Tight Lower Bound of Optimal Non-Coherent Detection for FSK Modulated AF Cooperative Communications in Rayleigh Fading Channels

Jian Tian, Qi Zhang, and Fengqi Yu

Abstract: **When wireless channels undergo fast fading, non-coherent frequency shift keying (FSK) (de)modulation schemes may be considered for amplify-and-forward (AF) cooperative communications. In this paper, we derive the bit-error-rate performance of partial non-coherent receiver as a lower bound of the optimal non-coherent receiver for FSK modulated AF cooperative communications. From the simulation and analytical results, it is found that the derived lower bound is very closed to simulation results. This result shows that knowing partial channel state information may not improve system performance significantly. On the other hand, conventional optimal non-coherent receiver involves complicated integration operation. To address the above complexity issue, we also propose a near optimal non-coherent receiver which does not involve integration operation. Simulation results have shown that the performance gap between the proposed near optimal receiver and the optimal receiver is small.**

Index Terms: **Amplify-and-forward (AF), cooperative communications, heterogeneous networks (HetNets), non-coherent detection, relay.**

I. INTRODUCTION

Recently, heterogeneous networks (HetNets) have received significant attention as an efficient way to improve the capacity and coverage in wireless networks [1]–[3]. For HetNets, the communications through cooperation between the macro evolved Node-Bs (eNBs) and home evolved Node-Bs (HeNBs) can improve system performance [4]. Cooperative communications are able to provide extra spatial diversity for conventional single antenna transceivers to combat fading in wireless networks [5]–[15]. In cooperative communications, the cooperative nodes relay the signals according to different relaying protocols, such as amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF) [5]–[8]. Among them, AF protocol is widely employed in the situations where the relaying nodes have limited ability of signal processing.

The coherent detection for AF cooperative communications, studied in [9], requires perfect channel state information (CSI) known at the destination. To obtain CSI, we should transmit extra pilot symbols and estimate the wireless channels, which reduces the network throughput and increases the system complexity. Especially when the wireless channels vary rapidly, the coherent detection at the destination is almost impossible.

When the channels undergo fast fading, non-coherent modulation and detection for AF cooperative communications may be utilized [10]–[15]. In [11], Annavajjala *et al.* proposed the optimal non-coherent detector for frequency shift keying (FSK) modulated AF cooperative communications in a Rayleigh fading environment. The optimal non-coherent detector in [11] involves integration operation, which causes the system performance analysis is very difficult. So far, to our best knowledge, the exact bit-error-rate (BER) performance or even the tight upper or lower bound has not been derived.

On the other hand, the integration operation involved in the proposed optimal detector in [11] makes the receiver for FSK modulated AF cooperative communications have high complexity. To address the complexity issue in AF cooperative communications, Zhu *et al.* proposed suboptimal non-coherent detection schemes for on-off keying (OOK) modulated AF cooperative communications where Jensen's inequality is employed to approximate the product of two independent zero-mean complex Gaussian random variables [12].

In this paper, we first derive the BER performance of partial non-coherent receiver as a lower bound of optimal detector for FSK modulated AF cooperative communication in Rayleigh fading channels. Then, we propose a near optimal non-coherent detection scheme. The proposed near optimal detection scheme does not involve complicated numerical integration. It will be shown in the Section V that compared with the optimal detection scheme which requires integration, the proposed suboptimal scheme suffers negligible performance degradation.

The rest of this paper is organized as follows. Section II describes the system mode of cooperative communication networks. In Section III, we derive the BER performance of partial non-coherent detector as a lower bound of optimal non-coherent detector. In Section IV, we propose the near optimal non-coherent detector. Simulated and theoretical results are presented and discussed in Section V. Finally, conclusions are given in Section VI.

II. SYSTEM MODEL

In a flat Rayleigh fading environment, we consider a three-node wireless cooperative communications network where node

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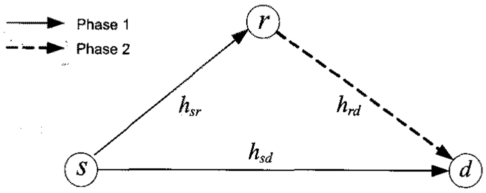

Fig. 1. System model for cooperative communication network.

s communicates with node d at the assistance of node r, as illustrated in Fig. 1. In this cooperative communications network, each node works in half-duplex mode. This is because that each node cannot transmit and receive signals simultaneously on the same frequency band. Thus, the information transmission from the node s to node d is divided into two phases. In the first phase, node s broadcasts its signals. In the second phase, node r retransmits the received signals from node s to node d.

