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Assessment of Mathematical Creativity in Mathematical Modeling[1]

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In mathematical modeling tasks, where students are exposed to model-eliciting for real and open problems, students are supposed to formulate and use a variety of mathematical skills and tools at hand to achieve feasible and meaningful solutions using appropriate problem solving strategies.
In contrast to problem solving activities in conventional math classes, math modeling tasks call for varieties of mathematical ability including mathematical creativity.
Mathematical creativity encompasses complex and compound traits. Many researchers suggest the exhaustive list of criterions of mathematical creativity. With regard to the research considering the possibility of enhancing creativity via math modeling instruction, a quantitative scheme to scale and calibrate the creativity was investigated and the assessment of math modeling activity was suggested for practical purposes.

Keywords: mathematical models, problem solving, assessment of mathematical models, mathematical creativity, collective creativity, multiple correspondence analysis
MESC Classification: D60
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1. INTRODUCTION

Mathematical modeling is as old as mathematics itself. Since Archimedes of Syracuse declared, "Give me a place to stand and I will move the earth", mathematical modeling has been dealing with an integral real world application. More generally, the relationship between mathematics and the extra-mathematical world (sometimes also called the "real world") is preferably called, "in the wild".

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In upper secondary mathematics education, the pedagogical advantages of mathematical modeling experience are abundant and clear. According to the experience of mathematics teachers, students are found to be more motivated by real world problems than abstract word problems in math classes. In the process of mathematical modeling, we can ask and answer important scientific, economic, social, and political issues and analyze the claims that policy makers present to the public.

Mathematical modeling skills and attitudes also help students become more questioning and inquisitive. They become active learners of mathematics and related studies. By the time students complete a couple of mathematical modeling experiences, they are able to:

(1) Understand, develop designs, work with abstractions, and create representations.
(2) Predict outcomes, optimize over constraints and make proper decisions.

There exists an additional demand upon the mathematical modeling curriculum. It is to re-establish the value of liberal arts learning strategies. Each discipline of mathematical modeling represents a different approach to looking at and understanding our world from new perspective. As a set of equations or another mathematical structure which are mental images of reality, mathematical modeling calls for a certain level of creativity from students.

There have been rare studies for mathematical creativity in high school classrooms. It is only during last ten years that professionals of mathematics education have showed a renewed interest in mathematical creativity in problem solving or mathematical modeling. Mathematical modeling activities for students and their deliberate implementation would make a reliable framework for the assessment of creativity. With regard to the research considering how to enhance creativity via mathematical modeling instruction, a quantitative scheme to scale and calibrate the creativity is investigated and the assessment of math modeling activity was suggested for practical purposes. In particular, three questions will be explored:

(1) What is the criterion of mathematical modeling assessment in high school level?
(2) How can we identify and calibrate the creativity reflected in mathematical modeling?
(3) How can we assess collective creativity in the group based mathematical modeling?

2. MATHEMATICAL MODELING VS PROBLEM SOLVING

A model is a simplified representation of certain aspects of the real world, capturing the essence of its phenomenon. In this respect, mathematical modeling is the process of creating a model using mathematical relations and concepts such as variables, operators, functions, equations, vectors, etc.

According to my experience, high schools students find mathematical modeling difficult even if they are well equipped with the necessary skills and concepts needed to tackle the problems at high school level. They find open modeling problems extremely difficult because of the complexity of the real problems which never identify which components of variables are most influential among other different negligible variables. Good mathematical models are produced when students become comfortable with uncertainty while striving for clarity in their descriptions and analyses. They learn to accept that creativity and effective communication are essential part of active learning and discovery.

Mathematics education has been providing students with extensive ‘exercises’, that encouraged students preparing for its application to the uncertain and challenging real problems. An important and interesting issue regarding math modeling in math education is the claim that there is a borderline between those who comprehend mathematics as a fragmented body of knowledge and those who have a cohesive view of it. Cohesive viewers of mathematics are found to show more competence in solving problems and gaining knowledge about the real world (Crawford *et al.*, 1994).

