{sum}^{(2)} - \sum_{n \in \Omega_2^B} P_{th}^{(2)}$.

Both the proof and the operating algorithm for Theorem 2 are similar to those for Theorem 1. We have already introduced the inner-layer optimization of p_n when $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$ are fixed. The remaining task is to optimize $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$, which belong to the outer-layer optimization. The optimal values of $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$ can be found by an exhaustive search over $[0, \; P_{\max}]$. However, it can be easily proved (and is further confirmed by the simulations in the next section) that problem 2 is convex for $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$, so we can apply the highly efficient bisection method to update the sum-power variables. From the above analysis, we conclude that the total complexity of the joint optimization method for problem 2 is $O\left((|\Omega_1|^2 + |\Omega_2|^2) \log_2(1/\varepsilon)\right)$, where ε is the convergence precision of $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$. This is acceptable in multiuser heterogeneous networks.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we provide numerical results for the power allocation method considering the limited sensing area of the CUs and the combined PU protection. We assume that the symbol duration is 100 ms, and the total bandwidth of the system is 6 MHz, which is equally divided into eight independent subbands [19]. The radius of the effective sensing area (area 1) is d = 300 m, and the required BER of the CU is 10^{-3}. We assume that the PUs are uniformly distributed in area 1, and CU-Rx is uniformly distributed within a 30-m radius of CU-Tx. The path-loss exponent is $\alpha = 4$. We do not consider the shadowing effect, and we model the multipath fading of all links as a three-path Rayleigh with an exponential power delay profile. We assume that the noise power σ^2 and the PU-interference power σ_p^2 at CU-Rx are both -100 dBW, the interference-power threshold of each PU is -90 dBW, and the subchannel occupancy probability of each PU is $Q_d = 0.3$.

Firstly, we set the maximum transmission power of the CUs to 0 dBW (1 W) and evaluate the convexity of problem 2 for $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$. In Fig. 2, the maximum sum-rate of the CUs per symbol per Hz is plotted versus $P_{sum}^{(1)}$ (or $P_{\max} - P_{sum}^{(1)}$) for one random realization of all the links. We use exhaustive search to list all the possible values within the interval $[0, \; P_{\max}]$. This figure also reveals the characteristics of the outer-layer optimization. The figure shows that the maximum sum-rate is a concave function of $P_{sum}^{(1)}$, and a uniquely optimal point ($P_{sum}^{(1)} = 0.342$ W, $P_{sum}^{(2)} = 0.658$ W) of set Ω_1 and Ω_2 is the final maximum sum-rate of problem 2. Through multiple simulations with different realizations, we discover that all the sum-rate curves reveal similar concave characteristics of $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$. Hence, in the simulations, we use the bisection method instead of exhaustive search to optimize $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$, averaging the results of 10,000 independent experiments.

Fig. 3 shows the average maximum sum-rate of CU achieved by our joint method and OSA[4]. The random-choosing $P_{sum}^{(1)}$ and $P_{sum}^{(2)}$ method and the interior point method for subproblem 1 are presented for comparison (see Table 2). The figure shows that the joint optimization method significantly outperforms OSA in sum-rate for the entire $P_{\max}$ range (−30–30 dBW), especially when $P_{\max} \ge 10$ dBW. This is because when $P_{\max}$ is sufficiently large, the system with combined protection has more flexibility to divide power between the two types of subchannels (occupied or not occupied by PUs), while the rate-increasing speed of the OSA mode is relatively low since limited num-

---

[3] Note that e in the expression $O(|\Omega_1|^{3.5} \log_2(1/e))$ represents the convergence precision of the interior point algorithm, and $|\Omega_1|$ represents the cardinality of set Ω_1.

[4] As discussed in the Introduction, since SS mode does not give the leisure spectrum using the protection principle, we do not compare the performance of SS with our method, but we have borrowed the SS idea when PU presents in our combined protection mechanism.

Table 2. Different power allocation schemes.

