오차 엔트로피 기준에 근거한 결정 궤환 등화 알고리듬

Decision Feedback Equalizer Algorithms based on Error Entropy Criterion

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요 약

다중경로 페이딩과 충격성 잡음에 의한 채널 왜곡을 보상하기 위하여 오차 엔트로피 최소화 (MEE)에 근거한 결정 궤환 등화 (DFE) 알고리듬을 제안하였다. MEE 성능기준이 아직 결정 궤환 구조나 충격성 잡음환경에 대해 연구된 바가 없다. 결정 궤환 구조의 등화기의 가중치에 대해 오차 엔트로피를 최소화함으로써 제안된 알고리듬은 심각한 다중경로와 충격성 잡음 환경에서 탁월한 잔여 심볼간 간섭제거능력을 보였다.

ABSTRACT

For compensation of channel distortion from multipath fading and impulsive noise, a decision feedback equalizer (DFE) algorithm based on minimization of Error entropy (MEE) is proposed. The MEE criterion has not been studied for DFE structures and impulsive noise environments either. By minimizing the error entropy with respect to equalizer weight based on decision feedback structures, the proposed decision feedback algorithm has shown to have superior capability of residual intersymbol interference cancellation in simulation environments with severe multipath and impulsive noise.

☞ keyword : Entropy(엔트로피), MEE, decision-feedback(결정궤환), impulsive noise(충격성 잡음), multipath(다중경로), equalizer(등화기), ITL

1. INTRODUCTION

In many communication systems such as satellite-mobile radio link, power line digital subscriber line systems and even in underwater communication channels, multipath fading and impulsive noise frequently occur [1-3].

As a typical adaptive equalizer algorithm For counteracting multipath fading, the least mean square (LMS) algorithm [4] employing the mean squared error (MSE) criterion has been being widely used due to its simplicity but MSE-based algorithms are highly sensitive to large error values from impulsive noise.

Unlike MSE based approaches that utilize instant error power, information-theoretic learning (ITL) methods, introduced by Princepe [5], are based on a combination of a nonparametric probability density function (PDF) estimator and a procedure to compute error entropy. As an entropy-related cost function, minimization of error entropy (MEE) has been developed by Erdogmus and Principe and it can be implemented by maximization of information potential [6]. MEE has shown superior performance as an alternative to MSE in supervised channel equalization applications [6,7]. However, the MEE based algorithms have not been studied for channels with impulsive noise or feedback approaches.

In this paper, we investigate the performance of supervised linear MEE algorithm for channels distorted by multipath fading and impulsive noise. And also we propose a MEE algorithm with decision feedback (MEE-DF) for enhanced

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performance against impulsive noise and strong multi-path fading.

2. MSE CRITERIONAND LMS ALGORITHM

As a linear structure for equalization, a tapped delay line (TDL) is usually used. Letting the TDL have L taps (weights) and weight vector W_k with L elements at symbol time k, the input vector $\boldsymbol{X}_k=[x_k, x_{k-1}, x_{k-2},..., x_{k-L+1}]^T$ and W_k produce an output sample y_k, that is, $y_k=\boldsymbol{W}_k^T\boldsymbol{X}_k$. Using d_k, a training symbol at time k or the desired value, we obtain an error sample $e_k = d_k - y_k$. The MSE criterion is statistical average of error power described as

$$MSE = E[e_k^2] \tag{1}$$

For practical reasons, instead of estimating the expected value of error power, we can use the instant squared error (SE) as a cost function.

$$SE = e_k^2 \tag{2}$$

Minimizing the cost function (2) using the gradient $\frac{\partial e_k^2}{\partial \boldsymbol{W}}$ and steepest descent method, we have the well known LMS algorithm [4].

$$\boldsymbol{W}_{k+1} = \boldsymbol{W}_k + 2\mu_{LMS} e_k \boldsymbol{X}_k \tag{3}$$

where the step size μ_{LMS} controls convergence speed and stability of the algorithm.

It is noticeable that the LMS algorithm is directly affected by error values, and in impulsive noise environments large error values induced from impulsive noise can make the weight adjustment process unstable. Using a very small step size can be a solution for stability.