In this paper, the binary information symbols transmitted from node s, denoted as b_s, are modulated through binary FSK modulation. For binary FSK modulation scheme, if $b_s = 1$, the node s will transmit $\sqrt{E_s}e^{i2\pi f_1 t}$ and if $b_s = 0$, the node s will transmit $\sqrt{E_s}e^{i2\pi f_2 t}$, where f_1 and f_2 are orthogonal frequency tones and i denotes $\sqrt{-1}$. We employ the low-pass equivalent complex-valued model to describe the received signals which have passed through the matched filter [11]. Thus, the FSK modulated signals transmitted from node s, denoted as $\mathbf{x}_s$, are expressed as

$$\mathbf{x}_s = \begin{cases} \mathbf{x}_1 = \left(\sqrt{E_s}, 0\right)^\dagger; & \text{if } b_s = 1 \\ \mathbf{x}_0 = \left(0, \sqrt{E_s}\right)^\dagger; & \text{if } b_s = 0 \end{cases} \tag{1}$$

in which $\dagger$ denotes the transpose and conjugate operation. The relationships between the transmitted and received signals at the relay and the destination are as follows.

$$\mathbf{y}_{sd} = h_{sd}\mathbf{x}_s + \mathbf{n}_{sd} \tag{2}$$
$$\mathbf{y}_{sr} = h_{sr}\mathbf{x}_s + \mathbf{n}_{sr} \tag{3}$$
$$\mathbf{y}_{rd} = A_r h_{rd}\mathbf{y}_{sr} + \mathbf{n}_{rd}. \tag{4}$$

In (2)–(4), the vector $\mathbf{y}_{pq}$ denotes the received signal at node q transmitted from node p, $p, q \in \{s, r, d\}$; h_{pq} denotes the channel fading coefficient from node p to node q; and the vector $\mathbf{n}_{pq}$ denotes the additive white Gaussian noise (AWGN) at node q when node p transmits signals to node q. The channel fading coefficient h_{pq} is circular symmetric complex Gaussian random variable with variance Ω_{pq}. Same as the channel fading coefficient, the entries of the noise vector $\mathbf{n}_{pq}$ are also circular symmetric complex Gaussian random variables, whose variance is N_o. In (2)–(4), the fading coefficients $\{h_{sd}, h_{sr}, h_{rd}\}$ are mutually independent. Throughout this paper, the instantaneous channel fading coefficients $\{h_{sd}, h_{sr}, h_{rd}\}$ are assumed to be unknown at any node. Hence node d has to employ non-coherent detection. In (4), A_r is the amplification factor which satisfies the constraint

$$A_r^2 \mathrm{E}\left[\mathbf{y}_{sr}^\dagger \mathbf{y}_{sr}\right] = E_r \tag{5}$$

where $\mathrm{E}[\cdot]$ denotes the expectation of $[\cdot]$ and E_r denotes the transmission power per symbol at node r. Thus,

$$A_r = \left(\frac{E_r}{E_s\Omega_{sr} + 2N_o}\right)^{\frac{1}{2}}. \tag{6}$$

It is worth noting that the relaying node r amplifies and retransmits the signals and noises over two frequencies. Thus, in the denominator of (6), the noise power is doubled.

Without loss of generality, we assume that node s transmits $\mathbf{x}_1$. At the receiver of node d, the received signals transmitted from node s over frequencies f_1 and f_2 are

$$y_{sd,1} = \sqrt{E_s}h_{sd} + n_{sd,1} \tag{7}$$

and

$$y_{sd,2} = n_{sd,2}, \tag{8}$$

respectively. In (7)–(8), $y_{sd,1}$ and $y_{sd,2}$ are zero mean complex Gaussian variables whose variances are $N_o(1 + \gamma\Omega_{sd})$ and N_o, respectively, where

$$\gamma = \frac{E_s}{N_o}. \tag{9}$$

Since the signals at the frequencies f_1 and f_2 are orthogonal with each other, the conditional joint probability density function (PDF) of $\mathbf{y}_{sd}$ on $\mathbf{x}_1$ is as follows.

