신뢰성응용연구 신뢰성기술
제11권, 제1호. pp.43-57, 2011

Warranty cost modeling using the parametric method

Minjae Park

Department of Industrial and Systems Engineering, Rutgers University
Piscataway, New Jersey, USA

Abstract

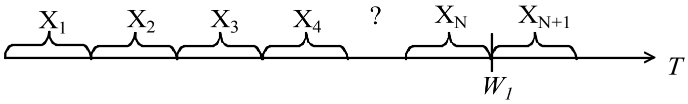
In the paper, we consider two-dimensional warranty policy with failure times and repair times. The failure times are considered within the warranty period and the repair times are considered within the repair time limit. Under the renewable warranty policy and non-renewable warranty policy, we consider the number of warranty services in the censored area by warranty period and repair time limit to conduct warranty cost analysis. We investigate the field data to check their dependency and implement our proposed approaches to conduct warranty cost analysis using the parametric methods. Numerical examples are discussed to demonstrate the applicability of the methodologies and results based on the proposed approach in the paper.

Keywords: Parametric; Random variable; Renewable warranty; Two-dimensional; Warranty period

논문접수일 : 2010년 12월 23일 논문수정일 : 2011년 01월 05일 게재확정일 : 2011년 02월 19일

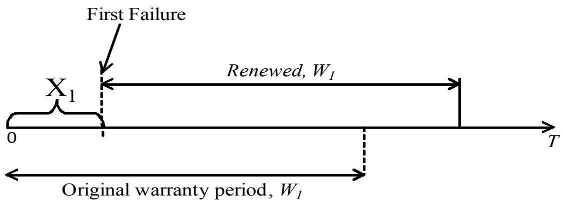
1. INTRODUCTION

A warranty is important to the manufacturer as well as the customer of any commercial product since it provides protection to both parties. As for the customer, a warranty provides a resource for dealing with items that fail due to the uncertainty of the product's performance and unreliable products. For the manufacturer, it provides protection since the warranty terms explicitly limit the responsibility of a manufacturer in terms of both time and type of product failure. Because of the role of the warranty, manufacturers have developed various types of warranty policy to grab the interest of the customers. However, manufacturers cannot extend the warranty period without limit and maximize warranty benefits because of the cost related to it.
One of the basic characteristics of warranties is whether they are renewable or not. For a regular renewable policy with warranty period, whenever a product fails in the warranty period, a customer is compensated according to the terms of the warranty contract and the warranty policy is renewed for another period. As a result, a warranty cycle starting from the point of sale, ending at the warranty expiration date, is a random variable whose value depends on the warranty period, the total number of failures under the warranty and the actual failure inter-arrival times. Under a renewing warranty, the product which fails during its warranty period is replaced by a new one at a cost to the manufacturer or at a pro-rated cost to the user and the warranty is renewed. Under a non-renewing warranty, the manufacturer guarantees a satisfactory service only during the original warranty period. Renewable warranties are usually given to the non-repairable and inexpensive products such as home appliances and so on. Compared to the renewable warranties, the period of non-renewable warranties is relatively longer. So this might be one of possible reasons why such policies are not as popular as non-renewable ones for warranty issuers (Bai (2004)). The warranty policy is separated into renewable warranty and non-renewable warranty in terms of policy's renewability. Under the renewable warranty, whenever product fails, the warranty period would be renewed. If there is no failure in the (renewed) warranty period, then the warranty period ends. Let W_1 be the warranty period. First, we consider one-dimensional warranty policy. Let X_i be the renewal inter-failure interval between the $(i-1)^{th}$ and i^{th} failure. Under the non-renewable warranty policy, warranty period is fixed and even though a failure of product happens, the warranty policy is not renewed. The number of failures within the warranty period is a r.v. N. So a r.v. N is the minimum $i \geq 1$ such as $\sum_{k=1}^{i+1} X_k > W_1, i=1,2,\cdots$. And when $X_1 > W_1$, then i is 0. It is described in Figure 1.



