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Properties of fuzzy (*r*, *s*)-semi-irresolute Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract

In this paper, we introduce the concept of fuzzy (r, s)-semi-irresolute mappings on intuitionistic fuzzy topological spaces in Sostak's sense, which is a generalization of the concept of fuzzy semi-irresolute mappings introduced by S. Malakar. The characterizations for the fuzzy (r, s)-semi-irresolute mappings are obtained by terms of semi-interior, semi-θ-interior, semi-clopen, and regular semi-open.

Keywords : fuzzy (r, s)-semi-cluster point, fuzzy (r, s)-semi-irresolute.

1. Introduction and Preliminaries

The concept of fuzzy set was introduced by Zadeh [1]. Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [3], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [4], and by Ramadan [5].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [6]. Recently, Çoker and his colleagues [7, 8] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [9] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. S. Malakar [10] introduced the concept of fuzzy semi-irresolute mappings, and Seong Hoon Cho and Jae Keun Park [11] established some other properties of fuzzy semi-irresolute mappings on Chang's fuzzy topological spaces.

In this paper, we introduce the concept of fuzzy (r, s)-semi-irresolute mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The characterizations for the fuzzy (r, s)-semi-irresolute mappings are obtained by terms of semi-interior, semi-θ-interior, semi-clopen, and regular semi-open.

Definition 1.1. ([9]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a mapping $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties:

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

Then $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Theorem 1.2. ([12]) (1) The fuzzy (r, s)-closure of a fuzzy (r, s)-open set is fuzzy (r, s)-regular closed for each $(r, s) \in I \otimes I$.
(2) The fuzzy (r, s)-interior of a fuzzy (r, s)-closed set is fuzzy (r, s)-regular open for each $(r, s) \in I \otimes I$.

Definition 1.3. ([12]) Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is called a *fuzzy (r, s)-irresolute* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-semiopen set in X for each fuzzy (r, s)-semiopen set B in Y.

Definition 1.4. ([10]) A function $f : X \to Y$ is said to be *fuzzy semi-irresolute* iff for any fuzzy singleton x_α in X and for any fuzzy semi-open set V in Y containing $f(x_\alpha)$, there exists a fuzzy semi-open set U in X containing x_α such that $f(U) \leq \text{scl}(V)$.

For the nonstandard definitions and notations we refer to [13, 14, 15, 12, 16].

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2. Fuzzy (r, s)-semi-irresolute Mappings

Now, we define the notion of fuzzy (r, s)-semi-irresolute mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and investigate some of their characteristic properties.

Definition 2.1. Let A and B be intuitionistic fuzzy sets in a SoIFTS $(X, \mathcal{T})$ and $(r, s) \in I \otimes I$. Then

(1) B is called a *fuzzy* (r, s)*-quasi-neighborhood* of A if there is a fuzzy (r, s)-open set C in X such that $A\mathrm{q}C \subseteq B$.

(2) A is said to be a *fuzzy* (r, s)*-quasi-semineighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r, s)-semiopen set B in X such that $x_{(\alpha,\beta)}\mathrm{q}B \subseteq A$. The family of all fuzzy (r, s)-semiopen quasi-semineighborhoods of $x_{(\alpha,\beta)}$ will be denoted by $N_{\mathrm{S}}^{\mathrm{q}}(x_{(\alpha,\beta)})$.

(3) $x_{(\alpha,\beta)}$ is said to be a *fuzzy* (r, s)*-semi-cluster* point of A if for each $B \in N_{\mathrm{S}}^{\mathrm{q}}(x_{(\alpha,\beta)})$, $B\mathrm{q}A$.

(4) $x_{(\alpha,\beta)}$ is said to be a *fuzzy* (r, s)*-semi-θ-cluster* point of A if for each $B \in N_{\mathrm{S}}^{\mathrm{q}}(x_{(\alpha,\beta)})$, $\mathrm{scl}(B, r, s)\mathrm{q}A$.

(5) The *fuzzy* (r, s)*-semi-θ-closure* of A, denoted by $\mathrm{scl}_\theta(A, r, s)$, is the union of all fuzzy (r, s)-semi-θ-cluster points of A.