Another negative implication for the ‘pan cooked exemplary exercises’ was studied extensively claiming the difference between exercises and problems.

In the process of solving exercises, students only learn imitative reasoning and copy or follow a model or an example without any attempts of originality. Learning difficulties are partly related to the reduction of complexity that appears as a procedural focus on facts (Jesper Bosen, 2006).

In this case, the strategy students learn would be the memory recalling only, which is an algorithmic procedure lacking in relational reasoning or heuristics.

Whenever you solve a real world problem, you have to create a model of that problem. It is crucial to make the distinction between the model and the problem. Every model throws something away - otherwise it would be too complex to deal with. We always work with simplified form of things. Every answer we obtain is a solution only to the model under certain conditions. Zbigniew & David (2002) provide a good example. “Suppose you throw a ball into the air. What is its trajectory?” Most high school students would answer, “Parabola.” That answer is correct if the earth is flat, otherwise the correct answer is ellipse. However, it is another simplification without the consideration of air resistance, friction and even the shape of the stone. Probability problems have different answers according to the different types of sample spaces, too.

The term modeling refers to applying a mathematical model in a problem solving situation. As a less ‘automatic’ act, modeling can be defined as the process of organizing and describing a situation or phenomenon by using a mathematical model (or models) or 'mathematizing' the situation by perceiving it through mathematical lenses (Greer,1997).

Problem solving itself has provided a lot of research topics in mathematics education since Gyorgy Polya's book "How to Solve It". It stands as one of the most important contributions to problem-solving literature in the twentieth century. It provides most of the techniques and procedures including how to make analogies, use auxiliary devises, work backwards from the goal to the given, and so on. Despite of the richness of literature for heuristic problem solving strategies in mathematics education, it often fails to provide the proper guidance on when and when not to apply it. Most often, students are not well prepared and equipped with proper knowledge to make decisions about whether a particular method is appropriate or not. This is undoubtedly because of the calculator and computer revolution. Superficial understanding and application of problem strategies with deterministic algorithms prevails.

In solving complex problems, it is natural for problem solvers to guess at the solution. Some guess seem more intuitive than others. Sometimes you have to try really hard to reject your intuition. Even seemingly straightforward problems demand serious scrutiny. Try the following problems:

(1) In ancient Greece, Zeus commissioned a black smith to make an iron ring that fits around the Earth exactly. However, he made a ring that was just one meter longer in circumference. If Zeus places the ring around the Earth, what kind of animal would be able to squeeze under the gap between the ring and the Earth? An ant? A mouse or cat?

(2) You drive a car from your home to your office at a constant speed of 60 km/h and return at the speed of 80km/h. What is the overall average speed?

(3) Which event is more likely: to throw at least one 6 when six dice are rolled, or to throw at least two 6's when twelve dices are rolled?

Usually, our intuition fails until we set up mathematical models, Sometimes the counterintuitive solution deviates dramatically. The problems above would not reveal a direct answer until they are modeled mathematically and solved with proper interpretation.

3. ASSESSMENT OF MATHEMATICAL MODELING

Niss (2006) pointed out that assessment of mathematical modeling could be misleading and problematic, since application and modeling qualifications are difficult to assess only by traditional evaluation tools. He further clarified that there is a need to move away from conventional mode of traditional assessment. The assessment of mathematical modeling should be based upon the intricately various components in a complex structure. This implies that assessment takes a considerable time and cannot be standardized easily. It does not imply that assessment cannot be constructed on a sound foundation of reflection and reasoning. As we assess mathematical models, we have to remember that the modeling activity is not a routine task like producing a one-word or one-number answer to a simple question. Instead, students are expected to interpret complex phenomena and formulate a mathematical description, procedure, or methods. For students working with mathematical modeling, it is like a horse with wings. How can we assess their design of the wings? On the basis of six components of the mathematical modeling activities the following should be reflected for evaluation and assessment:

(1) Model construction.
(2) Self assessment.
(3) Reality.
(4) Model documentation.
(5) Construct share ability and re-usability.
(6) Effective prototype.