<Figure 1> One-dimensional Warranty services model under the non-renewable policy

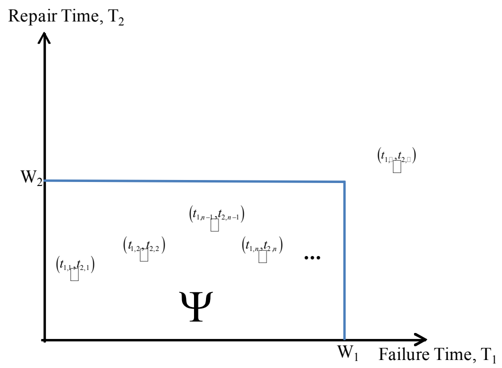
Similarly, under the renewable warranty policy, whenever a product failure happens, the warranty period is renewed. If there is no failures in the (renewed) warranty period, then the warranty period would end. Under the renewable warranty policy, a r.v. N is the minimum $i \geq 0$ such as $X_{i+1} > W_1, i = 1, 2, \cdots$.



<Figure 2> One-dimensional Warranty services model under the renewable policy

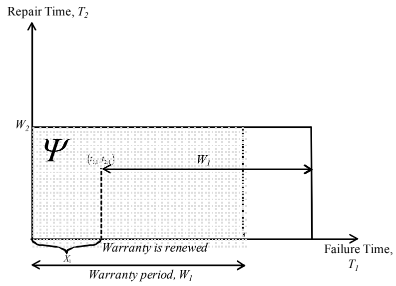
One dimensional warranty is characterized by the warranty period, which is defined in terms of a single variable. Single variable could be time, age or usage. In the case of two-dimensional warranties, there are two dimensions to express warranty policies. One is representing time and the other representing item usage. As a result, many different types of warranties may be defined based on the characteristics of warranty policies (Blischke (1994)). And many researchers have studied the cost analysis based on two dimensional warranty (Manna, *et al.* (2008), Baik, *et al.* (2004), Chen and Popova (2002), Chukova, *et al.* (2006), Chukova, *et al.* (2007), Chukova and Johnston (2006), Iskandar and Murthy (2003), Iskandar, *et al.* (2005), Jung and Bai (2007), Yun and Kang (2007)). Yun and Kang (2007) examine new warranty servicing strategy, considering imperfect repair with a two-dimensional warranty. Baik, *et al.* (2004) study two-dimensional failure modeling for a system where degradation is due to age and usage with minimal repair. In this study, under the two-dimensional warranty policy, two kinds of warranty services are considered. One is a repair service and the other is a replacement service. In general, when there is a failed component, the repair service is considered first, and replaced only when it cannot be repairable. Also, if failed products were delivered to the warranty service centers, they should return them back to the customers within a certain threshold of time for customers' satisfaction. Therefore, repair warranty service time limits are considered. Since there are two dimensions such as failure times and repair times, we call it a two-dimensional warranty.

Similarly to one-dimensional warranty policy, we consider the number of warranty services which is r.v. N, in the warranty period. Under the non-renewable warranty policy, warranty period is fixed and even though a failure of product happens, the warranty policy is not renewed. While trying to repair a failed system, if a repair time exceeds the time limit, then it is replaced, rather than continuing for repair. W1 represents the warranty period and W2 represents a time limit for the repair service. The horizontal axis is the failure time $T_{1i}, i = 1, 2, \cdots$, and the vertical axis is the repair time $T_{2i}, i = 1, 2, \cdots$, which is assumed to be independent of failure times. We consider repair times which are less than the repair time limit and repair times are not included in the warranty period for the customer's satisfaction. The number of failures within the warranty period is a r.v. N. So a r.v. N is the minimum $i \geq 1$ such as $\sum_{k=1}^{i+1} X_k > W_1$ or $Y_{i+1} > W_2$, $i = 1, 2, \cdots$. And when $X_1 > W_1$ or $Y_1 > W_2$ then i is 0. It is described in Figure 3.



<Figure 3> Two-dimensional warranty services model under non-renewable policy

Similarly under the renewable warranty policy, whenever a product failure happens, the warranty period is renewed. If there is no failure in the (renewed) warranty period, then the warranty period would end. If the number of failures happens in the warranty period is N then under the renewable warranty policy, a r.v. N is the minimum $i \geq 0$ such as $X_{k+1} > W_1$ or $Y_{i+1} > W_2, i = 1, 2, \cdots$. It is described in Figure 4.



