JOURNAL OF THE KOREAN INSTITUTE OF ELECTROMAGNETIC ENGINEERING AND SCIENCE, VOL. 11, NO. 3, SEP. 2011 JKIEES 2011-11-3-03

http://dx.doi.org/10.5515/JKIEES.2011.11.3.156

Analysis of Magnetically Coupled Wireless Power Transmission for Maximum Efficiency

Chungju Kim · Bomson Lee

Abstract

We have proposed and analyzed an equivalent circuit for a magnetically coupled wireless power transmission (WPT) system between two loop resonators by considering its coupling coefficient and radiation-related parameters. A complete formulation is provided for all the necessary circuit parameters. The mechanism of radiation loss is sufficiently explained. The circuit and electromagnetic (EM) simulation results have been shown to be in good agreement. Based on the proposed circuit formulation, a specific load impedance for maximum WPT efficiency was found to exist. The proposed modeling of the WPT in terms of circuit characterizations provides sufficient insight into the problems associated with WPT.

Key words: Wireless Power Transmission (WPT), Magnetic Coupled, Radiation Loss, Efficiency.

Ⅰ. Introduction

We live in a world of wireless communication. A huge amount of information is wirelessly communicated between mobile terminals. Now, the increasing requirement for the wireless transfer of electric power is making wireless power transmission (WPT) technology increasingly important. The concept of WPT was initiated by N. Tesla in 1914 [1], but until recently, it had not resulted in any practical application, due to its low efficiency. In 2007, magnetically coupled WPT technology based on the coupled-mode theory [2] was first investigated by Prof. Soljacic and his research group at Massachusetts Institute of Technology (MIT) [3] and recently, considerable research on magnetically coupled WPT has being conducted [4]~[9]. Prof. Soljacic and his team used coupled-mode theory to analyze the scheme. Based on the analysis, they successfully lit a 60 W bulb at a distance of 7 feet (more than 2 meters) by using helical coils of high Q. The efficiency was reported to be 60 % at 9.9 MHz. Most of the papers related to WPT focus on the power transmission efficiency. The mechanism of radiation loss and the coupling coefficient have rarely been explained in detail. The radiation loss is surely the most crucial limiting factor in WPT and it warrants an in-depth analysis.

In this paper, in order to investigate the mechanism of WPT in a more detail, we focus on the analysis of a WPT system using an equivalent circuit. Based on the equivalent circuit, key parameters-such as WPT efficiency and the radiation loss rate are properly defined and derived.

Furthemore, a method for extracting both from electromagnetic (EM) simulations or measurements is proposed. Finally, the proposed modeling for WPT is validated by comparing the circuit and the EM simulations.

Ⅱ. Analysis of an Equivalent Circuit for WPT using Two Loops

Commonly, WPT using magnetic coupling is realized using two resonant loops facing each other. One loop is connected to an AC power source and the other loop is connected to the load. Power is wirelessly transmitted from one loop to the other as a result of magnetic coupling between the two resonant structures with the same resonant frequency.

Fig. 1 shows the equivalent circuit for a magnetically coupled WPT that considers radiation effects. V_1 is the voltage source for the WPT system. R_1 and R_2 are the conductor loss resistances, R_{r1} and R_{r2} are the resistances accounting for radiation loss, and R_L is the load resistance.

L_1 and L_2 are the inductances, and C_1 and C_2 are the capacitances, for the first and second loops, respectively. M is the mutual inductance between the two loops. I_1 and I_2 are the currents flowing on each loop.

Using KVL's, we obtain

Manuscript received September 1, 2011 ; revised September 16, 2011. (ID No. 20110901-4J)

Department of Electronics and Radio Engineering, Kyung Hee University, Yongin, Korea.