3. ERROR ENTROPY CRITERION AND MEE ALGORITHM

The MSE criterion, which uses only second order statistics, is adequate under the assumptions of linearity and Gaussianity. When the noise is not Gaussian, a criterion considering all the higher order statistics of the error signal would be more appropriate. Entropy is a scalar quantity that provides a measure for the average information contained in a given PDF $f_E(e)$. When error entropy is minimized, the error distribution of adaptive systems is concentrated and all higher order moments are minimized.

The Shannon's entropy [8] is in general hard to estimate and minimize since it involves the integral of the logarithm of the PDF. As another useful alternative definition of entropy, Renyi's quadratic error entropy which is effectively used in ITL methods is defined as

$$H(e) = -\log(\int f_E(\xi)^2 d\xi). \tag{4}$$

To investigate how the entropy is associated with a set of data samples without pre-specifying the form of the PDF, we employ the Parzen estimator [5] with Gaussian kernel $G_\sigma(\cdot)$ and a block of N past error samples as

$$f_E(e) = \frac{1}{N}\sum_{i=k-N+1}^{k} G_\sigma(e-e_i)$$
$$= \frac{1}{N}\sum_{i=k-N+1}^{k} \frac{1}{\sigma\sqrt{2\pi}}\exp[\frac{-(e-e_i)^2}{2\sigma^2}] \tag{5}$$

Then the combination of Reny's quadratic entropy with the Parzen window leads to an estimation of entropy by computing interactions among pairs of error samples.

$$H(e) = -\log(\int f_E(\xi)^2 d\xi)$$
$$= -\log\left[\frac{1}{N^2}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k} G_{\sigma\sqrt{2}}(e_j - e_i)\right] \quad (6)$$

We can see that data samples (actually their locations) act as physical particles since the Gaussian kernel is always positive and is inversely proportional to the distance between data samples (an exponential decay with the distance square). From these observations, the Gaussian kernel may be considered to create a potential field and the argument of log [.] in (6) can be regarded as the sum of all pairs of interactions and is called information potential, IP_e [5].

$$IP_e = \frac{1}{N^2}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k} G_{\sigma\sqrt{2}}(e_j - e_i) \quad (7)$$

Substituting the information potential IP_e, for $\int f_E^2(\xi)d\xi$ in (6), we have

$$H(e) = -\log(IP_e), \quad (8)$$

Obviously, minimizing the error entropy $H(e)$ is equivalent to maximizing the information potential IP_e. This criterion minimizing error entropy is referred to as MEE [6,7].

$$\text{MEE} = \min_W H(e) = \max_W IP_e \quad (9)$$

By applying gradient ascent method to maximization of IP_e, MEE algorithm with step size μ_{MEE} becomes

$$\boldsymbol{W}_{k+1} = \boldsymbol{W}_k + \frac{\mu_{MEE}}{2\sigma^2 N^2}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k}(e_j - e_i)$$
$$\cdot G_{\sigma\sqrt{2}}(e_j - e_i)[\boldsymbol{X}_j - \boldsymbol{X}_i]. \quad (10)$$

4. MEE ALGORITHM WITH DECISION FEEDBACK

In order for the MEE algorithm (10) to be stretched and applied to structures with DF that consist of a feed-forward filter with weight vector W_k^F and a feedback filter with weight vector W_k^B, the algorithm has to be augmented with DF part using produced decisions $\hat{d}_k$. While the feed-forward filter receives input x_k to produce output y_k, the feedback filter receives the sequence of decisions.

Let the number of weights in feed-forward and feedback filter section be A and B, respectively. Then output y_k of the TDL equalizer with decision feedback becomes

$$y_k = \sum_{a=0}^{A-1} w_{k,a}^F x_{k-a} + \sum_{b=0}^{B-1} w_{k,b}^B \hat{d}_{k-b-1} \quad (11)$$

where $\{w_{k,0}^F, w_{k,1}^F, w_{k,2}^F, \ldots, w_{k,A-1}^F\}$ are elements of feed-forward weight vector W_k^F, $\{w_{k,0}^B, w_{k,1}^B, w_{k,2}^B, \ldots, w_{k,B-1}^B\}$ are elements of feedback weight vector W_k^B. The elements of vector $\hat{D}_{k-1}$, $\{\hat{d}_{k-1}, \hat{d}_{k-2}, \ldots, \hat{d}_{k-B}\}$ are previously detected symbols.