<Figure 4> Two-dimensional Warranty services model under renewable policy

The failure times and repair times and the warranty period and the repair time limits are assumed to be all not fixed but random variables. And we investigate how many times the warranty services happen in the censored area and using the frequency of the warranty services, we find out the distribution of the number of failures and conduct warranty cost analysis.

2. WARRANTY COST MODELING : PARAMETRIC METHOD

Under one-dimensional warranty and two-dimensional warranty, we propose the warranty model and investigate the warranty cost analysis. Customers select their product's warranty period. If they want to have longer warranty period, they would pay more warranty fee. Also, based on the locations and times, the warranty period could be different. Therefore, warranty period would not be fixed but could be a random variable.

2.1 Two-dimensional modeling with the renewable warranty

In this subsection, whenever the product fails, it would be repaired and the warranty period is renewed. If there is no failure in the (renewed) warranty period, then the warranty would be expired as described in Figure 4.

Additionally, the repair time exceeds the repair time limit then a manufacturer is supposed to provide a replacement service and the failed product's warranty would be expired too. It describes two-dimensional warranty policy. In the one-dimensional warranty policy, we do not consider the repair time limit. We consider warranty period as one dimension. Let X_i be the i^{th} inter-failure interval between failures and W_1 be the warranty period. In the one dimensional warranty policy, the number of events can be given by

$$N = \min[i \geq 0 : X_{i+1} > W_1] \tag{1}$$

Let T_X be $(X_1, X_2, \cdots, X_N)$ and have a c.d.f. H_X. It means T_X: $\max(X_1, X_2, \cdots, X_N) \sim H_X$. The probability $P(N=n)$ that there is N number of warranty services in the warranty period is given by

$$\begin{aligned} P(N=n) &= P(X_1 < W_1, X_2 < W_1, \cdots, X_n < W_1) \\ &= \int_{-\infty}^{\infty} P(X_1 < w_1, X_2 < w_1, \cdots, X_n < w_1 | W_1 = w_1) g_1(w_1) dw_1 \\ &= \int_{-\infty}^{\infty} P(T_X < w_1 | W_1 = w_1) g_1(w_1) dw_1 \\ &= \int_{-\infty}^{\infty} H_X(w_1) dG_1(w_1) \end{aligned} \tag{2}$$

The r.v. T_x is independent of the random variable W_1. If inter-failure intervals $X_i, i = 1, 2, \cdots, N$ follow same cdf $F(t)$ then $H_X(t)$ is given by

$$H_X(t) = P(T_X \leq t) = (F(t))^N \tag{3}$$

If they follow different cdf $F_i(t), i = 1, 2, \cdots, N$ then $H(t)$ is given by

$$H_X(t) = P(T_X \leq t) = \prod_{i=1}^{N} (F(t))^i \tag{4}$$

All these random variables are mutually independent. Then the density function of the random variable T is given as

$$h_X(t) = n(F(t))^{n-1} f(t) \tag{5}$$

If the inter-failure intervals follow same distribution, i.e. $X_1, X_2, \cdots, X_N \sim F$, and W_1 follows F_{w_1} distribution, i.e. $W_1 \sim F_{w_1}$ then

$$\begin{aligned} P(N=n) &= P(X_1 < W_1, X_2 < W_1, \cdots, X_n < W_1) \\ &= \int_{-\infty}^{\infty} P(X_1 < w_1, X_2 < w_1, \cdots, X_n < w_1 | W_1 = w_1) f_{W_1}(w_1) dw_1 \\ &= \int_{-\infty}^{\infty} F(w_1)^n dF_{W_1}(w_1) \end{aligned} \tag{6}$$

If $F = F_{W_1}$ then

$$P(N=n) = \frac{1}{n+1} \tag{7}$$

By a similar way, we can obtain $P(N=n)$ in terms of repair times and repair time limits. Under the renewable warranty policy, the number of events under the two-dimensional warranty policy can be given by

$$N = \min[j \geq 0 : X_{j+1} > W_1 \text{ or } Y_{j+1} > W_2] \tag{8}$$

We consider two dimensional warranty modeling. If there is no failure in the warranty period or if the repair time exceeds the repair time limit, then the failed product's warranty would be finished. Then, $P(N=n)$ is the probability that there are N number of warranty services in the warranty period. Let T_Y be $\max(Y_1, Y_2, \cdots, Y_N)$ and have a c.d.f.H_Y. $P(N=n)$ is given by

$$\begin{aligned} P(N=n) &= P(X_1 < W_1, X_2 < W_1, \cdots, X_n < W_1, Y_1 < W_2, Y_2 < W_2, \cdots, Y_n < W_2) \\ &= P(T_X < W_1, T_Y < W_2) \end{aligned} \tag{9}$$