(6) A is called a *fuzzy* (r, s)*-semi-θ-closed* set if $A = \mathrm{scl}_\theta(A, r, s)$.

(7) A set is called *fuzzy* (r, s)*-semi-θ-open* if it is the complement of a fuzzy (r, s)-semi-θ-closed set.

Theorem 2.2. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then $\mathrm{scl}(A, r, s)$ is the union of all fuzzy (r, s)-semi-cluster points of A.

Proof. Let $\mathrm{SCP}(A, r, s)$ is the union of all fuzzy (r, s)-semi-cluster points of A. Suppose that there is an intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X such that $x_{(\alpha,\beta)} \in \mathrm{scl}(A, r, s)$ and $x_{(\alpha,\beta)} \notin \mathrm{SCP}(A, r, s)$. Then there is a $B \in N_{\mathrm{S}}^{\mathrm{q}}(x_{(\alpha,\beta)})$ such that $B\tilde{\mathrm{q}}A$. Then there exists a fuzzy (r, s)-semiopen set C in X such that $x_{(\alpha,\beta)}\mathrm{q}C \subseteq B$. Since $C \subseteq B$, we have $C\tilde{\mathrm{q}}A$, and hence $A \subseteq C^c$. Since C^c is fuzzy (r, s)-semiclosed, we obtain $\mathrm{scl}(A, r, s) \subseteq C^c$. As $x_{(\alpha,\beta)} \notin C^c$, we obtain $x_{(\alpha,\beta)} \notin \mathrm{scl}(A, r, s)$, which is a contradiction. Hence $\mathrm{scl}(A, r, s) \subseteq \mathrm{SCP}(A, r, s)$.

Conversely, assume that $x_{(\alpha,\beta)} \notin \mathrm{scl}(A, r, s)$. Then there is a fuzzy (r, s)-semiclosed set B such that $A \subseteq B$ and $x_{(\alpha,\beta)} \notin B$. Then $x_{(\alpha,\beta)}\mathrm{q}B^c$, B^c is fuzzy (r, s)-semiopen, and $B^c\tilde{\mathrm{q}}A$. Thus $x_{(\alpha,\beta)} \notin \mathrm{SCP}(A, r, s)$. Hence $\mathrm{SCP}(A, r, s) \subseteq \mathrm{scl}(A, r, s)$. □

By the above theorem, we see that $\mathrm{scl}(A, r, s) \subseteq \mathrm{scl}_\theta(A, r, s)$ for any intuitionistic fuzzy set A in X.

Definition 2.3. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is said to be *fuzzy* (r, s)*-semi-irresolute* if for each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-semiopen set B in Y with $f(x_{(\alpha,\beta)}) \in B$, there is a fuzzy (r, s)-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq \mathrm{scl}(B, r, s)$.

Remark 2.4. It is clear that every fuzzy (r, s)-irresolute mapping is fuzzy (r, s)-semi-irresolute. But the following example shows that the converse need not be true.

Example 2.5. Let $X = \{x\}$ and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.7, 0.2), \quad A_2(x) = (0.6, 0.3).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$. Then A_2 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set in $(X, \mathcal{U})$. Note $\mathrm{scl}(B, \frac{1}{2}, \frac{1}{3}) = \underline{1}$ in $(X, \mathcal{U})$. Thus f is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-irresolute mapping. But f is not fuzzy $(\frac{1}{2}, \frac{1}{3})$-irresolute, because $f^{-1}(A_2) = A_2$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set in $(X, \mathcal{T})$.

Lemma 2.6. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then for any fuzzy (r, s)-semiopen set A in X, $\mathrm{scl}(A, r, s) = \mathrm{scl}_\theta(A, r, s)$.