These will ensure the solution to include a description, justification, usefulness and feasibility of models. The documentation of a model and an effective prototype will assess the students' ability to externalize their ideas. In the process of assessment of math modeling, the modeler's self assessment is essential, as it helps students to understand and evaluate the model itself. Clearly, students who cannot recognize high-quality work produced by their peers (or by themselves) have little claim to soundly-based knowledge (Sriraman & Lesh, 2006). Through this interplay, the students can learn to identify the criteria for a qualitatively good performance, which would be scrutinized by evaluators. A quite systematic approach of assessment is worth being proposed with the research based on the mathematical modeling perspective (MMP). Sriraman & Lesh (2006) claims:

> MMP research suggests that the models that students develop involve a series of Iterative Design Cycles similar to design science professionals. In order to develop artifacts and designs that are sufficiently powerful, sharable, and reusable, it usually is necessary for designers to go through a series of design cycles in which trial products are iteratively tested and revised for specified purposes. Then, the development cycles automatically generate auditable trails of documentation which reveal significant information about the products that evolve.

On the basis of the test theory, Wu &Adams (2006) examined students' responses to mathematics problem-solving tasks and applied a general multidimensional IRT model at the response category level. In doing so, cognitive processes were identified and modeled through item response modeling to extract more information than would be provided using conventional practices in scoring items. According to them, problem-solving framework was theoretically grounded and the framework was then used as the basis for item development drawing upon research in mathematics education and cognitive psychology. It was also demonstrated that multidimensional IRT models were powerful tools for extracting information from a limited number of item responses. A problem-solving profile for each student could be constructed from the results of IRT scaling. Their theoretical profile of the students will make a good tool for the evaluation of mathematical modeling:

> The model of within-item dimensionality enables us to build a profile for each student, identifying areas of strength and weakness. For example, some students have a good grasp of mathematical concepts but fall down in extracting all information from the question, while others need to improve their mathematical knowledge and skills. Item Response Theory enables us to provide such profiles for students from a relatively short test, as we extract as much information as possible from all item responses, and not just correct or incorrect answers.

4. CREATIVITY, COMMUNICATION AND SEARCH OF INFORMATION

Creativity in mathematics is often confined to eminent or professional mathematicians, which is not suitable for this paper. According to Haylock (1997), there are at least two major ways in which the term is used:

(1) Thinking that is divergent and overcomes fixation.
(2) Thinking behind the product that is perceived as grandiose by a large group of people.

Since the creativity was put into agenda in mathematics education, lot of feasible literature for the definition of creativity and mathematical creativity was introduced and discussed. As a whole, creativity does not seem to have an authoritative and clear definition to explain it properly.

However, there is a unanimous premise for creativity researchers, whether they are qualitative protagonist or quantitative-factor analysts. There exists a problem of defining creativity: It is regarded as a compound of hypothetical ingredients and it should be studied holistically. Even if it could be only a Holy Grail, it also should be possible for educators of mathematics to imagine the glimpse as the model of ‘creativeness’ to nurture the prospective students. The identification of creative potential in mathematics education worthily demands a challenge.

More practically, Sriraman (2006) defined the mathematical creativity at the school levels as:

(1) The process that results in unusual (novel) and/or insightful solution(s) to a given

problem or analogous problem, and/or

(2) The formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle (this bears resemblance to Kuhn's ideas).