$$P\left(\mathbf{y}_{sd} \left| \mathbf{x}_1\right.\right) = P\left(y_{sd,1} \left| \mathbf{x}_1\right.\right) P\left(y_{sd,2} \left| \mathbf{x}_1\right.\right) \tag{10}$$

where

$$P\left(y_{sd,1} \left| \mathbf{x}_1\right.\right) = \frac{1}{\pi N_o(1 + \gamma\Omega_{sd})} \exp\left(-\frac{|y_{sd,1}|^2}{N_o(1 + \gamma\Omega_{sd})}\right), \tag{11}$$

$$P\left(y_{sd,2} \left| \mathbf{x}_1\right.\right) = \frac{1}{\pi N_o} \exp\left(-\frac{|y_{sd,2}|^2}{N_o}\right). \tag{12}$$

At the receiver of node d, with the cooperative transmission of node r, the received signals at frequencies f_1 and f_2 are

$$y_{rd,1} = A_r h_{rd}\left(h_{sr}\sqrt{E_s} + n_{sr,1}\right) + n_{rd,1} \tag{13}$$

and

$$y_{rd,2} = A_r h_{rd} n_{sr,2} + n_{rd,2}, \tag{14}$$

respectively, where $n_{rd,1}$ and $n_{rd,2}$ denote the low-pass equivalent noises over frequencies f_1 and f_2 at node d when receiving the relaying signals, respectively. Thus, the optimal detector under maximum likelihood rule is as in [11, eq. 23].

III. BER PERFORMANCE LOWER BOUND OF THE OPTIMAL DETECTOR

In this section, we will derive the BER performance lower bound of the optimal non-coherent detector for FSK modulated AF cooperative communications in Rayleigh fading channels.

Since the optimal detector for FSK modulated AF cooperative communications involves integration operation [11], the exact analytical BER performance is difficult to obtain. In this paper, we will show that with the partial CSI h_{rd}, the optimal non-coherent detector does not involve integration operation. Therefore, we propose to derive the system BER performance with partial CSI h_{rd} which is the lower bound of the actual system performance.

Without loss of generality, we assume $\mathbf{x}_1$ is transmitted. We rewrite (4) as follows.

$$\mathbf{y}_{rd} = A_r h_{rd} h_{sr} \mathbf{x}_s + \mathbf{n}_{srd} \tag{15}$$

where

$$\mathbf{n}_{srd} = A_r h_{rd} \mathbf{n}_{sr} + \mathbf{n}_{rd}. \tag{16}$$

If the partial CSI h_{rd} is known at node d, $\mathbf{n}_{srd}$ is a complex Gaussian random vector with mean vector $(0,0)^\dagger$ and variance matrix $\left(1 + A_r^2 |h_{rd}|^2\right) \mathbf{I}$, where $\mathbf{I}$ is a 2×2 identity matrix. Thus, the conditional joint PDF of $\mathbf{y}_{rd}$ on $\mathbf{x}_1$ and h_{rd} is as follows.

$$P\left(\mathbf{y}_{rd} | \mathbf{x}_1, h_{rd}\right) = P\left(y_{rd,1} | \mathbf{x}_1, h_{rd}\right) P\left(y_{rd,2} | \mathbf{x}_1, h_{rd}\right) \tag{17}$$

where $y_{rd,1}$ and $y_{rd,2}$ denote the received low-pass equivalent signals from node r at node d over frequencies f_1 and f_2, respectively. In (17),

$$\begin{aligned} P(y_{rd,1} | \mathbf{x}_1, h_{rd}) &= \frac{1}{\pi N_o \left(1 + (1+\gamma\Omega_{sr}) A_r^2 |h_{rd}|^2\right)} \\ &\cdot \exp\left(- \frac{|y_{rd,1}|^2}{N_o \left(1 + (1+\gamma\Omega_{sr}) A_r^2 |h_{rd}|^2\right)}\right) \end{aligned} \tag{18}$$

$$\begin{aligned} P(y_{rd,2} | \mathbf{x}_1, h_{rd}) &= \frac{1}{\pi N_o (1 + A_r^2 |h_{rd}|^2)} \\ &\cdot \exp\left(- \frac{|y_{rd,2}|^2}{N_o \left(1 + A_r^2 |h_{rd}|^2\right)}\right). \end{aligned} \tag{19}$$