Proof. Clearly $\mathrm{scl}(A, r, s) \subseteq \mathrm{scl}_\theta(A, r, s)$. Let $x_{(\alpha,\beta)} \notin \mathrm{scl}(A, r, s)$. Then there is a $B \in N_{\mathrm{S}}^{\mathrm{q}}(x_{(\alpha,\beta)})$ such that $B\tilde{\mathrm{q}}A$. This means $B \subseteq A^c$, and hence $\mathrm{scl}(B, r, s) \subseteq \mathrm{scl}(A^c, r, s) = A^c$. Thus $\mathrm{scl}(B, r, s)\tilde{\mathrm{q}}A$, and so $x_{(\alpha,\beta)} \notin \mathrm{scl}_\theta(A, r, s)$. Hence $\mathrm{scl}_\theta(A, r, s) \subseteq \mathrm{scl}(A, r, s)$. □

Corollary 2.7. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. If A is fuzzy (r, s)-semi-clopen in X, then A is a fuzzy (r, s)-semi-θ-closed set.

Proof. By Lemma 2.6, we have $A = \mathrm{scl}(A, r, s) = \mathrm{scl}_\theta(A, r, s)$. Thus A is fuzzy (r, s)-semi-θ-closed. □

Lemma 2.8. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. If B is fuzzy (r, s)-semiopen in X such that $A\tilde{\mathrm{q}}B$, then $\mathrm{scl}(A, r, s)\tilde{\mathrm{q}}B$.

Proof. Since $A\tilde{q}B$, we have $A \subseteq B^c$, and hence $\mathrm{scl}(A,r,s) \subseteq \mathrm{scl}(B^c,r,s) = B^c$. Thus $\mathrm{scl}(A,r,s)\tilde{q}B$. □

Lemma 2.9. Let $f : X \to Y$ be a mapping and A and B intuitionistic fuzzy sets in Y. If $A\tilde{q}B$, then $f^{-1}(A)\tilde{q}f^{-1}(B)$.

Proof. Suppose that $f^{-1}(A)qf^{-1}(B)$. Then there is an $x \in X$ such that $\mu_A(f(x)) > \gamma_B(f(x))$ or $\gamma_A(f(x)) < \mu_B(f(x))$. Thus AqB, which is a contradiction. □

Theorem 2.10. Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is fuzzy (r,s)-semi-irresolute.

(2) For each fuzzy (r,s)-semiopen set B in Y,
$$f^{-1}(B) \subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(B,r,s)),r,s).$$

(3) For each fuzzy (r,s)-semiopen set B in Y,
$$\mathrm{scl}(f^{-1}(B),r,s) \subseteq f^{-1}(\mathrm{scl}(B,r,s)).$$

Proof. (1) ⇒ (2) Let B be a fuzzy (r,s)-semiopen set in Y and $x_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(x_{(\alpha,\beta)}) \in B$. Since f is fuzzy (r,s)-semi-irresolute, there is a fuzzy (r,s)-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq \mathrm{scl}(B,r,s)$. Thus $x_{(\alpha,\beta)} \in A \subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(B,r,s)),r,s)$. Hence we have
$$\begin{aligned} f^{-1}(B) &= \bigcup\{x_{(\alpha,\beta)} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(B,r,s)),r,s). \end{aligned}$$

(2) ⇒ (3) Let B be a fuzzy (r,s)-semiopen set in Y. If $x_{(\alpha,\beta)} \notin f^{-1}(\mathrm{scl}(B,r,s))$, then $f(x_{(\alpha,\beta)}) \notin \mathrm{scl}(B,r,s)$. Hence there is a $C \in N_s^q(f(x_{(\alpha,\beta)}))$ such that $C\tilde{q}B$. By Lemma 2.8, we obtain $\mathrm{scl}(C,r,s)\tilde{q}B$. Since $C \in N_s^q(f(x_{(\alpha,\beta)}))$, we have $x_{(\alpha,\beta)}qf^{-1}(C)$. By (2), we obtain $f^{-1}(C) \subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(C,r,s)),r,s)$, and hence
$$x_{(\alpha,\beta)}q\mathrm{sint}(f^{-1}(\mathrm{scl}(C,r,s)),r,s).$$
Since $\mathrm{scl}(C,r,s)\tilde{q}B$, by Lemma 2.9, we have $f^{-1}(\mathrm{scl}(C,r,s))\tilde{q}f^{-1}(B)$, and so
$$\mathrm{sint}(f^{-1}(\mathrm{scl}(C,r,s)),r,s)\tilde{q}f^{-1}(B).$$
Thus $x_{(\alpha,\beta)} \notin \mathrm{scl}(f^{-1}(B),r,s)$. Therefore $\mathrm{scl}(f^{-1}(B),r,s) \subseteq f^{-1}(\mathrm{scl}(B,r,s))$.