In a mathematical modeling activity, the problem posed to students is solved most efficiently in group settings. Especially, the challenging problems can't be handled by an individual because such problems call for diverse capacities like searching for a feasible solution out of curriculum, web searching, determining the level of available mathematical conception, elaborate scaffolding, writing reports, etc. In a collaborate activity, the communication among group members contributes a lot for productive solution in mathematical modeling. According to Martin Towers & Pirie (2006), mathematical understanding is most effectively emerged from coactions. Brainstorming among group members is another important ingredient that makes the mathematical modeling activity more fruitful. For the collaborating mathematical modeling activity, the optimal number of members is four as is suggested from HiMCM (High school Competition in Mathematical Modeling). The efficient and effective cooperation for model developing, problem solving, reporting, coordinating and web searching as a whole converges into an elegant model, with collective creativity. It is also meaningful to include the ability to search and process the information available on web sites and archives of references for creativity. The capacity for systematic information searching consists of creativity components like flexibility and fluency. The theoretical approach of multiple solution tasks (MTS) setting is well studied by Leikin (2009).

5. METHODOLOGY FOR COLLECTIVE CREATIVITY CALIBRATION

Many of the researches have been conducted to identify mathematical creativity which focused on the device and development of scale instruments. And most instruments were designed to explore the relationship between mathematical creativity mathematical achievement, also, attitude towards mathematics, self-perception of creative ability, gender, and teacher perception of mathematical talent and creative ability. Exhaustive analyses and results abound. Scoping all of these instruments is time consuming and subject to fallacious interpretation, limiting their utility.

This study sought a simpler means to identify the indicators of creative potential in mathematics and mathematical modeling based Leikin & Lev's (2007) multiple solution tasks as a "magnifying glass" and Larson's (2000) categorical approach of problem solving strategy. The conjecture I would like suggest is as follows:

(1) Mathematical creativity is reflected in the process of mathematical modeling ac-

tivities.

(2) Well defined problem solving strategies constitute the relevant components of creativity.
(3) The mathematical creativity for groups can be assessed collectively as an additive form.
(4) The collective creativity is fully ignited by communication skills among group members.

For the overall assessment of mathematical creativity, the three stages scheme by Ervynck (1991) was applied where:

- Stage 0 is the Preliminary technical stage, which consists of "some kind of technical or practical application of mathematical rules and procedures without the user having any awareness of theoretical foundation".
- Stage 1 is the algorithmic activity, which consists primarily of "performing mathematical techniques such as explicitly applying the algorithm repeatedly.
- Stage 2 is referred to as creative (conceptual, constructive) activity. This is the stage in which true mathematical creativity occurs and consists of non algorithmic decision making. "The decisions that have to taken many of a widely divergent nature and always involve a choice"

The stage 0 and stage 1 were evaluated on the basis of a conventional math test: The HYFL first year student's midterm achievement test and performance assessment (home works, quizzes, etc.) covering intermediate algebra and geometry were used.

Parallel to the holistic approach suggested by Niss and Jensen (2006) to modeling leading to overall estimates of modeling competence, Stage 2 was evaluated by essentially a geometric model in which there are three dimensions of creativity in mathematical modeling assessment: Novelty, Flexibility and Fluency. And, as the group of subordinate activities to be observed qualitatively, 14 strategies of problem solving were chosen as follows:

(1) Seeking patterns
(2) Using invariants
(3) Drawing pictures
(4) Formulating an equivalence problem
(5) Modifying problems
(6) Choosing effective notations
(7) Exploiting symmetry
(8) Dividing into cases
(9) Working backwards

(10) Arguing by contradiction
(11) Pursue parities
(12) Considering extreme cases
(13) Generalization and
(14) Exhaustive searches.

The strategies again are categorized into three components of mathematical creativity.

Novelty: Using invariants, Formulating an equivalence problems, Modifying problems, Exploit symmetry and Generalization.

Flexibility: Drawing pictures, Choosing effective notations, Working backwards, Pursue parities, Considering extreme cases.

Fluency: Seeking patterns, Dividing into cases, Arguing by contradiction and Exhaustive searches.