With the same method, we can achieve the conditional joint PDFs of $\mathbf{y}_{sd}$ and $\mathbf{y}_{rd}$ on $\mathbf{x}_0$. Therefore, with partial CSI h_{rd}, the optimal detector under maximum likelihood rule implements

$$\hat{b}_s = \begin{cases} 0; & \lambda_1 < \lambda_2 \\ 1; & \lambda_1 \geq \lambda_2 \end{cases} \tag{20}$$

where

$$\lambda_k = \xi \frac{|y_{rd,k}|^2}{N_o} + \frac{\gamma\Omega_{sd}}{1+\gamma\Omega_{sd}} \frac{|y_{sd,k}|^2}{N_o}, \ k = \{1,\, 2\} \tag{21}$$

in which

$$\xi = \frac{1}{1 + A_r^2 |h_{rd}|^2} - \frac{1}{1 + (1+\gamma\Omega_{sr}) A_r^2 |h_{rd}|^2}. \tag{22}$$

From (20)–(22), the optimal detector with partial CSI h_{rd} for FSK modulated AF cooperative communications does not involve integration operation. The BER performance of optimal detector with partial CSI is derived as follows.

When $\mathbf{x}_1$ is transmitted, λ_1 and λ_2 in (21) are the sum of weighted chi-square distributed random variables whose conditional PDFs on $\mathbf{x}_1$ and h_{rd} are as follows [16], [17].

$$P(\lambda_1 | \mathbf{x}_1, h_{rd}) = \frac{1}{\delta_{11} - \delta_{12}} \left[\exp\left(-\frac{\lambda_1}{\delta_{11}}\right) - \exp\left(-\frac{\lambda_1}{\delta_{12}}\right)\right], \tag{23}$$

$$P(\lambda_2 | \mathbf{x}_1, h_{rd}) = \frac{1}{\delta_{21} - \delta_{22}} \left[\exp\left(-\frac{\lambda_2}{\delta_{21}}\right) - \exp\left(-\frac{\lambda_2}{\delta_{22}}\right)\right] \tag{24}$$

where

$$\delta_{11} = \xi\left(1 + (1+\gamma\Omega_{sr}) A_r^2 |h_{rd}|^2\right), \tag{25}$$

$$\delta_{12} = \gamma\Omega_{sd}, \tag{26}$$

$$\delta_{21} = \xi(1 + A_r^2 |h_r d|^2), \tag{27}$$

$$\delta_{22} = \frac{\gamma\Omega_{sd}}{1+\gamma\Omega_{sd}}. \tag{28}$$

Thus, the conditional BER on h_{rd} is evaluated as follows.

$$\begin{aligned} &P\ \ (e|h_{rd}) = P(\lambda_1 < \lambda_2 | \mathbf{x}_1, h_{rd}) \\ &= \int_0^\infty \frac{1}{\delta_{11} - \delta_{12}} \left[\exp\left(-\frac{\lambda_1}{\delta_{11}}\right) - \exp\left(-\frac{\lambda_1}{\delta_{12}}\right)\right] \\ &\cdot \int_{\lambda_1}^\infty \frac{1}{\delta_{21} - \delta_{22}} \left[\exp\left(-\frac{\lambda_2}{\delta_{21}}\right) - \exp\left(-\frac{\lambda_2}{\delta_{22}}\right)\right] d\lambda_2 d\lambda_1. \end{aligned} \tag{29}$$

The integral is calculated as follows.

$$\begin{aligned} P(e|h_{rd}) = &\sum_{k=1}^{2} (-1)^{k-1} \frac{\delta_{1k}}{\delta_{11} - \delta_{12}} \\ &\cdot \left(\frac{\delta_{21}}{\delta_{21} - \delta_{22}} \frac{\delta_{21}}{\delta_{21} + \delta_{1k}} + \frac{\delta_{22}}{\delta_{22} - \delta_{21}} \frac{\delta_{22}}{\delta_{22} + \delta_{1k}} \right). \end{aligned} \tag{30}$$

We obtain the system BER performance by taking expectation over the exponentially distributed random variable $|h_{rd}|^2$,

$$P(e) = \int_0^\infty \frac{1}{\Omega_{rd}} \exp\left(-\frac{z}{\Omega_{rd}}\right) P(e|h_{rd}) \left|_{|h_{rd}|^2 = z} dz. \right. \tag{31}$$

It should be noted that a closed form expression of above integral can be trivially evaluated by employing partial fraction expansion and [18, eq. 3.353.5].