(3) ⇒ (1) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-semiopen set in Y with $f(x_{(\alpha,\beta)}) \in B$. Then $x_{(\alpha,\beta)} \in f^{-1}(B)$, and hence $x_{(\alpha,\beta)}\tilde{q}f^{-1}(B^c) = f^{-1}(\mathrm{scl}(B^c,r,s))$. By (3), we obtain $\mathrm{scl}(f^{-1}(\mathrm{sint}(B^c,r,s)),r,s)$
$$\begin{aligned} &\subseteq f^{-1}(\mathrm{scl}(\mathrm{sint}(B^c,r,s),r,s)) \\ &\subseteq f^{-1}(\mathrm{scl}(B^c,r,s)). \end{aligned}$$
Hence $x_{(\alpha,\beta)}\tilde{q}\mathrm{scl}(f^{-1}(\mathrm{sint}(B^c,r,s)),r,s)$, and thus
$$x_{(\alpha,\beta)} \in \mathrm{scl}(f^{-1}(\mathrm{sint}(B^c,r,s)),r,s)^c.$$
Put
$$\begin{aligned} A &= \mathrm{scl}(f^{-1}(\mathrm{sint}(B^c,r,s)),r,s)^c \\ &= \mathrm{sint}(f^{-1}(\mathrm{scl}(B,r,s)),r,s). \end{aligned}$$
Then A is fuzzy (r,s)-semiopen in X, $x_{(\alpha,\beta)} \in A$, and
$$\begin{aligned} f(A) &= f(\mathrm{sint}(f^{-1}(\mathrm{scl}(B,r,s)),r,s)) \\ &\subseteq f(f^{-1}(\mathrm{scl}(B,r,s))) \\ &\subseteq \mathrm{scl}(B,r,s). \end{aligned}$$
Therefore f is fuzzy (r,s)-semi-irresolute. □

Lemma 2.11. Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. If A is fuzzy (r,s)-semiopen, then $\mathrm{scl}(A,r,s)$ is fuzzy (r,s)-semi-clopen.

Proof. It suffices to show that $\mathrm{scl}(A,r,s)$ is fuzzy (r,s)-semiopen. Since A is fuzzy (r,s)-semiopen, we have $\mathrm{cl}(\mathrm{int}(A,r,s),r,s) \supseteq A$. Hence
$$\mathrm{cl}(\mathrm{int}(\mathrm{scl}(A,r,s),r,s),r,s) \supseteq A.$$
Since $\mathrm{cl}(\mathrm{int}(\mathrm{scl}(A,r,s),r,s),r,s)$ is fuzzy (r,s)-semiclosed, we obtain
$$\mathrm{cl}(\mathrm{int}(\mathrm{scl}(A,r,s),r,s),r,s) \supseteq \mathrm{scl}(A,r,s).$$
Therefore $\mathrm{scl}(A,r,s)$ is fuzzy (r,s)-semiopen. □

Theorem 2.12. Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is fuzzy (r,s)-semi-irresolute.

(2) For each fuzzy (r,s)-semi-clopen set B in Y, $f^{-1}(B)$ is fuzzy (r,s)-semi-clopen in X.

(3) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r,s)-semiopen set B in Y containing $f(x_{(\alpha,\beta)})$, there is a fuzzy (r,s)-semiopen set A in X containing $x_{(\alpha,\beta)}$ such that $f(\mathrm{scl}(A,r,s)) \subseteq \mathrm{scl}(B,r,s)$.