Also, each stage of mathematical modeling and strategy is assessed with four categorically ordinal scales according to the performance level of the examinee: Poor, Fair, Good, and Excellent.

In order to formulate the criteria, finished models with written interpretation of the model by participants were analyzed in detail on the basis of logical steps taken towards the accomplishment of the task. The specific trace of actions was observed to detect the relevant knowledge. A problem solving task should always have a context given by the topic of the modeling. Especially at group level, cluster of knowledge that typically is applied to problems related to that model is identified and scaled for each component that comprises creativity: Novelty, Fluency and Flexibility.

The assessment results were put into the MCA (Multiple Correspondence Analysis). Twenty-five Groups of four members from three classes from HYFL were assessed on the basis of the group activity which is composed of four sections (Algebra, Geometry, Dynamic and Stochastic) and a group project. Each group is supposed to solve an open problem by way of mathematical modeling. Out of these twenty-five observations, the proximities of conventional test scores and each defined component of creativity was calibrated and located on low dimensional spaces via MINITAB for the Multiple Correspondence Analysis.

6. DATA & MULTIPLE CORRESPONDENCE ANALYSIS

The purpose of present exploratory statistical analysis for the assessment of mathematical modeling is twofold: To calibrate and locate the traits of creativity reflected in mathematical modeling and to investigate the relationship between creativity components of mathematical modeling and conventional achievements of mathematics.

The midterm mathematics test results of HYFL participants were collected scaled by four ordinals like creativity component scales: Poor, Fair, Good, and Excellent. The data was constructed as the form (100×4) of contingency table according to the levels of 100 individual conventional achievements vs. conventional achievement and novelty, fluency and flexibility.

The configuration of conventional achievements of mathematics and creativity components is given in result (1) with [Figure1] and the configuration of conventional achievements of mathematics and creativity as an additive form of components is given in result (2) with [Figure2].

The Novelty, Fluency, Flexibility and Conventional test scores are ranked and categorized as

N0,N1,N2,N3, FL0, FL1, FL2, FL3,FX0,FX1,FX2,F3, C0,C1,C2,C3 and Achieve0, Achieve1, Achieve2, Achieve3 with additive form C=N+FL+FX

(1) Conventional Math Achievement, Novelty, Flexibility and Fluency

Analysis of indicator matrix

Axis	Inertia	Ratio	Cumulative	Histogram
1	0.5677	0.1892	0.1892	******************************
2	0.4466	0.1489	0.3381	***********************
3	0.4116	0.1372	0.4753	*********************
4	0.3699	0.1233	0.5986	*******************
5	0.2838	0.0946	0.6932	**************
6	0.2331	0.0777	0.7709	************
7	0.1958	0.0653	0.8362	**********
8	0.1535	0.0512	0.8874	********
9	0.1311	0.0437	0.9311	******
10	0.1075	0.0358	0.9669	*****
11	0.0540	0.0180	0.9849	**
12	0.0452	0.0151	1.0000	
Total	3.0000			

The proximities between equal levels of different components indicate that the same levels appear together in the observations. Novelty and flexibility tend to be positioned along the second principal axis. The highest level of novelty, fluency and flexibility sticks together, which means the mutual dependency to each other components and conventional mathematics achievement is strong. However, considering the low cumulative proportion of inertia, which is usual in MCA, the dependence level should be carefully interpreted. In contrast, the low levels of creativity components tend to be irregularly positioned which indicates that the low achievement of conventional mathematics makes identification of creativity relatively difficult. On the contrary, all the excellent levels of components are extremely positioned cohesively with less “pulling effect of second principal axis”.