Since the derived expression for the bound is complicated, we analyze the asymptotic lower bound to show the effectiveness of the optimal receiver. For simplicity, we normalize the AWGN such that $N_o = 1$. When $\gamma \to \infty$, the conditional PDF $P(\lambda_1 | \mathbf{x}_1, h_{rd})$ becomes

$$P(\lambda_1 | \mathbf{x}_1, h_{rd}) = \frac{1}{\tilde{\delta}_{11} - \tilde{\delta}_{12}} \left[\exp\left(-\frac{\lambda_1}{\tilde{\delta}_{11}}\right) - \exp\left(-\frac{\lambda_1}{\tilde{\delta}_{12}}\right)\right] \tag{32}$$

where

$$\tilde{\delta}_{11} = \frac{|h_{rd}|^2 A_r^2 \Omega_{sr} \gamma}{(1 + |h_{rd}|^2 A_r^2)} \tag{33}$$

$$\tilde{\delta}_{12} = \gamma\Omega_{sd}. \tag{34}$$

When $\gamma \to \infty$, λ_2 is the sum of two independent chi-square distributed random variables with identical distribution, whose conditional distribution $P(\lambda_2 | \mathbf{x}_1, h_{rd})$ is as follows.

$$P(\lambda_2 | \mathbf{x}_1, h_{rd}) = \lambda_2 \exp(-\lambda_2). \tag{35}$$

After some mathematical manipulations, we have

$$\lim_{\gamma\to\infty} P(e) = \frac{1}{\gamma^2}\int_0^\infty \frac{3(1+zA_r^2)}{\Omega_{sd}\Omega_{sr}\Omega_{rd}A_r^2 z}\exp\left(-\frac{z}{\Omega_{rd}}\right)dz. \quad (36)$$

From the above equation, it is found that the partial non-coherent receiver can achieve full diversity.

IV. NEAR OPTIMAL NON-COHERENT RECEIVER

Since the optimal receiver involves integration operation which is complicated [11, eq. (23)] and partial non-coherent receiver needs channel estimation. We propose a near optimal non-coherent detection scheme in this section.

In (13)–(14), the expressions for $y_{rd,1}$ and $y_{rd,2}$ are a little bit complicated. To simplify the expressions, we rewrite (13) as follows.

$$y_{rd,1} - n_{rd,1} = A_r h_{rd}\left(\sqrt{E_s}h_{sr} + n_{sr,1}\right) \quad (37)$$

where $y_{rd,1} - n_{rd,1}$ is a product of two independent complex Gaussian random variables [12, eq. (9)]. The conditional PDF of $y_{rd,1}$ on $\mathbf{x}_1$ and $n_{rd,1}$ can be expressed as

$$P(y_{rd,1}|\mathbf{x}_1, n_{rd,1}) = \frac{2}{\pi\rho_{r,1}}K_0\left(\frac{2}{\sqrt{\rho_{r,1}}}|y_{rd,1} - n_{rd,1}|\right) \quad (38)$$

where

$$\rho_{r,1} = A_r^2\Omega_{rd}(E_s\Omega_{sr} + N_o) \quad (39)$$

and $K_0(\cdot)$ is the zero-order modified Bessel function of the second kind. Thus, the PDF of the $y_{rd,1}$ on $\mathbf{x}_1$ can be achieved by taking expectation of expression (38) over $n_{rd,1}$, i.e.,

$$P(y_{rd,1}|\mathbf{x}_1) = \mathrm{E}_{n_{rd,1}}\left[\frac{2}{\pi\rho_{r,1}}K_0\left(\frac{2}{\sqrt{\rho_{r,1}}}|y_{rd,1} - n_{rd,1}|\right)\right]. \quad (40)$$

Unfortunately, to best of our knowledge, there is no closed form expression available for above-mentioned expectation. As an alternative, we will employ the Jensen's inequality to approximate the expression (40). It can be shown that $K_0(\sqrt{x})$ is a convex function because its second-order derivative is greater than or equal to zero, i.e.,

$$\frac{d^2}{dx^2}K_0(\sqrt{x}) = \frac{\sqrt{x}K_0(\sqrt{x}) + 2K_1(\sqrt{x})}{4x^{3/2}} \geq 0 \quad (41)$$

where $K_1(\cdot)$ is the first order modified Bessel function of the second kind. By applying Jensen's inequality to (40), we have

$$\begin{aligned} P(y_{rd,1}|\mathbf{x}_1) &\geq \frac{2}{\pi\rho_{r,1}}K_0\left(\frac{2}{\sqrt{\rho_{r,1}}}\sqrt{\mathrm{E}_{n_{rd,1}}[|y_{rd,1} - n_{rd,1}|^2]}\right) \\ &= \frac{2}{\pi\rho_{r,1}}K_0\left(\frac{2}{\sqrt{\rho_{r,1}}}\sqrt{|y_{rd,1}|^2 + N_o}\right). \end{aligned} \quad (42)$$