The feedback filter plays a role of removing the residual ISI from the present estimate which is caused by previously detected symbols [4]. It is noticeable that incorrect decisions can cause error propagation because the decisions are fed back into feedback filter. Though errors from AWGN do not have disastrous effects on the performance, strong impulsive noise induces substantial error propagation. Conventional algorithms highly dependent on instant error power can not cope with this problem. Therefore equalizers with DF for impulsive noise environments need robust algorithms against strong impulsive noise.

As described before, the Gaussian kernel produces an exponential decay with distance square of data sample pairs. Large error samples are mainly induced from impulsive noise, so the distance of their pairs is very likely to be large. From this observation, we may decide that the argument of Gaussian kernel in MEE algorithm (10) has the effect of cutting out large error values from impulsive noise.

Now the filter weights are adjusted recursively to minimize the cost function (9) using the calculated error $e_k = d_k - y_k$ in training mode and $e_k = \hat{d}_k - y_k$ in decision directed mode. Then the feed-forward weight vector W_k^F and the feedback weight vector W_k^B are updated based on steepest ascent method with the step size μ_{MEE-DF} and the gradients

$$\frac{\partial IP_e}{\partial \boldsymbol{W}^F} = \frac{1}{2\sigma^2 N^2} \sum_{i=k-N+1}^{k} \sum_{j=k-N+1}^{k} (e_j - e_i) \cdot G_{\sigma\sqrt{2}}(e_j - e_i)[\boldsymbol{X}_j - \boldsymbol{X}_i] \tag{12}$$

$$\frac{\partial IP_e}{\partial \boldsymbol{W}^B} = \frac{1}{2\sigma^2 N^2} \sum_{i=k-N+1}^{k} \sum_{j=k-N+1}^{k} (e_j - e_i) \cdot G_{\sigma\sqrt{2}}(e_j - e_i)[\hat{\boldsymbol{D}}_{j-1} - \hat{\boldsymbol{D}}_{i-1}] \tag{13}$$

$$\boldsymbol{W}_{k+1}^F = \boldsymbol{W}_k^F + \mu_{MEE-DF} \frac{\partial IP_e}{\partial \boldsymbol{W}^F} \tag{14}$$

$$\boldsymbol{W}_{k+1}^B = \boldsymbol{W}_k^B + \mu_{MEE-DF} \frac{\partial IP_e}{\partial \boldsymbol{W}^B} \tag{15}$$

In the expression of weight elements,

$$w_{k+1,p}^F = w_{k,p}^F + \frac{\mu_{MEE-DF}}{2\sigma^2 N^2} \sum_{i=k-N+1}^{k} \sum_{j=k-N+1}^{k} (e_j - e_i) \cdot G_{\sigma\sqrt{2}}(e_j - e_i)[x_{j-p} - x_{i-p}] \tag{16}$$

$$w_{k+1,q}^B = w_{k,q}^B + \frac{\mu_{MEE-DF}}{2\sigma^2 N^2} \sum_{i=k-N+1}^{k} \sum_{j=k-N+1}^{k} (e_j - e_i) \cdot G_{\sigma\sqrt{2}}(e_j - e_i)[\hat{d}_{j-1-q} - \hat{d}_{i-1-q}] \tag{17}$$