Corresponding Author : Bomson Lee (e-mail : bomson@khu.ac.kr)

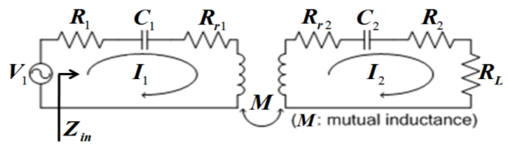


Fig. 1. Equivalent circuit for magnetically coupled loops considering radiation loss.

$$\left(R_1 + R_{r1} + j\omega L_1 + \frac{1}{j\omega C_1}\right)I_1 - j\omega M I_2 = V_1 \tag{1}$$

and

$$\left(R_2 + R_{r2} + R_L + j\omega L_2 + \frac{1}{j\omega C_2}\right)I_2 - j\omega M I_1 = 0\,. \tag{2}$$

When each loop is resonant at an angular frequency ω_0, the current on each loop can be expressed as

$$I_1 = \frac{\left(R_2 + R_{r2} + R_L\right)}{(R_1 + R_{r1})\left(R_2 + R_{r2} + R_L\right) + \omega_0{}^2 M^2} V_1\,, \tag{3}$$

$$I_2 = \frac{j\omega_0 M}{(R_1 + R_{r1})\left(R_2 + R_{r2} + R_L\right) + \omega_0{}^2 M^2} V_1. \tag{4}$$

We can see that when M reaches a maximum, the current flowing on loop 1 becomes minimum. When M=0, the two loops are isolated from each other, $I_1 = V_1/(R_1 + R_{r1})$, and I_2=0.

The coupling coefficient k between the two loops is given by

$$k = \frac{M}{\sqrt{L_1 L_2}}\,. \tag{5}$$

The loss rate η_l is defined as the ratio of the dissipated power (in R_1, R_{r1}, R_2, and R_{r2}) and the input power. The transmission rate η_t (or WPT efficiency) is defined as the ratio of the delivered power in R_L and the input power.

If the two loops are identical and assumed to be lossless for simplicity (R_1=R_2=0, R_{r1}=R_{r2}=R_r, L_1=L_2=L, C_1=C_2, and M=kL), based on the equivalent circuit (Fig. 1) and S-parameters obtained from the EM simulation (or measurements), the loss rate η_l and transmission rate η_t are given by

$$\eta_l = \frac{(R_r + R_L)R_r + \dfrac{\omega_0{}^2 k^2 L^2}{(R_r + R_L)} R_r}{(R_r + R_L)R_r + \omega_0{}^2 k^2 L^2} = \frac{1 - |S_{11}|^2 - |S_{21}|^2}{1 - |S_{11}|^2} \tag{6}$$

and

$$\eta_t = \frac{\dfrac{R_L}{R_r + R_L}}{(R_r + R_L)R_r + \omega_0{}^2 k^2 L^2} = \frac{|S_{21}|^2}{1 - |S_{11}|^2}\,. \tag{7}$$

Note that $\eta_l + \eta_t = 1$. The loss rate η_l may now be understood as being the radiation rate since R_1=R_2=0. Another term for the transmission rate η_t is the WPT efficiency. From equations (6) and (7), we can see that the loss rate is 1 and the transmission rate is 0 when the coupling coefficient k is 0 (when the two loops are far apart). The opposite is true when k is 1 (when the two loops are very close to each other).

The input impedance Z_{in} in Fig. 1 at the resonant angular frequency ω_0 is given by

$$Z_{in} = \frac{V_1}{I_1} = \frac{R_r(R_r + R_L) + \omega_0{}^2 k^2 L^2}{R_r + R_L}\,. \tag{8}$$

From this equation, we can see that if the separation between the two loops is large (k=0), Z_{in} reduces to the radiation resistance R_r of loop 1 only.

Solving equations (6) (or (7)) and (8) simultaneously, the coupling coefficient k and radiation resistance R_r can be expressed as

$$R_r = \frac{R_L Z_{in} \eta_l}{R_L + Z_{in} - Z_{in}\eta_l} \tag{9}$$

and

$$k = \frac{(R_L + Z_{in})\sqrt{R_L Z_{in} - R_l Z_{in} \eta_l}}{\omega_0 L\left(R_L + Z_{in} - Z_{in}\eta_l\right)} \tag{10}$$

where the loss rate η_l and input impedance Z_{in} are obtained from the EM simulations (or measurements).