Applying the same mathematical manipulations as those for $y_{rd,1}$, we can get the PDF of $y_{rd,2}$

$$P(y_{rd,2}|\mathbf{x}_1) \geq \frac{2}{\pi\rho_{r,2}}K_0\left(\frac{2}{\sqrt{\rho_{r,2}}}\sqrt{|y_{rd,2}|^2 + N_o}\right) \quad (43)$$

where

$$\rho_{r,2} = A_r^2\Omega_{rd}N_o. \quad (44)$$

Because f_1 and f_2 are orthogonal frequency tones, the conditional joint PDF of $\mathbf{y}_{rd}$ is

$$P(\mathbf{y}_{rd}|\mathbf{x}_1) = P(y_{rd,1}|\mathbf{x}_1)P(y_{rd,2}|\mathbf{x}_1). \quad (45)$$

Similarly, if node s transmits $\mathbf{x}_0$, the conditional PDFs $P(y_{sd,1}|\mathbf{x}_0)$ and $P(y_{sd,2}|\mathbf{x}_0)$ can be obtained by exchanging $y_{sd,1}$ and $y_{sd,2}$ in (11)-(12). The conditional PDFs $P(y_{rd,1}|\mathbf{x}_0)$ and $P(y_{rd,2}|\mathbf{x}_0)$ can be obtained by exchanging $y_{rd,1}$ and $y_{rd,2}$ in (42)–(43). Thus, according to maximum likelihood rule, we can achieve the estimate of transmitted information symbol, $\hat{b}_s$, as follows.

$$\hat{b}_s = \begin{cases} 0; & \mathsf{F}(y_{sd,1}, y_{sd,2}) + \mathsf{H}(y_{rd,1}, y_{rd,2}) > 0 \\ 1; & \mathsf{F}(y_{sd,1}, y_{sd,2}) + \mathsf{H}(y_{rd,1}, y_{rd,2}) < 0 \end{cases} \quad (46)$$

where

$$\begin{aligned} \mathsf{F}(y_{sd,1}, y_{sd,2}) &= \log\frac{p(y_{sd,1}|\mathbf{x}_0)p(y_{sd,2}|\mathbf{x}_0)}{p(y_{sd,1}|\mathbf{x}_1)p(y_{sd,2}|\mathbf{x}_1)} \\ &= \frac{\gamma_{sd}}{N_o(1+\gamma_{sd})}\left(|y_{sd,2}|^2 - |y_{sd,1}|^2\right) \end{aligned} \quad (47)$$

and

$$\begin{aligned} \mathsf{H}(y_{rd,1}, y_{rd,2}) &= \log\frac{p(y_{rd,1}|\mathbf{x}_0)p(y_{rd,2}|\mathbf{x}_0)}{p(y_{rd,1}|\mathbf{x}_1)p(y_{rd,2}|\mathbf{x}_1)} \\ &= \log\frac{K_0\left(\frac{2}{\sqrt{\rho_{r,1}}}\sqrt{|y_{rd,2}|^2 + N_o}\right)}{K_0\left(\frac{2}{\sqrt{\rho_{r,1}}}\sqrt{|y_{rd,1}|^2 + N_o}\right)} \\ &\quad + \log\frac{K_0\left(\frac{2}{\sqrt{\rho_{r,2}}}\sqrt{|y_{rd,1}|^2 + N_o}\right)}{K_0\left(\frac{2}{\sqrt{\rho_{r,2}}}\sqrt{|y_{rd,2}|^2 + N_o}\right)}. \end{aligned} \quad (48)$$

V. SIMULATED AND THEORETICAL RESULTS

In this section, we will compare the simulation results with the theoretically derived BER performance lower bounds of the optimal non-coherent detector for FSK modulated AF cooperative communications in Rayleigh fading channels. We will also provide simulation results to illustrate the BER performance of our proposed near optimal detector.