Proof. (1) ⇒ (2) Let B be a fuzzy (r,s)-semi-clopen set in Y. Since f is fuzzy (r,s)-semi-irresolute, by Theorem 2.10,
$$f^{-1}(B) \subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(B,r,s)),r,s) = \mathrm{sint}(f^{-1}(B),r,s).$$

Hence $f^{-1}(B)$ is fuzzy (r,s)-semiopen in X. Similarly, by Theorem 2.10, we have

$$\mathrm{scl}(f^{-1}(B),r,s) \subseteq f^{-1}(\mathrm{scl}(B,r,s)) = f^{-1}(B).$$

Thus $f^{-1}(B)$ is fuzzy (r,s)-semiclosed.

(2) $\Rightarrow$ (1) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-semiopen set in Y with $f(x_{(\alpha,\beta)}) \in B$. Then $x_{(\alpha,\beta)} \in f^{-1}(B) \subseteq f^{-1}(\mathrm{scl}(B,r,s))$. Put $A = f^{-1}(\mathrm{scl}(B,r,s))$. Then by (2), A is fuzzy (r,s)-semiopen in X, $x_{(\alpha,\beta)} \in A$, and

$$f(A) = f(f^{-1}(\mathrm{scl}(B,r,s))) \subseteq \mathrm{scl}(B,r,s).$$

Hence f is fuzzy (r,s)-semi-irresolute.

(2) $\Rightarrow$ (3) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-semiopen set in Y with $f(x_{(\alpha,\beta)}) \in B$. Then $x_{(\alpha,\beta)} \in f^{-1}(B) \subseteq f^{-1}(\mathrm{scl}(B,r,s))$. Put $A = f^{-1}(\mathrm{scl}(B,r,s))$. Then by (2), A is fuzzy (r,s)-semi-clopen in X. Also, $x_{(\alpha,\beta)} \in A$ and

$$\begin{aligned} f(\mathrm{scl}(A,r,s)) = f(A) &= f(f^{-1}(\mathrm{scl}(B,r,s))) \\ &\subseteq \mathrm{scl}(B,r,s). \end{aligned}$$

(3) $\Rightarrow$ (1) Trivial. □

Definition 2.13. Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then A is said to be *fuzzy (r,s)-regular semiopen* if there is a fuzzy (r,s)-regular open set B in X such that $B \subseteq A \subseteq \mathrm{cl}(B,r,s)$.

It is clear that every fuzzy (r,s)-regular semiopen set is fuzzy (r,s)-semiopen. However, the following example shows that the converse need not be true.

Example 2.14. Let $X = \{x\}$ and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.5, 0.4), \quad A_2(x) = (0.6, 0.2).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then $\mathcal{T}$ is SoIFT on X. It is easy to show that A_2 is fuzzy $(\frac{1}{2},\frac{1}{3})$-semiopen in X. Note that

$$\mathrm{int}(\mathrm{cl}(A_1, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = \mathrm{int}(\underline{1}, \frac{1}{2}, \frac{1}{3}) = \underline{1} \neq A_1.$$

Hence the only fuzzy $(\frac{1}{2},\frac{1}{3})$-regular open sets in X are $\underline{0}$ and $\underline{1}$. Thus A_2 is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-regular semiopen set in X.

Definition 2.15. Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then the *fuzzy (r,s)-semi-θ-interior* is defined by $\mathrm{sint}_\theta(A,r,s) = \mathrm{scl}_\theta(A^c,r,s)^c$.

Lemma 2.16. Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then

(1) $\mathrm{sint}_\theta(A^c,r,s) = \mathrm{scl}_\theta(A,r,s)^c$.

(2) $\mathrm{scl}_\theta(A^c,r,s) = \mathrm{sint}_\theta(A,r,s)^c$.

Proof. Straightforward. □

Theorem 2.17. Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then A is a fuzzy (r,s)-semi-θ-open set if and only if $A = \mathrm{sint}_\theta(A,r,s)$.

Proof. Straightforward. □

Lemma 2.18. Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then the following statements are equivalent:

(1) A is fuzzy (r,s)-regular semiopen.