The coupling coefficient k in (6) may also be obtained from the EM-simulated or measured S_{21} [10] using

$$k = \frac{f_h^2 - f_l^2}{f_h^2 + f_l^2}\,. \tag{11}$$

If k and R_r are fixed, the load resistance R_L for maximum power transmission can be given by

$$R_L = \sqrt{R_r^2 + \omega_0^2 k^2 L^2}\,. \tag{12}$$

Equations (6) and (7) can be rearranged using the Q-factor of each loop as

$$\eta_l = \frac{1 + \dfrac{\left(1 + \beta_e\right)^2}{k^2 Q_1 Q_2}}{\left\{1 + \dfrac{1}{k^2 Q_1 Q_2}\left(1 + \beta_e\right)\right\}\left(1 + \beta_e\right)} \tag{13}$$

and

$$\eta_t = \frac{\beta_e}{\left\{1 + \dfrac{1}{k^2 Q_1 Q_2}\left(1 + \beta_e\right)\right\}\left(1 + \beta_e\right)} \tag{14}$$

where Q_1 and Q_2 are the Q-factors of the two loops

$(Q_1 = \omega_0 L_1 / R_{r1}, Q_2 = \omega_0 L_2 / R_{r2})$ and $\beta_e = R_L/R_r$. From these equations, we can see that the transmission rate increases as Q_1 and Q_2 increase.

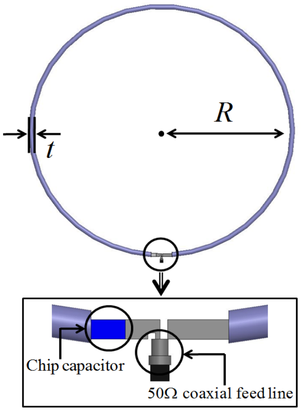
Ⅲ. Circuit and EM Simulations

Fig. 2(a) shows the geometry of the loop. The radius R and thickness t of the perfect electronic conductor (PEC) loop are 250 mm and 10 mm, respectively.

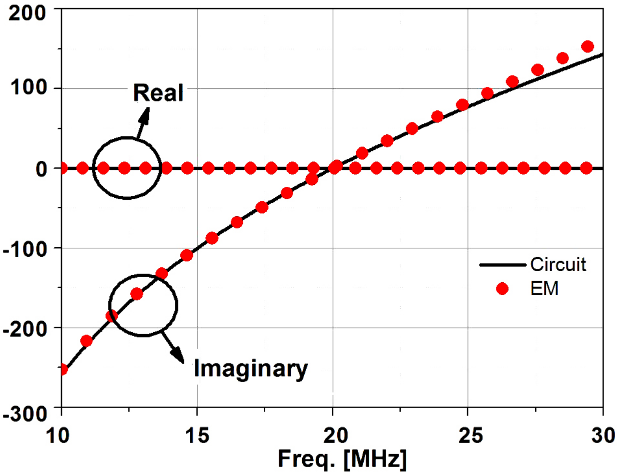
Since PEC is assumed for the loops, the conductor loss resistance R_1 is zero. The inductance of the loop is about 1.37 uH. We use the 46.2 pF capacitor to resonate this loop at 20 MHz. At 20 MHz, the real and imaginary parts of the loop input impedance are about 0.46 Ω and 0 Ω, respectively.

This loop is fed by a 50 Ω coaxial line for all EM simulations in this study. However, any reference impedances other than 50 Ω may be used. It is noted that equations (6) and (7) hold true for any reference impedances.

We obtained the S-parameters based on circuit/EM simulations using two loops with the same dimensions as shown in Fig. 2(a). Since the two loops are identical, $R_{r1} = R_{r2} = R_r$, $L_1 = L_2 = L$, $C_1 = C_2$, and port impedance is 50 Ω.



(a) Geometry of the designed loop



(b) Input impedance of the designed loop

Fig. 2. Geometry and input impedance of the designed loop.