In Fig. 2, we compare simulation results (denoted as "Simu" in the legend) and theoretically derived lower bounds (denoted as "LB" in the legend) of the optimal non-coherent detector for FSK modulated AF cooperative communications. For cooperative transmission, the transmitted power E_r is defined as $E_r = E_s$. In the simulations, the fading variances are assigned by adopting a path loss model of the form $\Omega_{pq} = L_{pq}^{-4}$, where L_{pq} denotes the distance between node p and node q, $p, q \in \{s, r, d\}$. The variance of Rayleigh fading coefficient from node s to node d is normalized such that $\Omega_{sd} = 1$. In Fig. 2, we will consider three cases: (1) The distances from node r to node s and node d are equal, i.e., $L_{sr} = L_{rd} = 0.6L_{sd}$; (2) the node r locates close to node d, i.e., $L_{sr} = 0.7L_{sd}$ and $L_{rd} = 0.5L_{sd}$; (3) the node r locates close to node s, i.e., $L_{sr} = 0.5L_{sd}$ and

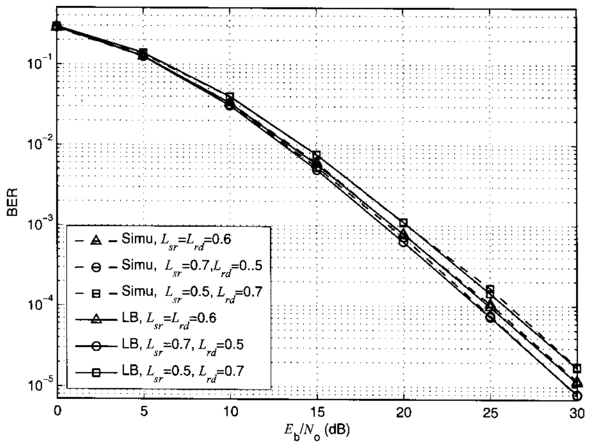

Fig. 2. BER versus E_b/N_o; simulated results and theoretically derived lower bounds comparison of optimal detector for FSK modulated AF cooperative communications.

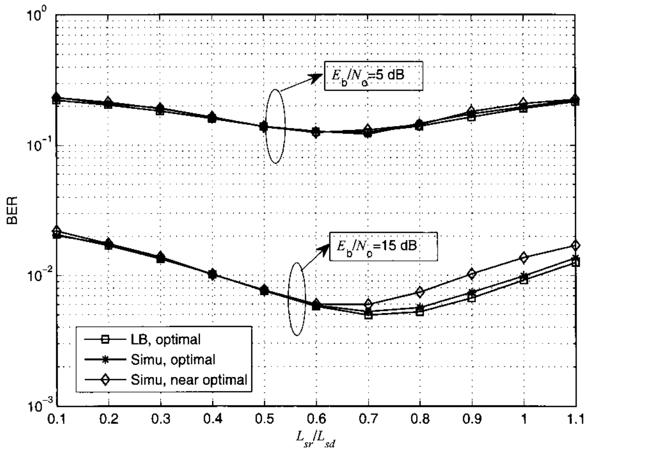

Fig. 4. BER versus L_{sr}; the impact of location of node r on the BER performance of optimal detector for FSK modulated AF cooperative communications.

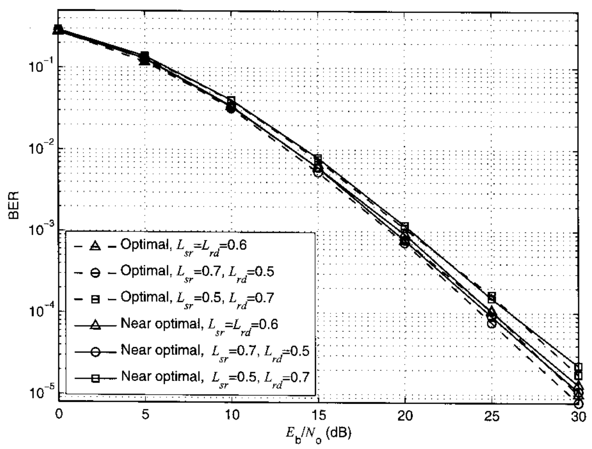

Fig. 3. BER versus E_b/N_o; performance comparison of proposed near optimal non-coherent detector with the optimal detector for FSK modulated AF cooperative communications.