(2) A^c is fuzzy (r,s)-regular semiopen.

Proof. (1) $\Rightarrow$ (2) Let A be a fuzzy (r,s)-regular semiopen set in X. Then there is a fuzzy (r,s)-regular open set B in X such that $B \subseteq A \subseteq \mathrm{cl}(B,r,s)$. Since B is fuzzy (r,s)-open, $\mathrm{cl}(B,r,s)$ is fuzzy (r,s)-regular closed. Since

$$\mathrm{int}(B^c,r,s) \subseteq A^c \subseteq B^c = \mathrm{cl}(\mathrm{int}(B^c,r,s),r,s),$$

A^c is fuzzy (r,s)-regular semiopen.

(2) $\Rightarrow$ (1) Similar to the above. □

Lemma 2.19. Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then the following statements are equivalent:

(1) A is fuzzy (r,s)-regular semiopen.

(2) $A = \mathrm{scl}(\mathrm{sint}(A,r,s),r,s)$.

(3) A is fuzzy (r,s)-semi-clopen.

Proof. (1) $\Rightarrow$ (2) Since A is fuzzy (r,s)-regular semiopen, we have $A = \mathrm{sint}(A,r,s)$. By Lemma 2.18, A^c is fuzzy (r,s)-regular semiopen. Thus A is fuzzy (r,s)-semiclosed, and hence $A = \mathrm{scl}(\mathrm{sint}(A,r,s),r,s)$.

(2) $\Rightarrow$ (1) Let $A = \mathrm{scl}(\mathrm{sint}(A,r,s),r,s)$. Put $B = \mathrm{sint}(A,r,s)$. Then $B \subseteq A \subseteq \mathrm{scl}(B,r,s)$. Since B is fuzzy (r,s)-semiopen, we have

$$B \subseteq A \subseteq \mathrm{scl}(\mathrm{cl}(\mathrm{int}(B,r,s),r,s),r,s),$$

and hence

$$\text{sint}(\text{int}(\text{cl}(B^c, r, s), r, s), r, s) \subseteq A^c \subseteq B^c = \text{scl}(A^c, r, s).$$

Note that
$\text{int}(\text{cl}(B^c, r, s), r, s)$

$$\begin{aligned} &= \text{sint}(\text{int}(\text{cl}(B^c, r, s), r, s), r, s) \\ &\subseteq A^c \subseteq \text{scl}(A^c, r, s) \subseteq \text{cl}(A^c, r, s) \\ &\subseteq \text{cl}(\text{cl}(\text{int}(A^c, r, s), r, s), r, s) \\ &= \text{cl}(\text{int}(A^c, r, s), r, s) \\ &\subseteq \text{cl}(\text{int}(\text{cl}(\text{scl}(A^c, r, s), r, s), r, s), r, s) \\ &= \text{cl}(\text{int}(\text{cl}(B^c, r, s), r, s), r, s). \end{aligned}$$

Since $\text{int}(\text{cl}(B^c, r, s), r, s)$ is fuzzy (r, s)-regular open, A^c is fuzzy (r, s)-regular semiopen. Hence by Lemma 2.18, A is fuzzy (r, s)-regular semiopen.

(1) $\Rightarrow$ (3) Let A be a fuzzy (r, s)-regular semiopen set. Then A is fuzzy (r, s)-semiopen. Also, A^c is fuzzy (r, s)-regular semiopen, and hence A^c is fuzzy (r, s)-semiopen. Thus A is fuzzy (r, s)-semi-clopen.

(3) $\Rightarrow$ (2) Since A is fuzzy (r, s)-semi-clopen, we obtain

$$A = \text{scl}(\text{sint}(A, r, s), r, s).$$

□

Lemma 2.20. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is fuzzy (r, s)-regular semiopen if and only if A is a fuzzy (r, s)-semi-θ-clopen set.