The probability with different warranty policies that there are N number of failures in the warranty period is given by

$$\begin{aligned} P(N=n) &= P(X_1 < W_1, X_2 < W_1, \cdots, X_n < W_1, Y_1 < W_2, Y_2 < W_2, \cdots, Y_n < W_2) \\ &= \iint P(T_X \leq w_1, T_Y \leq w_2 | W_1 = w_1, W_2 = w_2) f_{W_1}(w_1) g_{W_2}(w_2) dw_1 dw_2 \\ &= \iint H_{X,Y}(w_1, w_2) dF_{W_1}(w_1) dG_{W_2}(w_2) \end{aligned} \tag{10}$$

If $T_x < W_1$ and $T_y < W_2$ are independent, the eq. (10) can become

$$P(N=n) = \iint H_X(w_1) H_Y(w_2) dF_{W_1}(w_1) dG_{W_2}(w_2) \tag{11}$$

If X_i follows F_i distributions and Y_i follows G_i distributions then eq. (11) becomes

$$P(N=n) = \int_{-\infty}^{\infty} \prod_{i=1}^{n} F_i(w_1) dF_{W_1}(w_1) \int_{-\infty}^{\infty} \prod_{i=1}^{n} G_i(w_1) dG_{W_2}(w_2) \tag{12}$$

If $H_X = H_Y = F_{W_1} = G_{W_2}$ then $P(N=n) = \left(\frac{1}{n+1}\right)^2$.

If $T_x < W_1$ and $T_y < W_2$ are dependent, we can conduct warranty analysis using bivariate distribution. We can think that the manufacturers have a couple of categorized groups for different warranty periods. For example, Hyundai, Korean auto company, sell their cars to many countries such as U.S.A., European countries and Asian countries and they sell their cars with different warranty periods based on the locations. Additionally, we can think of different times. These days, warranty is 10 years and 100,000 miles warranty but they began to service this warranty in early 1990. So, we consider different groups of warranty policy. Let m be the number of different groups. In this case, we think the number of failures is r.v. N which is similar to eqs. (1) &

(8). Regarding about the m numbers of groupings, a r.v. N is given by

$$N_j = \min[i \geq 0 : X_{i+1,j} > W_{1,j} \text{ or } Y_{i,j} > W_{2,j}], j = 1,2,\cdots,m \tag{13}$$

Under the renewable warranty policy, the probability with different warranty policies that there are N_j number of failures in the warranty period is given by

$$P(N_j = n_j) = P(T_{X,j} \leq W_{1,j} \text{ or } T_{Y,j} \leq W_{2,j}), j = 1,2,\cdots,m \tag{14}$$

When inter-failure intervals are i.i.d, then the probability is given

$$\begin{aligned} P(N = n) &= P(N_j = n_j, j = 1,2,\cdots,m) \\ &= \prod_{j=1}^{m} P(T_{X,j} \leq W_{1,j} \text{ or } T_{Y,j} \leq W_{2,j}) \end{aligned} \tag{15}$$

If X_i is following uniform distribution, then the probability that N^{th} failures interval is less than the warranty period is given by

$$\begin{aligned} \prod_{j=1}^{m} P(T_{X,j} \leq W_{1,j}) &= \prod_{j=1}^{m} \int P(T_{X,j} \leq w | W_{1,j} = w) f_{W_{1,j}}(w) dw \\ &= \prod_{j=1}^{m} \int H_{X,j}(w) dF_{W_{1,j}}(w) \end{aligned} \tag{16}$$

If $T_{X,j} \leq W_{1,j}$ and $T_{Y,j} \leq W_{2,j}$ are independent, then

$$\begin{aligned} P(N = n) &= \prod_{j=1}^{m} P(T_{X,j} \leq W_{1,j}, T_{Y,j} \leq W_{2,j}) \\ &= \prod_{j=1}^{m} \int H_{X,j}(w) dF_{W_{1,j}}(w) \prod_{j=1}^{m} \int H_{Y,j}(w) dG_{W_{2,j}}(w) \end{aligned} \tag{17}$$