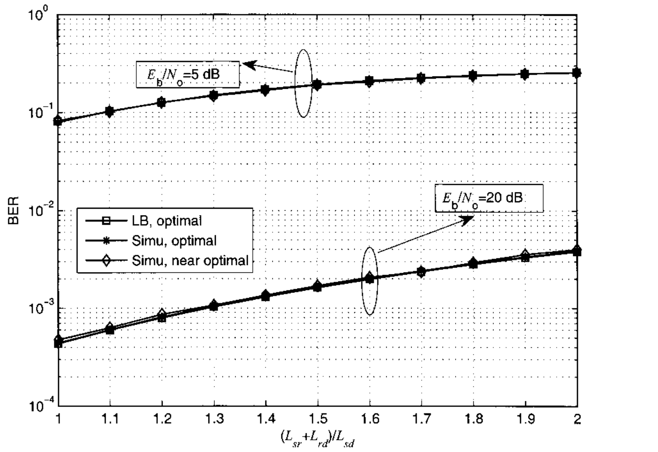

Fig. 5. BER versus $L_{sr} + L_{rd}$ with $L_{sr} = L_{rd}$; the impact of location of node r on the BER performance of optimal detector for FSK modulated AF cooperative communications.

$L_{rd} = 0.7L_{sd}$. From Fig. 2, it is found that the theoretically derived lower bounds are very close to the simulation results.

In Fig. 3, we compare the BER performances of proposed near optimal non-coherent detection scheme with the optimal detection scheme. It is shown in Fig. 3 that when the BER is 10^{-4}, the performance gaps between the proposed near optimal and optimal detection scheme are negligible. When the BER is about 10^{-5}, the performance gaps are smaller than 0.5 dB. The observed performance gap is due to the approximation by employing Jensen's inequality.

In Figs. 4 and 5, the effect of location of node r on the BER performances of proposed near optimal non-coherent detector and optimal detector for FSK modulated AF cooperative communications is shown. In Fig. 4, the sum of distances, L_{sr} and L_{rd}, is set to be $1.2L_{sd}$. It is shown from Fig. 4 that it may not the best choice to select the relaying node which locates at the equal distances to node s and node d. In order to achieve better BER performance, we may select the relaying node with the distance L_{sr} is a little bit larger than L_{rd}. In Fig. 4, we also compare the BER performances of proposed near optimal non-coherent detector and the optimal detector. It is found from Fig. 4 that when node r is at different locations, the BER performance of proposed near optimal detector is close to that of the optimal detector, especially, when $L_{sr} \leq L_{rd}$.

In Fig. 5, we compare the BER performances of proposed near optimal non-coherent detector and optimal detector when $L_{sr} = L_{rd}$ and the sum $L_{sr} + L_{rd}$ is different. It is found from Fig. 5, at any distance, the proposed near optimal detector has almost the same BER performance with the optimal detector.

In Figs. 4 and 5, we also present theoretically derived lower bounds of the optimal detector for FSK modulated AF cooperative communications. It is found from Figs. 4 and 5 that with different locations of node r, the derived lower bounds provide a very good prediction for the simulation results.

The results in Fig. 2, Fig. 4, and Fig. 5 show that knowing the partial CSI h_{rd} may not improve system performance significantly. This is because the known CSI generally helps improve system performance significantly through coherent detection. However, with the partial CSI h_{rd}, we cannot employ the coherent detection since h_{sr} in (15) is unknown. With the partial CSI h_{rd}, the performance improvement is due to the known power of $\mathbf{n}_{srd}$. Thus, if the variance of h_{rd} increases when the relaying node approaches the destination, the above-mentioned performance improvement increases accordingly as shown in Fig. 4.

VI. CONCLUSION

In this paper, we theoretically derive the BER performance of partial non-coherent receiver as a lower bound of optimal non-coherent receiver for FSK modulated AF cooperative communications in Rayleigh fading channels. It is found that the derived lower bound agrees with simulated results with very high accuracy. We have also proposed a near optimal non-coherent detection scheme. The proposed detection scheme does not involve complicated integration operation. It is shown through simulation results that the proposed near optimal detector approaches the BER performance of optimal detector.

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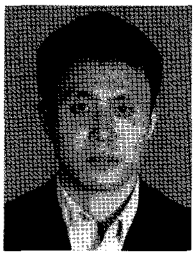

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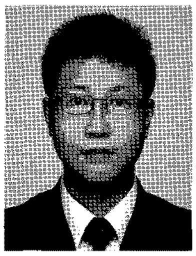

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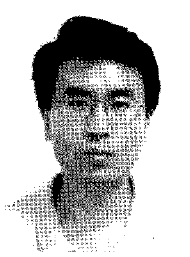

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