Proof. ($\Rightarrow$) Let A be fuzzy (r, s)-regular semiopen. Then A is fuzzy (r, s)-semiopen, and by Lemma 2.6, we have

$$A = \text{scl}(A, r, s) = \text{scl}_\theta(A, r, s).$$

Hence A is fuzzy (r, s)-semi-θ-closed. Since A^c is fuzzy (r, s)-semiopen, we obtain

$$A^c = \text{scl}(A^c, r, s) = \text{scl}_\theta(A^c, r, s).$$

Thus A^c is fuzzy (r, s)-semi-θ-closed. Hence A is a fuzzy (r, s)-semi-θ-open set.

($\Leftarrow$) Let A be a fuzzy (r, s)-semi-θ-clopen set. Then A is fuzzy (r, s)-semi-clopen. Thus by Lemma 2.19, A is a fuzzy (r, s)-regular semiopen set. □

Lemma 2.21. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. If A is a fuzzy (r, s)-semiopen set in X, then $\text{scl}(A, r, s)$ is fuzzy (r, s)-regular semiopen in X.

Proof. By Lemma 2.11, $\text{scl}(A, r, s)$ is fuzzy (r, s)-semi-clopen. Hence by Lemma 2.19, $\text{scl}(A, r, s)$ is fuzzy (r, s)-regular semiopen in X. □

Theorem 2.22. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is fuzzy (r, s)-semi-irresolute.

(2) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-regular semiopen set B in Y containing $f(x_{(\alpha,\beta)})$, there is a fuzzy (r, s)-regular semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

(3) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-regular semiopen set B in Y containing $f(x_{(\alpha,\beta)})$, there is a fuzzy (r, s)-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(\text{scl}(A, r, s)) \subseteq B$.

Proof. (1) $\Rightarrow$ (2) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r, s)-regular semiopen set containing $f(x_{(\alpha,\beta)})$. Then B is a fuzzy (r, s)-semiopen set in Y containing $f(x_{(\alpha,\beta)})$. By Theorem 2.12, there is a fuzzy (r, s)-semiopen set A such that $x_{(\alpha,\beta)} \in A$ and $f(\text{scl}(A, r, s)) \subseteq \text{scl}(B, r, s) = B$. Let $C = \text{scl}(A, r, s)$. Then C is fuzzy (r, s)-regular semiopen such that $x_{(\alpha,\beta)} \in C$, and $f(C) \subseteq B$.

(2) $\Rightarrow$ (3) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r, s)-regular semiopen set in Y containing $f(x_{(\alpha,\beta)})$. By (2), there is a fuzzy (r, s)-regular semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. Since A is fuzzy (r, s)-semiclosed, we have $f(A) = f(\text{scl}(A, r, s)) \subseteq B$.

(3) $\Rightarrow$ (1) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r, s)-semiopen set in Y containing $f(x_{(\alpha,\beta)})$. Then $\text{scl}(B, r, s)$ is a fuzzy (r, s)-regular semiopen set containing $f(x_{(\alpha,\beta)})$. By (3), there is a fuzzy (r, s)-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and

$$f(\text{scl}(A, r, s)) \subseteq \text{scl}(B, r, s).$$

Thus f is fuzzy (r, s)-semi-irresolute. □

Theorem 2.23. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is fuzzy (r, s)-semi-irresolute.

(2) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-semi-θ-clopen set B containing $f(x_{(\alpha,\beta)})$, there is a fuzzy (r, s)-semi-θ-clopen set A such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

(3) For each fuzzy (r, s)-regular semiopen set B in Y, $f^{-1}(B)$ is fuzzy (r, s)-regular semiopen in X.

(4) For each fuzzy (r,s)-semiopen set B in Y,

$$f^{-1}(B) \subseteq \text{sint}_\theta(f^{-1}(\text{scl}_\theta(B,r,s)),r,s).$$

(5) For each fuzzy (r,s)-semiclosed set B in Y,

$$\text{scl}_\theta(f^{-1}(\text{sint}_\theta(B,r,s)),r,s) \subseteq f^{-1}(B).$$

(6) For each fuzzy (r,s)-semiopen set B in Y,

$$\text{scl}_\theta(f^{-1}(B),r,s) \subseteq f^{-1}(\text{scl}_\theta(B,r,s)).$$

Proof. (1) $\Leftrightarrow$ (2) It follows from Lemma 2.20 and Theorem 2.22.