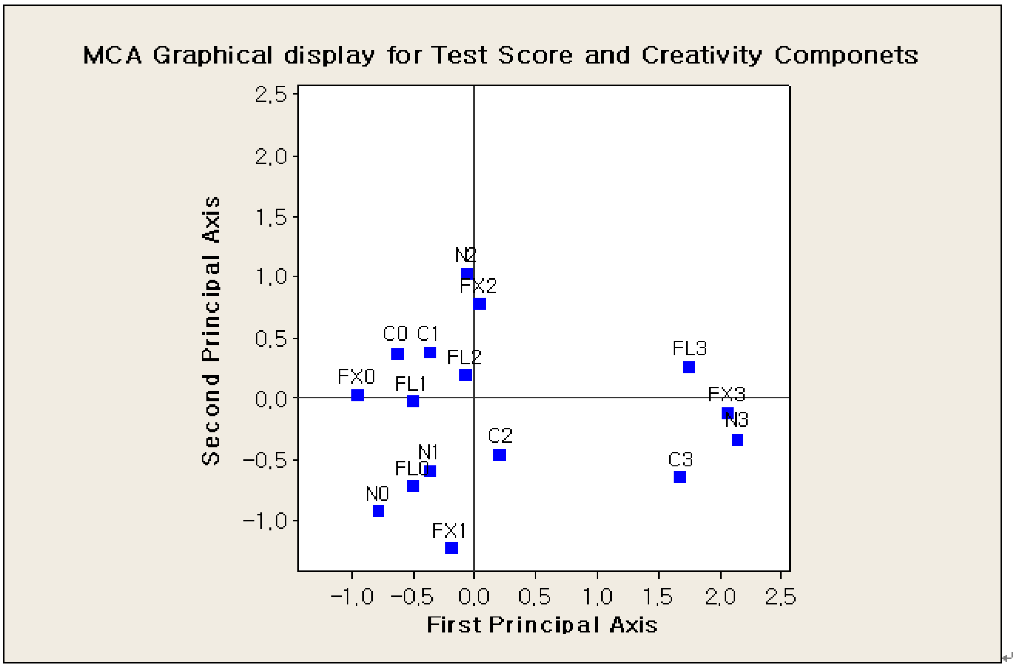

Figure 1. Graphical display for Test Score and Creativity Components

(2) Conventional Math Achievement and Additive Creativity

Analysis of indicator matrix

Axis	Inertia	Ratio	Cumulative	Histogram
1	0.8720	0.2907	0.2907	******************************
2	0.7624	0.2541	0.5448	**************************
3	0.5753	0.1918	0.7366	*******************
4	0.4247	0.1416	0.8782	**************
5	0.2376	0.0792	0.9573	********
6	0.1280	0.0427	1.0000	****
Total	3.0000			