Legend in the figure	**Inner-layer optimization**	**Outer-layer optimization**
Interior point+ bisection method	Uses interior point method to solve subproblem 1 and 2	Uses bisection method to update $P_{sum}^{(1)}$, $P_{sum}^{(2)}$
Joint optimization method	Uses CW-F algorithm to solve subproblem 1 and 2	Uses bisection method to update $P_{sum}^{(1)}$, $P_{sum}^{(2)}$
CW-FA $+P_{sum}^{(1)}$, $P_{sum}^{(2)}$ **random**	Uses CW-F algorithm to solve subproblem 1 and 2	Uses random choosing for $P_{sum}^{(1)}$, and $P_{sum}^{(2)} = P_{\max} - P_{sum}^{(1)}$
OSA mode	Uses non-PU-occupation subbands only	

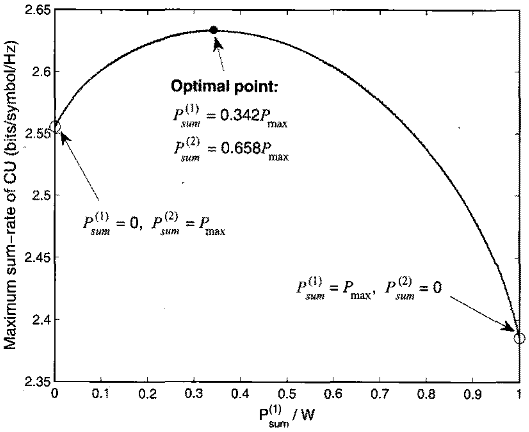


Fig. 2. Maximum sum-rate of CU versus $P_{sum}^{(1)}$ ($P_{\max} = 0$ dBW, $P_{th}^{(1)} = -90$ dBW)

bers of subchannels can be used. An interesting phenomenon can be observed in Fig. 3: When $P_{\max} \geq 16$ dBW, even the "CW-FA + $P_{sum}^{(1)}$, $P_{sum}^{(2)}$ random" scheme achieves a transmission rate higher than that of the OSA mode. This is also because the combined protection has more freedom in channel selection and power allocation. The interior point + bisection method has the same rate as the joint optimization method. This is expected since both the interior point algorithm and CW-FA can find the optimal solution of the convex subproblem.

Finally, Figs. 4 and 5 show the average maximum sum-rate of the CUs per symbol per Hz versus the CUs' required BER and the number of subbands, respectively. These two figures show that the rate performance of the proposed combined protection and joint optimization method is better than the performance of OSA for different BER values and numbers of subchannels. This illustrates the superiority of our scheme for different values of the system parameters.

V. CONCLUSION

Using a multi-area heterogeneous network model and the limited effective spectrum-sensing area of the CUs, we have proposed a combined interference-avoidance principle to protect the primary-network communication. We use dual decomposition to transform the original power allocation problem into a two-layer optimization. For the inner-layer optimization, we give the optimal solution theorem and develop a CW-F algorithm with low complexity to find the solution. For the outer-layer optimization, we use the bisection method to update the sum-power of the channel sets. Numerical results show that the power allocation scheme performs significantly better than the OSA mode and other heuristic approaches in term of the average maximum sum-rate for different values of the parameters. In future research, we will explore the following two questions: 1) In a multi-CU environment, when the PUs appear in different areas corresponding to different CUs, how does combined protection influence the primary network? 2) How to find a tradeoff between the maximizing the rate of cognitive systems and minimizing the collision probability of heterogeneous users, when there is a false-alarm probability and missed-detection probability in the effective sensing area?

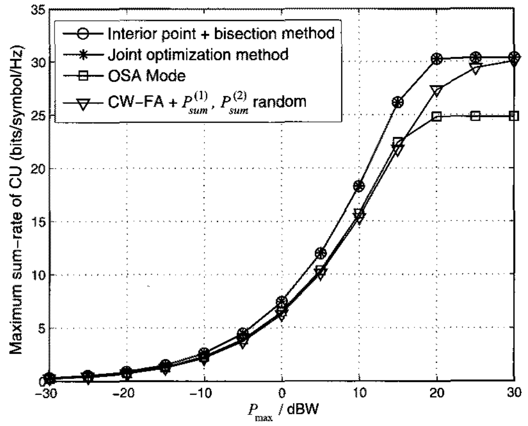


Fig. 3. Maximum average sum-rate of CU versus $P_{\max}$ ($P_{\max} = 0$ dBW, $P_{th}^{(1)} = -90$ dBW)