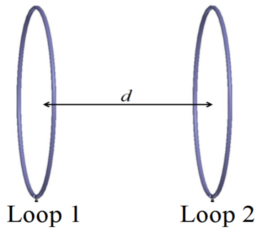
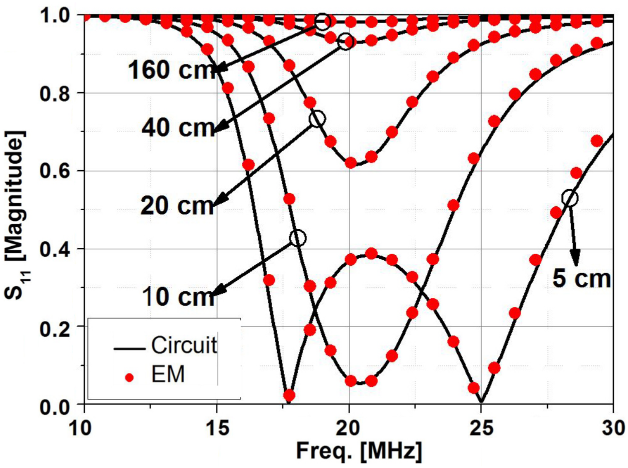


Fig. 3. Wireless power transmission system.

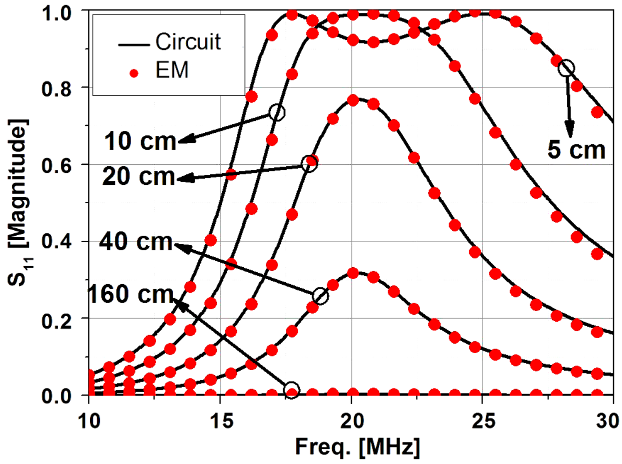
For the circuit and EM simulations, ADS and HFSS are used, respectively.

Fig. 3 shows the schematic of a wireless power transmission system consisting of two loops. The two identical loops face each other. The symbol d in Fig. 3 represents the distance between the two loop structures.

Fig. 4(a) and (b) show the magnitudes of S_{11} and S_{21}



(a) Magnitude of S_{11}



(b) Magnitude of S_{21}

Fig. 4. S-parameters for different distances d.

obtained from the circuit and EM simulations when the distance between the two loops increases from 10 cm to 160 cm. The circuit and EM simulation results are shown to be in good agreement. For this specific case, Z_{in} given in (8) is about 44 Ω when d is 10cm. Thus, the loop is shown to match well to the 50 Ω coaxial feed. When Z_{in} is greater than the feed impedance (when there is strong coupling), two peaks are usually observed [10].

The loss rate η_l and transmission rate η_t can be calculated by substituting these S-parameters in equation (6) and (7).

Fig. 5 shows the loss rate η_l given by (6) and the transmission rate η_t given by (7) with varying distances at 20 MHz. η_l increases and η_t decreases as the distance increases. The sum of η_l and η_t is shown to be 1. We can see that when the distance between the two loops is 10 cm, the WPT efficiency (η_t) is 98.1 %. When the distance is 140 cm, η_t is 0.1 %. Fig. 6 shows the coupling coefficient k given by (10) and the radiation resistance R_r (9) as the distance changes from 10 cm to 180 cm at 20 MHz. R_r is shown to converge to 0.46 Ω when d is very large, which is the radiation resistance of the single loop. This convergence gives us more confidence in the proposed modeling. It is notable that the variance of R_r relative to the radiation resistance of 0.46 Ω is small.