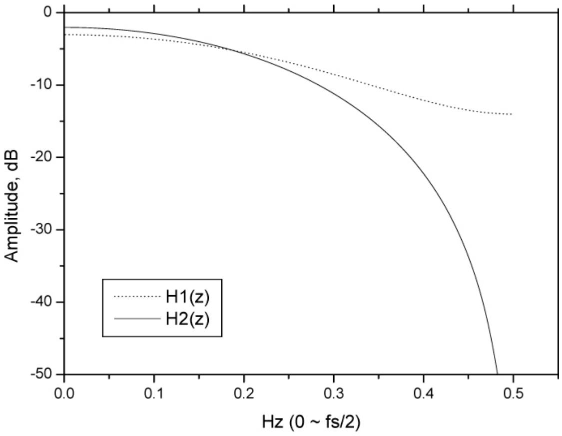
5. RESULTS AND DISCUSSION

As s figure of merit, we compare MSE convergence and steady state error probability of the proposed MEE-DF algorithm, linear MEE, LMS and LMS-DF in the multipath channel environments with impulsive noise. The multipath channel models have the following transfer functions, where $H_2(z)$ results in severe intersymbol interference and exhibits spectrum nulls [4].

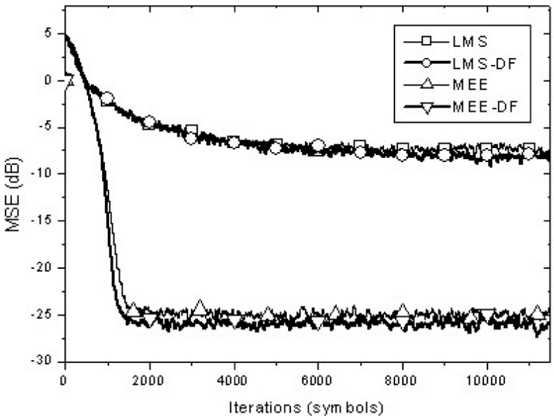
$$H_1(z) = 0.26 + 0.93z^{-1} + 0.26z^{-2} \tag{18}$$

$$H_2(z) = 0.407 + 0.815z^{-1} + 407z^{-2} \tag{19}$$

Background Gaussian noise and impulsive noise (zero-mean and white) are added to the channel output. The overall channel noise has the following probability density in (20) [9,10]. The value σ_2^2 is the variance of impulse noise plus background noise and σ_1 represents the standard deviation of background noise. In this simulation $\varepsilon = 0.03$, $\sigma_1^2 =$ 0.001, and $\sigma_2^2 = 50.001$.



(Fig. 1) Amplitude spectrum for the channels $H_1(z)$ and $H_2(z)$.



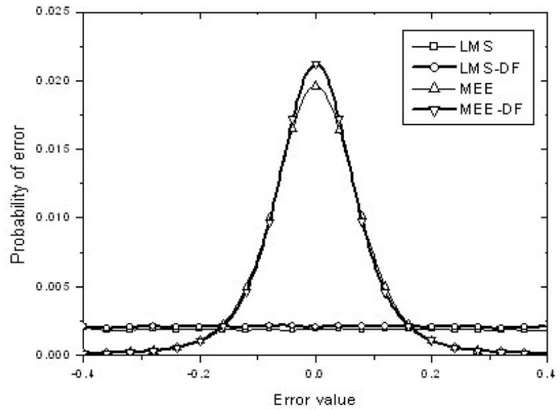
(Fig. 2) MSE convergence performance for $H_1(z)$.

$$f_{NOISE}(n) = \frac{\varepsilon}{\sigma_2\sqrt{2\pi}}\exp[\frac{-n^2}{2\sigma_2^2}] + \frac{1-\varepsilon}{\sigma_1\sqrt{2\pi}}\exp[\frac{-n^2}{2\sigma_1^2}] \quad (20)$$

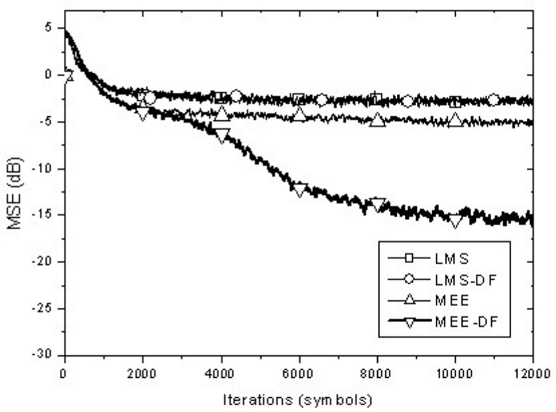
The numbers of feed-forward and feedback filter weights are A=7 and B=4, respectively. The linear algorithms have corresponding number of weights 11. The 4 PAM random symbols {-3, -1, 1, 3}are transmitted to the channel. The step-size is set to $\mu_{MEE} = \mu_{MEE-DF}$ = 0.01, and $\mu_{LMS} = \mu_{LMS-DF}$ = 0.0002 for both channel models. Data-block size N for MEE and MEE-DF is 20 and the kernel size σ is 0.7. The parameters are chosen to show the lowest steady-state MSE.