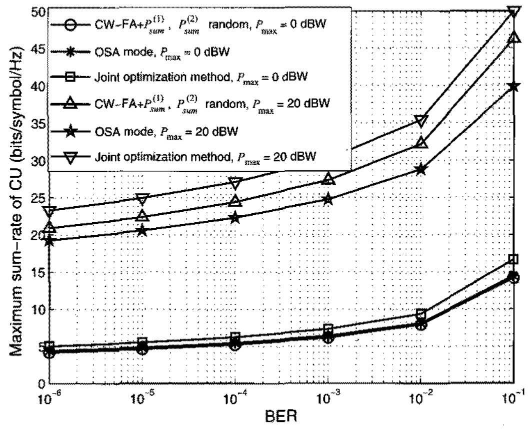


Fig. 4. Maximum average sum-rate of CU versus required BER ($P_{\max} =$ 0 dBW and 20 dBW, $P_{th}^{(1)} = -90$ dBW).

APPENDIX

Proof of Theorem 1

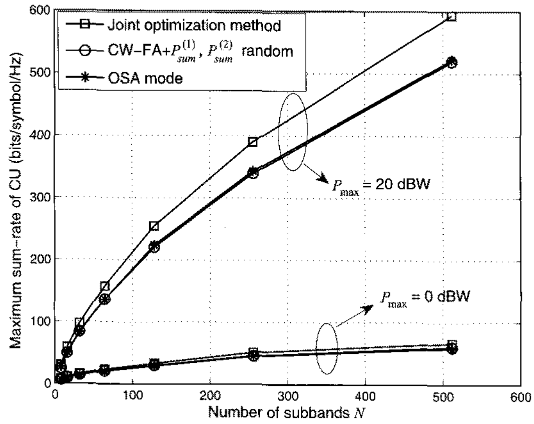


Fig. 5. Maximum average sum-rate of CU versus number of subchannels N ($P_{\max} = 0$ dBW and 20 dBW, $P_{th}^{(1)} = -90$ dBW)

First, we write the Lagrangian function of subproblem 1 as

$$L\left(\mathbf{P}_1^*, \lambda, \mu_n, v_n\right) = \sum_{n\in\Omega_1} \log_2\left(1 + \frac{p_n h_n}{\Gamma(\sigma^2+\sigma_p^2)}\right) - \lambda\left(\sum_{n=1}^{N} p_n - P_{sum}^{(1)}\right) - \sum_{n\in\Omega_1} \mu_n\left(p_n - \frac{P_{th}^{(1)}}{g_n}\right) + \sum_{n\in\Omega_1} v_n p_n \tag{11}$$

where $\lambda \geq 0$, $\mu_n \geq 0$, $v_n \geq 0$, $n \in \Omega_1$ are Lagrange coefficients corresponding to the constraints. When a problem is strictly convex, the KKT conditions are necessary and sufficient for the optimal power allocation. The KKT conditions of subproblem 1 are

$$\frac{\partial L(\mathbf{P}_1^*, \lambda, \mu_n, v_n)}{\partial p_n} = \frac{\frac{1}{\ln 2}}{\frac{p_n + \Gamma(\sigma^2+\sigma_p^2)}{h_n}} - \lambda - \mu_n + v_n = 0, \quad \forall n \in \Omega_1 \tag{12}$$

$$\lambda\left(\sum_{n\in\Omega_1} p_n - P_{sum}^{(1)}\right) = 0 \tag{13}$$

$$\mu_n\left(p_n - \frac{P_{th}^{(1)}}{g_n}\right) = 0, \quad \forall n \in \Omega_1 \tag{14}$$

$$v_n p_n = 0, \quad \forall n \in \Omega_1 \tag{15}$$

and constraints C1 and C4. We now give the derivation of (8) and (9). From (12), we get

$$p_n = \frac{\frac{1}{\ln 2}}{\lambda + \mu_n - v_n} - \frac{\Gamma(\sigma^2+\sigma_p^2)}{h_n}, \quad \forall n \in \Omega_1. \tag{16}$$

1) For $\forall n \in \Omega_1^B$, clearly that $p_n = P_{th}^{(1)}\big/g_n$, since $p_n g_n > P_{th}^{(1)}$ is not allowed $p_n g_n > P_{th}^{(1)}$ when PUs exist, and if $p_n g_n < P_{th}^{(1)}$, subchannel n would belong to set Ω_1^A.