The expected number of failures is given by

$$\begin{aligned} E(N) &= \sum_{n=1}^{\infty} n P(N = n) \\ &= \sum_{n=1}^{\infty} n \prod_{j=1}^{m} \int H_{X,j}(w) dF_{W_{1,j}}(w) \prod_{j=1}^{m} \int H_{Y,j}(w) dG_{W_{2,j}}(w) \end{aligned} \tag{18}$$

The variance of the number of failures is given by

$$\begin{aligned} Var(N) &= \sum_{n=1}^{\infty} n^2 \prod_{j=1}^{m} \int H_{X,j}(w) dF_{W_{1,j}}(w) \prod_{j=1}^{m} \int H_{Y,j}(w) dG_{W_{2,j}}(w) \\ &\quad - [\sum_{n=1}^{\infty} n \prod_{j=1}^{m} \int H_{X,j}(w) dF_{W_{1,j}}(w) \prod_{j=1}^{m} \int H_{Y,j}(w) dG_{W_{2,j}}(w)]^2 \end{aligned} \tag{19}$$

2.2 Two-dimensional modeling with the non-renewable warranty

In the one dimensional warranty policy, the number of events N can be given by

$$N = \min[i \geq 1 : \sum_{k=1}^{i+1} X_k > W_1] \tag{20}$$

Let X_i be i^{th} inter-failure interval and W_1 be the warranty period. S_N denotes summation by N^{th} inter-failure intervals. The probability that there are N number of warranty service in the warranty period, is given by

$$\begin{aligned} P(N=n) &= P\left(\sum_{i=1}^{n} X_i \le W_1, \sum_{i=1}^{n+1} X_i > W_1\right) \\ &= P(S_n \le W_1, S_{n+1} > W_1) \\ &= \int_0^{\infty} P(S_n \le w_1, S_{n+1} > w_1 | W_1 = w_1) f_{W_1}(w_1) dw_1 \\ &= \int_0^{\infty} \left(F_S(w_1) - F_S^2(w_1)\right) dF_{W_1}(w_1) \end{aligned} \tag{21}$$

If W_1 and S_n have the same distribution F, eq. (21) can be written as follows:

$$P(N=n) = \frac{5}{6} - \left(\frac{F(0)^2}{2} + \frac{F(0)^3}{3}\right) \tag{22}$$

Similarly, the number of events in terms of repair time limit W_2 can be given by

$$N = \min[i \ge 1 : Y_{j+1} > W_2] \tag{23}$$

We consider the repair times Y and its repair time limit W_2 for customer's satisfaction. If warranty service providers could not fix the failed products within the repair time limit, they are providing replacement service, not continuing to fix the failed products. This is another dimension. So in a similar ways for failure times and warranty period, we can obtain the probability that there are N numbers of repair services can be given by

$$\begin{aligned} P(N=n) &= P(Y_n \le W_2, Y_{n+1} > W_2) \\ &= \int_0^{w_2} \left(G_Y(w_2) - F_Y^2(w_2)\right) dG_{W_2}(w_2) \end{aligned} \tag{24}$$

If W and Y have the same distribution G, eq. (24) can be written as follows:

$$P(N=n) = \frac{5}{6} - \left(\frac{G(0)^2}{2} + \frac{G(0)^3}{3}\right) \tag{25}$$

Under the non-renewable warranty policy, the number of events under the two-dimensional warranty policy, can be given by

$$N = \min\left[j \ge 1 : \sum_{k=1}^{j+1} X_k > W_1 \text{ or } Y_{j+1} > W_2\right] \tag{26}$$

the probability with different warranty policies that there are N number of failures in the warranty period is given by

$$\begin{aligned} P(N=n) &= P\left(\sum_{k=1}^{n} X_k \le W_1, Y_n \le W_2\right) \\ &= P\left(S_n \le W_1, Y_n \le W_2\right) \\ &= \iint P(S_n \le w_1, Y_n \le w_2 | W_1 = w_1, W_2 = w_2) f_{W_1}(w_1) g_{W_2}(w_2) dw_1 dw_2 \end{aligned} \tag{27}$$