(1) $\Rightarrow$ (3) Let B be a fuzzy (r,s)-regular semiopen set in Y. By Theorem 2.10,

$$\text{scl}(f^{-1}(B),r,s) \subseteq f^{-1}(\text{scl}(B,r,s)) = f^{-1}(B),$$

and hence $f^{-1}(B)$ is fuzzy (r,s)-semiclosed. Similarly, $f^{-1}(B)$ is fuzzy (r,s)-semiopen. Therefore $f^{-1}(B)$ is fuzzy (r,s)-regular semiopen in X.

(3) $\Rightarrow$ (4) Let B be a fuzzy (r,s)-semiopen set in Y. Then $\text{scl}_\theta(B,r,s) = \text{scl}(B,r,s)$ is fuzzy (r,s)-regular semiopen. By (3), $f^{-1}(\text{scl}_\theta(B,r,s))$ is fuzzy (r,s)-regular semiopen. By Lemma 2.20, $f^{-1}(\text{scl}_\theta(B,r,s))$ is fuzzy (r,s)-semi-θ-clopen. Since $f^{-1}(B) \subseteq f^{-1}(\text{scl}_\theta(B,r,s))$, we have

$$f^{-1}(B) \subseteq \text{sint}_\theta(f^{-1}(\text{scl}_\theta(B,r,s)),r,s).$$

(4) $\Rightarrow$ (5) Let B be a fuzzy (r,s)-semiclosed set in Y. Then B^c is fuzzy (r,s)-semiopen. By (4), we have

$$f^{-1}(B^c) \subseteq \text{sint}_\theta(f^{-1}(\text{scl}_\theta(B^c,r,s)),r,s).$$

Thus by Lemma 2.16, $\text{scl}_\theta(f^{-1}(\text{sint}_\theta(B,r,s)),r,s) \subseteq f^{-1}(B)$.

(5) $\Rightarrow$ (6) Let B be a fuzzy (r,s)-semiopen set in Y. Then $\text{scl}(B,r,s)$ is fuzzy (r,s)-regular semiopen, and hence $\text{scl}(B,r,s)$ is fuzzy (r,s)-semi-θ-clopen. By (5), we obtain

$$\begin{aligned}
&\text{scl}_\theta(f^{-1}(B),r,s)\\
&\quad\subseteq \text{scl}_\theta(f^{-1}(\text{scl}(B,r,s)),r,s)\\
&\quad= \text{scl}_\theta(f^{-1}(\text{sint}_\theta(\text{scl}(B,r,s),r,s)),r,s)\\
&\quad\subseteq f^{-1}(\text{scl}(B,r,s)) = f^{-1}(\text{scl}_\theta(B,r,s)).
\end{aligned}$$

(6) $\Rightarrow$ (3) Let B be a fuzzy (r,s)-regular semiopen set in Y. Then B is fuzzy (r,s)-semi-clopen. By (6), we have

$$\begin{aligned}
\text{scl}_\theta(f^{-1}(B),r,s) &\subseteq f^{-1}(\text{scl}_\theta(B,r,s))\\
&= f^{-1}(\text{scl}(B,r,s)) = f^{-1}(B),
\end{aligned}$$

and hence $f^{-1}(B) = \text{scl}_\theta(f^{-1}(B),r,s)$. Thus $f^{-1}(B)$ is a fuzzy (r,s)-semi-θ-closed set in X. Since B is fuzzy (r,s)-regular semiopen, B^c is a fuzzy (r,s)-regular semiopen set. Hence $f^{-1}(B^c)$ is fuzzy (r,s)-semi-θ-closed in X. Thus $f^{-1}(B)$ is a fuzzy (r,s)-semi-θ-open set in X. Therefore $f^{-1}(B)$ is fuzzy (r,s)-regular semiopen.

(3) $\Rightarrow$ (1) It follows from Lemma 2.19 and Theorem 2.12. □

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