2) For $\forall n \in \Omega_1^A$, from $p_n g_n < P_{th}^{(1)}$ and (14), we can get $\mu_n = 0$, so (16) can be transformed to

$$p_n = \frac{\frac{1}{\ln 2}}{\lambda - v_n} - \frac{\Gamma(\sigma^2+\sigma_p^2)}{h_n}. \tag{17}$$

It can be discussed that when $v_n > 0$, from (15), it follows that $p_n = 0$. On the other hand, $p_n > 0$ when $v_n = 0$, therefore, (17) reduces to $p_n = 1/\lambda \ln 2 - (\Gamma(\sigma^2+\sigma_p^2)/h_n)$. In conclusion, we obtain

$$p_n = \left(\frac{1}{\lambda \ln 2} - \frac{\Gamma(\sigma^2+\sigma_p^2)}{h_n}\right)^+, \quad \forall n \in \Omega_1^A. \tag{18}$$

Set $\Delta = 1/\lambda \ln 2$, then from 1) and 2), we can derive (8).

3) From (18), we have $\lambda \geq \frac{h_n}{\Gamma(\sigma^2+\sigma_p^2)\ln 2} > 0$, so given (13), we obtain $\sum_{n\in\Omega_1} p_n = P_{sum}^{(1)}$, i.e.,

$$\begin{aligned} P_{sum}^{(1)} &= \sum_{n\in\Omega_1} p_n = \sum_{n\in\Omega_1^A} p_n + \sum_{n\in\Omega_1^B} p_n \\ &= \sum_{n\in\Omega_1^A}\left(\Delta - \frac{\Gamma(\delta^2+\delta_p^2)}{h_n}\right)^+ + \sum_{n\in\Omega_1^B} \frac{P_{th}^{(1)}}{g_n} \end{aligned} \tag{19}$$

which is also (9).

Now we have derived (8) and (9) from the KKT conditions, which confirm the 'only if' part (sufficient condition). Finally, since the KKT conditions are sufficient and necessary for the optimal power allocation, the 'if' part of Theorem 1 immediately follows. This completes the proof.

REFERENCES

[1] Federal Communications Commission, "FCC adopts rule changes for smart radios," in Cognitive Radio Technologies Proceeding (CRTP), ET Docket No. 03–08, 2005.

[2] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[3] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, 2009.

[4] J. Mitola, III, "Cognitive radio: An integrated agent architecture for software defined radio," Ph.D. dissertation, KTH, Stockholm, Sweden, 2000.

[5] DARPA XG WG, "The XG Architectural Framework V1.0," 2003.

[6] X. Kang, Y. C. Liang, A. Nallanathan, H. K. Garg and R. Zhang, "Optimal power allocation for fading channels in cognitive radio networks: Ergodic capacity and outage capacity," *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 940–950, Feb. 2009.

[7] L. B. Le and E. Hossain, "Resource allocation for spectrum underlay in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5306–5315, Dec. 2008.

[8] M. Hoyhtya, A. Hekkala, M. Katz, and A. Mammela, "Spectrum awareness: Techniques and challenges for active spectrum sensing," *Cognitive Wireless Networks*, Eds. Springer, 2007, ch. 18, pp. 353-372.

[9] Q. Zhao, L. Tong, A. Swami, and Y. Chen, "Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework," *IEEE J. Select. Areas Commun.*, vol. 25, no. 3, pp. 589–600, Apr. 2007.

[10] L. Musavian and S. Aissa, "Capacity and power allocation for spectrum-sharing communications in fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp.148–156, Jan. 2009.

[11] Y. Zhang and C. Leung, "Cross-layer resource allocation for mixed services in multiuser OFDM-based cognitive radio systems," *IEEE Trans. Veh. Technol.*, vol. 58, no. 8, pp. 4605–4619, Oct. 2009.