If they are independent the eq. (27) can be rewritten as

$$P(N=n) = \iint F_S(w_1) G_Y(w_2) dF_{W_1}(w_1) dG_{W_2}(w_2) \tag{28}$$

Under the non-renewable warranty policy, the probability with different warranty policies that there are N number of failures in the warranty period is given by

$$N = \min\left[j \ge 1 : \sum_{k=1}^{j+1} X_k > W_1 \text{ or } Y_{j+1} > W_2\right] \tag{29}$$

When inter-failure intervals are i.i.d and following same distribution, then the probability is given

$$\begin{aligned} P(N=n) &= P(N_j = n_j, j = 1,2,\cdots,m) \\ &= \prod_{j=1}^{m} P\left(S_{n_j,j} \le W_{1,j} \text{ or } Y_{n_j,j} \le W_{2,j}\right) \end{aligned} \tag{30}$$

The probability that N^{th} failures interval is less than the warranty period is given by

$$\begin{aligned} \prod_{j=1}^{m} P\left(S_{n_j,j} \le W_{1,j}\right) &= \prod_{j=1}^{m} \int P\left(S_{n_j,j} \le w | W_{1,j} = w\right) f_{W_{1,j}}(w) dw \\ &= \prod_{j=1}^{m} \int F_{S,j}(w) dF_{W_{1,j}}(w) \end{aligned} \tag{31}$$

If $S_{n_j,j} \le W_{1,j}$ and $Y_{n_j,j} \le W_{2,j}$ are independent, then

$$\begin{aligned} P(N=n) &= \prod_{j=1}^{m} P\left(S_{n_j,j} \le W_{1,j}\right) P\left(Y_{n_j,j} \le W_{2,j}\right) \\ &= \prod_{j=1}^{m} \int F_{S,j}(w) dF_{W_{1,j}}(w) \prod_{j=1}^{m} \int G_j(w) dG_{W_{2,j}}(w) \end{aligned} \tag{32}$$

The expected number of failures is given by

$$\begin{aligned} E(N) &= \sum_{n=1}^{\infty} n P(N=n) \\ &= \sum_{n=1}^{\infty} n \prod_{j=1}^{m} \int F_{S,j}(w) dF_{W_{1,j}}(w) \prod_{j=1}^{m} \int G_j(w) dG_{W_{2,j}}(w) \end{aligned} \tag{33}$$

The variance of the number of failures is given by

$$\begin{aligned} Var(N) &= \sum_{n=1}^{\infty} n^2 \prod_{j=1}^{m} \int F_{S,j}(w) dF_{W_{1,j}}(w) \prod_{j=1}^{m} \int G_j(w) dG_{W_{2,j}}(w) \\ &\quad - \left[\sum_{n=1}^{\infty} n \prod_{j=1}^{m} \int F_{S,j}(w) dF_{W_{1,j}}(w) \prod_{j=1}^{m} \int G_j(w) dG_{W_{2,j}}(w)\right]^2 \end{aligned} \tag{34}$$

3. Real Application and Numerical Examples

In South Korea, there are four nuclear sites and, in 2010, there are 20 nuclear power plants in operation with a total licensed output amount to 17,716 MWe (MegaWatt electrical) and 8 nuclear power plants under construction, for a total of 28 units in operation by the end of 2016 (Safety and Operational Status of Nuclear Power Plants in Korea(2008)). We investigate the field data to check their dependency and implement our proposed approaches to conduct warranty cost analysis.

Among 20 nuclear power reactors in the four nuclear plants in South Korea, we pick one nuclear plant which has three nuclear power reactors. It is summarized that 10 failure data for nuclear power plants for relatively recent events or failures in Table 1. It contains the failure data and the repair data.