In Table 1, we summarize the loss rate η_l, transmission rate η_t, coupling coefficient k, and radiation resistance R_r for different distances d.

Fig. 7 shows the transmission rate η_t (or WPT efficiency) as a function of the load impedance R_L when the distance between the two loops is 80 cm. For this case, k is 0.009 and R_r is 0.460 (See Table 1). This figure has been drawn based on (12). The load impedance R_L for a maximum WPT efficiency is about 1.52 Ω. When the load impedance is 1.52 Ω, the transmission rate η_t is about 56 %, which is about six times larger than when the load impedance is 50 Ω (η_t= 9.5 % as seen in Table

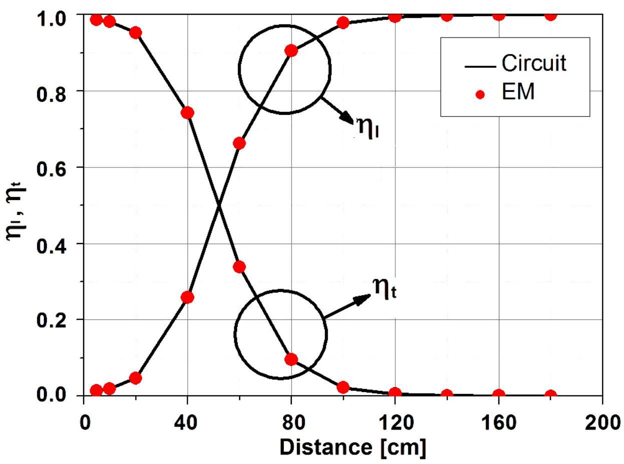


Fig. 5. Loss rate η_l and transmission rate η_t with varying distances at 20 MHz.

Table 1. η_l, η_l, k and R_r for different separations.

d (cm)	η_l (%)	η_l (%)	k	R_r (Ω)
5	1.3	98.7	0.428	0.450
10	1.9	98.1	0.272	0.450
20	4.7	95.3	0.138	0.451
40	25.8	74.2	0.048	0.453
60	66.2	33.8	0.020	0.459
80	90.5	9.5	0.009	0.460
100	97.8	2.2	0.004	0.467
120	99.4	0.6	0.002	0.462
140	99.9	0.1	0.001	0.461
160	1.0	0	0	0.460

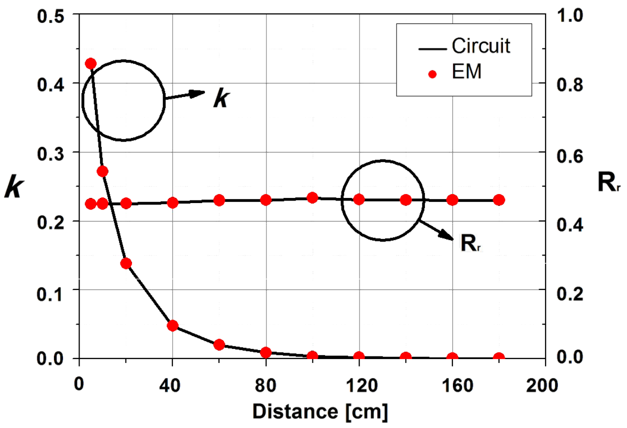


Fig. 6. Coupling coefficient k and radiation resistance R_r with varying distances at 20 MHz.

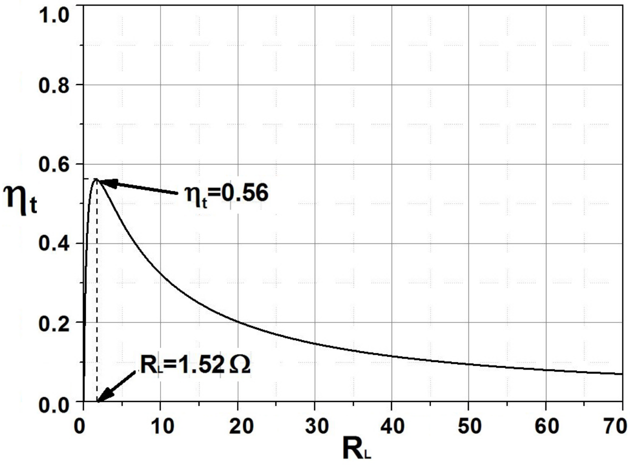


Fig. 7. Transmission rate η_t in accordance with load impedance R_L when the distance between the two loops is 80 cm (k=0.009, Rr=0.460 Ω).