The MSE performance for $H_1(z)$ in Fig. 2 shows that in impulsive noise environments, steady state MSEs of LMS and LMS-DF do not decrease below -6dB and -7dB, respectively. We can observe a very slow convergence and MSE performance enhancement of only about 1dB by employing DF. On the other hand the MEE converges rapidly to about -25dB and MEE-DF to -27dB of steady state MSE. Performance difference is seen clearly from the error PDF estimates in Fig. 3. The MEE-DF equalizer produces error distribution being the most concentrated around zero whereas LMS-type algorithms form almost flat shapes.

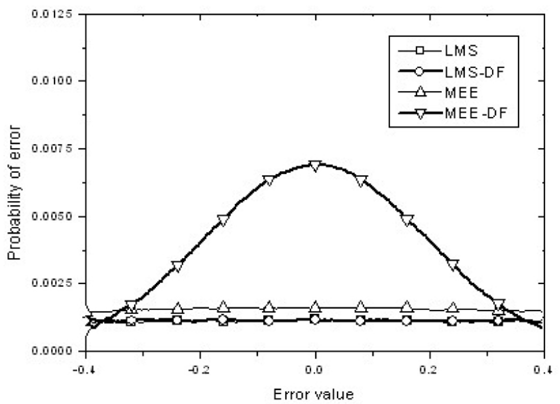
The performance in the worse channel $H_2(z)$ with impulsive noise shows prominent improvement in MEE-DF due to decision feedback in Fig. 4. As noticed in Fig. 1, the channel model $H_2(z)$ has a very deep null so the received signal is highly distorted by enormous intersymbol interference. In this case equalizer algorithms with DF and without DF produce large performance difference. While LMS and LMS-DF show similar degradation with -3dB of steady state MSE performance, MEE and MEE-DF result in a very large performance difference. The steady state MSE of MEE stays at -5dB, but MEE-DF yields -17dB. . The 12dB of performance enhancement has been brought by employing DF in MEE algorithm. Considering MEE and MEE-DF have the inherent immunity against impulsive noise, we can observe how much impact the decision feedback strategy has on MEE-type equalizer algorithms under severe multipath fading. In Fig. 5, performance differences are shown more clearly as MEE-DF produces error distribution well



(Fig. 3) Probability density for errors in $H_1(z)$.



(Fig. 4) MSE convergence performance for $H_2(z)$.



(Fig. 5) Probability density for errors in $H_2(z)$

concentrated around zero while the error samples of LMS, LMS-DF and even MEE appear not to gather around zero.

6. CONCLUSION

In this paper, a minimum-error-entropy algorithm with decision feedback is proposed to counteract multipath fading and impulsive noise. LMS-type algorithms based on conventional MSE criterion have no capability to cope with impulsive noise whether with DF or not since the cost function of squared error is highly sensitive to large instant error power caused by impulsive noise. It is revealed that MEE-type algorithms designed on purpose of concentrating error samples have the inherent immunity against impulsive noise. In simulations for severe channel models, the error samplesof LMS, LMS-DF and even MEE appear not to gather around zero. On the other hand MEE-DF produces error distribution well concentrated around zero and 12dB of performance enhancement has been yielded through employing DF in MEE algorithm. From the results,we may conclude that in severe multipath channels with impulsive noise, MEE does not function properly without DF and employing decision feedback in MEE algorithm can be a strong candidate for channel equalizers in severe multipath fading and impulsive noise.

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