<Table 1> Failure times and repair times for nuclear power plants (failure times : Days, repair times : hours)

No.	Reactor 1		Reactor 2		Reactor 3	
	Failure	Repair	Failure	Repair	Failure	Repair
1	465.43	197.83	34.85	276.92	218.85	29.25
2	717.26	202.50	383.85	85.13	12.04	278.87
3	7.00	641.87	188.86	310.25	110.94	5.25
4	39.11	372.58	666.26	316.00	278.84	35.67
...	...	...	...	...	...	...
10	174.62	3.79	666.06	447.79	633.10	622.08

From Operational Performance Information System for Nuclear Power Plant (http://opis.kins.re.kr/index.jsp?Lan=US)

The proposed approach has warranty model in the warranty period and the maintenance model in the post warranty period. But the real application covers only the warranty model. From the website (http://opis.kins.re.kr/index.jsp?Lan=US), we obtain the failure data and repair data but the maintenance data is not open to public. Every failure times and every repair times are assumed to be in the censored area. It indicates that the product failures happen in the warranty period and the repair times are less than the repair time limit.

We investigate the warranty cost analysis using repair times and failure times of the nuclear power plants in the warranty period. To conduct cost analysis, failure times and their repair times should be independent each other.

Using Kendall's τ method, we are going to test the hypothesis if the failure times and repair times are dependent. Kendall's rank correlation measures the strength of monotonic association between the failure times and repair times. It may also be noted that usual Pearson correlation is fairly robust and it usually agrees well in terms of statistical significance with results obtained using Kendall's rank correlation. Based on the result of Kendall's τ method using *R software* McLeod (2005), for Reactor 1, τ is -0.022 and the p value is 1. Therefore, at significant level α, it is concluded that the failed times and repair times are independent. Similarly, for Reactor 2, τ is 0.067 and the p value is 0.8618. For Reactor 3, τ is 0.33 and the p value is 0.2105. Because all Reactors' p values are larger than significant level α, it is concluded that the failure times and repair times for Reactor 2 and 3 are independent.

Given the field data from Table 1, we now want to figure out the best fit distributions for the repair times. Calculations are based on more than 10 distributions specified from computer software. Using the computer software, it shows that for Reactor 1's repair times, Weibull distribution, exponential and gamma distributions are best three well-fitted distribution by the order of log likelihood values. For Reactors 1, 2 and 3, the three best well fitted distributions are described in Table 2.

the pdfs of each distributions are as follows.

Weibull distribution with three parameters: $f(x)=\dfrac{\beta}{x-\gamma}\left(\dfrac{x-\gamma}{\eta}\right)^{\beta}e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}}$

Weibull distribution with two parameters: $f(x)=\dfrac{\beta}{x}\left(\dfrac{x}{\eta}\right)^{\beta}e^{-\left(\frac{x}{\eta}\right)^{\beta}}$

Exponential distribution with two parameters: $f(x)=\dfrac{1}{\lambda}e^{-\left(\frac{x-\eta}{\lambda}\right)}$

<Table 2> Two best fitted distributions of repair times for 3 reactors

Reactor 1	Reactor 2	Reactor 3
Weibull (3 Par.)	Weibull (3 Par.)	Weibull (3 Par.)
$\beta=0.72, \eta=303.98, \gamma=-3.75$	$\beta=3.14, \eta=356.86, \gamma=-58.74$	$\beta=0.64, \eta=131.41, \gamma=-5.20$
Exponential (2 Par.)	Weibull (2 Par.)	Exponential (2 Par.)
$\lambda=0.0029, \eta=345.99$	$\beta=2.4628, \eta=293.05$	$\lambda=0.0060, \eta=167.85$

For Reactor 1, Weibull with 3 parameters, exponential with 2 parameters and gamma distributions are well-fitted distributions. Based on the output, we choose the Weibull distribution for the repair times' pdf then conduct warranty cost analysis. After we use the computer software, we figure out that for Reactor 2, the best fitting distributions for the repair times are listed by Weibull distribution with three parameters (β=3.1424, η= 356.8643, γ=-58.7363), Weibull distribution with two parameters (β=2.4628, γ=293.05) and

Rayleigh distribution (β=2, η=253.2770). And the best fitting distributions for Reactor 3 are listed by Weibull distribution with three parameters (β=0.6428, η=131.4138, γ=5.1975), exponential distribution with two parameters (β=0.0060, η=167.85) and Weibull distribution with two parameters (β=0.8496, η=159.2770). Table 3 shows the expected number of failures under warranty for the limitation parameters and Table 4 shows the stadard deviation of the number of failures under warranty. Using Eqs. (18) & (19), we obtain the expected number of warranty services and its standard deviation.