1). This example choosing the correct load impedance, as given by (12), is very important for maximum WPT efficiency.

Ⅳ. Conclusion

We have proposed and analyzed an equivalent circuit for a magnetically coupled WPT between the loop resonators in terms of the coupling coefficient and radiation factors. All the necessary circuit parameters have been extracted through a complete formulation. The circuit and EM simulation results have been shown to be in good agreement. Based on the proposed circuit formulation, it has been found that there is a specific load impedance for maximum WPT efficiency. The proposed modeling of the WPT in terms of circuit characterizations provides sufficient physical insight into the problems often associated with WTP.

This research was supported by the KCC (Korea Communications Commission), Korea, under the R&D program supervised by the KCA (Korea Communications Agency) (KCA-2011-11911-01110)"

References

[1] N. Tesla, U.S. patent 1,119,732, 1914.

[2] H. Haus, *Waves and Fields in Optoelectronics*, Prentice Hall, Englewood Cliffs, NJ., 1984.

[3] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher and M. Solacic, "Wireless power transfer via strongly coupled magnetic resonances," *Science*, vol. 317, pp. 83-86, Jul. 2007.

[4] Gunyoung Kim, Bomson Lee, and Younkwon Jung, "Wireless power transmission between two metamaterial-inspired loops at 300 MHz," *Journal of The Korean Institute of Electromagnetic Engineering and Science*, vol. 10, no. 4, pp. 219-223, Dec. 2010.

[5] Chunlai Yu, Rengui Lu, Yinhua Mao, Litao Ren, and Chunbo Zhu, "Research on the model of magnetic-resonance based wireless energy transfer system," *Vehicle Power and Propulsion Conference*, pp. 414-418, Sep. 2009.

[6] I. Awai, T. Komori, "A simple and versatile design method of resonator-coupled wireless power transfer systerm," *Proc. ICCCAS2010*, Jul. 2010.

[7] Qiaowei Yuan, Qiang Chen, Long Li, and Kunio Sawaya, "Numerial analysis on transmission efficiency of evanescent resonant coupling wireless power transfer system," *IEEE Trans. Antennas Propag.*, vol. 58, no. 5, May 2010.

[8] Youn-Kwon Jung, Bomson Lee, "Metamaterial-inspired loop antennas for wireless power transmission," *Proc. Int. Symp. on AP-S*, pp. 1-4, Jun. 2010.

[9] Ikuo Awai, "New expressions for coupling coefficient between resonators," *IEICE Trans. Electron.*, E88-C, no. 12, pp. 2295-2301, Dec. 2005.

[10] J. Hong, M. J. Lancaster, *Microstrip Filters for RF/Microwave Applications*, Wiley & Sons., pp. 247-249, 2001.

Chungju Kim

received the B.S. degree in radio and communication engineering from Kyung Hee University, Yongin, Korea in 2010. From 2010, he is studying in master's course in electronics and radio engineering, Kyung Hee University, Yongin, Korea. His research interests include microwave antennas, metamaterial, and wireless power transmission.

Bomson Lee

received the B.S. degree in electrical engineering from Seoul National University, Seoul, Korea in 1982, and M.S and Ph.D. degrees in electrical engineering from University of Nebraska-Lincoln in 1991 and 1995, respectively. He is a professor in electronics and radio engineering at Kyung Hee University in Korea from 1995. He is now the editor-in-chief of the Journal of the Korean Institute of Electromagnetic Engineering and Science. His research interests include microwave antennas, metamaterial, RFID tags, microwave passive devices, and wireless power transmission.