[12] M. Almalfouh and L. Stuber, "Interference-aware power allocation in cognitive radio networks with imperfect spectrum sensing," in *Proc. IEEE ICC*, Cape Town, South Africa, pp. 1–6, May 2010.

[13] R. Wang, V. Lau, L. Lv, and B. Chen, "Joint cross-layer scheduling and spectrum sensing for OFDMA cognitive radio systems," *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, pp. 2410–2416, May 2009.

[14] W. Ren, Q. Zhao, and A. Swami, "Power control in cognitive radio networks: How to cross a multi-lane highway," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1283–1296, Sept. 2009.

[15] Y. Song and J. Xie, "Optimal power control for concurrent transmissions of location-aware mobile cognitive radio ad hoc networks," in *Proc. IEEE GLOBECOM*, Honolulu, USA, pp. 1–6, Nov. 2009.

[16] S. Srinivasa and S. Jafar, "The throughput potential of cognitive radio: A theoretical perspective," *IEEE Commun. Mag.*, vol. 45, no. 5, pp. 73–79, May 2007.

[17] T. A. Weiss and F. K. Jondral, "Spectrum pooling: An innovative strategy for the enhancement of spectrum efficiency," *IEEE Commun. Mag.*, vol. 42, no. 3, pp. S8–S14, Mar. 2004.

[18] L. Gao and S. Cui, "Multi-band power and rate control for cognitive radios with energy constraints: A dynamic programming approach," in *Proc. IEEE ICC*, Beijing, China, May 2008, pp. 3563–3567.

[19] "Draft standard for wireless regional area networks part 22: Cognitive wireless RAN medium access control (MAC) and physical layer (PHY) specifications: Policies and procedures for operation in the TV Bands," IEEE Std. 802.22, P802.22/D0.2, Nov. 2006.

[20] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.

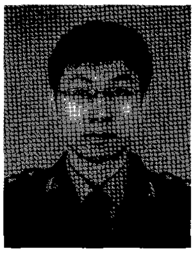
[21] Y. C. Liang, Y. Zeng, E. Peh, and A. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1326–1337, Apr. 2008.

[22] A. J. Goldsmith and S. G. Chua, "Variable-rate variable-power MQAM for fading channels," *IEEE Trans. Commun.*, vol. 45, no. 10, pp. 1218–1230, Oct. 1997.

[23] D. P. Palomar and M. Chiang, "A tutorial on decomposition methods for network utility maximization," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1439–1451, Aug. 2006.

[24] L. Zhang, Y. Xin, and Y. C. Liang, "Weighted sum rate optimization for cognitive radio MIMO broadcast channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2950–2959, June 2009.

[25] S. Boyd and L. Vandenberghe, *Convex Optimization.* Cambridge University Press, 2004.



Yuehuai Ma was born in Xuanhua, China, on April 12, 1981. He received the B.S. and M.S. degrees in Electrical Engineering and the Ph.D. degree in Military Communications from the PLA University of Science and Technology, China, in 2003, 2007, and 2011, respectively. His major interests are cognitive radio, spectrum management, cooperative communication, wireless signal processing, and etc.

Youyun Xu was born in 1966. He was graduated from Shanghai Jiao Tong University with Ph.D. Degree in Information and Communication Engineering in 1999. He is currently a Professor with Nanjing Institute of Communication Engineering, China. He is also a Part-Time Professor with the Institute of Wireless Communication Technology of Shanghai Jiao Tong University (SJTU), China. He has more than 20 years professional experience of teaching and researching in communication theory and engineering. Now, his research interests are focusing on new generation wireless mobile communication system (IMT-advanced and related), advanced channel coding and modulation techniques, multi-user information theory and radio resource management, wireless sensor networks, cognitive radio networks, etc. He is a Senior Member of IEEE, a Senior Member of Chinese Institute of Electronics.

Dongmei Zhang was born in 1972. She was graduated from Nanjing Institute of Communication Engineering with M.S. degree in Information and Communication Engineering in 2005. She is currently an Associate Professor with Nanjing Institute of Communication Engineering, PLA University of Science and Technology. Her research interests include mobile communication, cognitive radio, wireless resource management, and etc.