<Table 3> Expected number of warranty services under the warranty period

w	Reactor 1	Reactor 2	Reactor 3
0.1	0.0305	0.0090	0.0235
0.2	0.1206	0.0359	0.0929
0.3	0.2684	0.0807	0.2069
0.4	0.4718	0.1434	0.3642
0.5	0.7292	0.2239	0.5635
0.6	1.0388	0.3222	0.8035
0.7	1.3990	0.4383	1.0832
0.8	1.8082	0.5721	1.4014
0.9	2.2650	0.7235	1.7571
1	2.7679	0.8926	2.1491

<Table 4> Standard deviation of the number of warranty services under the warranty period

w	Reactor 1	Reactor 2	Reactor 3
0.1	1.2322	0.7876	1.1296
0.2	2.4498	1.5747	2.2473
0.3	3.6510	2.3612	3.3518
0.4	4.8344	3.1467	4.4422
0.5	5.9984	3.9309	5.5174
0.6	7.1415	4.7134	6.5767
0.7	8.2625	5.4940	7.6190
0.8	9.3600	6.2724	8.6437
0.9	10.4329	7.0482	9.6498
1	11.4800	7.8212	10.6367

4. Concluding Remarks

In this paper, we develop cost models for the warranty policy using the parametric method. The field data is investigated to check their dependency and implement our proposed approaches to conduct warranty cost analysis using the parametric methods. Two-dimensional warranty policy is investigated with failure times and repair times. The failure times are considered within the warranty period and the repair times are considered within the repair time limit. Two best fit distributions for failure data and repair data are obtained and based on the distributions, we have the expected number of warranty services and its variance. This information would be helpful for warranty policy makers to make important decisions for their companies. For future research topics, we can investigate warranty cost analysis using non-parametric methods. It would be interesting topics.

Reference

[1] J. Bai, "On the Study of Warranties for Repairable Complex Systems," in *Department of Industrial and System Engineering* vol. Ph.D Dissertation: Rutgers University, 2004.

[2] W. Blischke, *Warranty Cost Analysis*: CRC Press, 1994.

[3] D. Manna, S. Pal, and S. Sinha(2008). A note on calculating cost of two-dimensional warranty policy, *Computers & Industrial Engineering,* vol. 54, pp. 1071-1077.

[4] J. Baik, D. Murthy, and N. Jack(2004). Two-dimensional failure modeling with minimal repair, *Naval Research Logistics,* vol. 51, pp. 345-362.

[5] T. Chen and E. Popova(2002). Maintenance policies with two-dimensional warranty, *Reliability Engineering and System Safety,* vol. 77, pp. 61-69.

[6] S. Chukova, Y. Hayakawa, and M. Johnston, "Two-dimensional warranty: minimal/complete repair strategy," 2006.

[7] S. Chukova, Y. Hayakawa, and M. Johnston(2007). Optimal two-dimensional warranty repair strategy, *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability,* vol. 221, pp. 265-273.

[8] S. Chukova and M. Johnston(2006). Two-dimensional warranty repair strategy based on minimal and complete repairs, *Mathematical and computer modelling,* vol. 44, pp. 1133-1143.

[9] B. Iskandar and D. Murthy(2003). Repair-replace strategies for two-dimensional warranty policies, *Mathematical and computer modelling,* vol. 38, pp. 1233-1241.

[10] B. Iskandar, D. Murthy, and N. Jack(2005). A new repair-replace strategy for items sold with a two-dimensional warranty, *Computers and Operations Research,* vol. 32, pp. 669-682.

[11] M. Jung and D. Bai(2007). Analysis of field data under two-dimensional warranty, *Reliability Engineering and System Safety,* vol. 92, pp. 135-143.

[12] W. Yun and K. Kang(2007). Imperfect repair policies under two-dimensional warranty, *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability,* vol. 221, pp. 239-247.

[13] Safety and Operational Status of Nuclear Power Plants in Korea(2008), *Annual Report, Korea Institute of Nuclear Safety,* vol. 9.

[14] A. I. McLeod(2005). Kendall rank correlation and Mann-Kendall trend test, *R Package "Kendall", http://cran.r-project.org/web/packages/Kendall.*