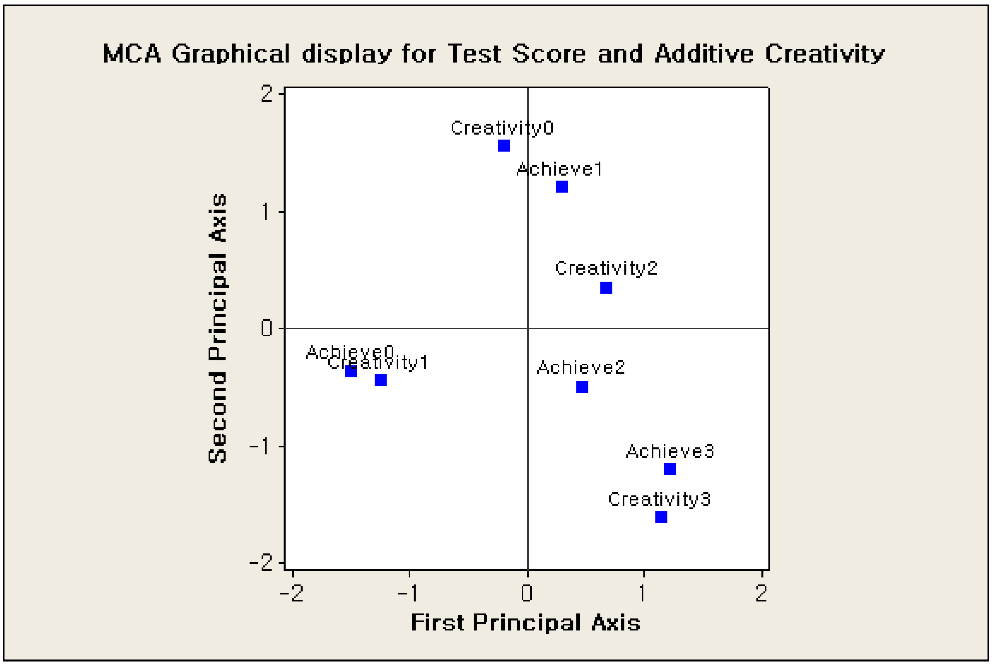

Figure 2. MCA Graphical display for Test Score and Additive Creativity

Also, the proximate between equal levels of “summed up creativity” and the conventional test score probably means that the groups observed with these different ingredients are themselves “similar” which could call for more critical definition of creativity itself. As is the case of result (1), low levels of creativity and test score show discrepancies between two identical levels. In this case, conventional achievements were approximately scaled through horizontal components and total creativity through vertical components.

As is indicated in (2), two major components constitute a significant predictor for the conventional achievement and the creativity in mathematical modeling explaining more than 50% of the inertia in scores. It could be consistently interpreted as the case of Eric Mann’s creativity research in 2009, which deduced 23% contribution of mathematical achievement to mathematical creativity.

CONCLUSION

The identification and calibration of mathematical creativity in mathematical modeling comprises an important aspect of mathematical modeling assessment. Apart from problem solving, the ability of eliciting mathematical model out of real world problems is in greater demand in this age of information & technological innovation. Students are exposed more and more to smart calculators and computers. Therefore, the mathematical modeling with its assessment scheme should be well established in mathematics curricula. The feasible and practical assessment of collective creativity also demands its role as a new way of understanding students as individuals as well as members of groups.

The assessment for mathematical modeling and problem solving should be dealt from different perspective. If a real-world problem is well presented and feasibly restated, high school students show creative solutions and demonstrate sheer potentialities. For the teachers committed to math modeling, the intriguing and challenging problems galore will prompt them also to develop well structured assessments of creativity and excellence standards with reliable calibration of mathematical creativity. The exploratory attempt of present study to scale and calibrate the creativity for mathematical modeling seems to have its intrinsic limitation with a low level of inertia from MCA, but more elaborate, synthetic and confirmative approach would improve the scaling instruments with full understanding of mathematical creativity traits in mathematical modeling.

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APPENDIX

Preliminary Questions : Show your works and justify your answers.

【Algebraic Section】

Find pairs of rational numbers as many as possible of which the sum of squares are one's.

【Geometric Section】

A line segment is said to be cut in extreme and mean ratio when as the whole is to the longer segment so is the longer segment to the shorter. Evaluate the mean ratio and give examples.

【Stochastic Section】

If a stick is broken at random, what are the averages length of shorter piece and longer piece?

【Dynamic Section】

Hanoi Tower is the stack of n disks arranged from largest on the bottom to smallest on top placed on a rod, together with two empty rods. What is the minimum number of moves required to move the tower from one rod to another, where moves are allowed only if they place smaller disks on top of larger disks.

MATHEMATICAL MODELING PROBLEM: CHOOSE A OR B

Problem A

The design of airline terminals varies widely. The sketches below show airline terminals from several cities. The designs are quite dissimilar. Some involve circular arcs; others are rectangular; some are quite irregular. Which is optimal for operations?

Develop a mathematical model for airport design and operation.

Use your model to argue for the optimality of your specified design with summary 100 words.

 Boston-Logan International	 Munich International	 Charlotte/Douglas International
 Ronald Reagan Washington International		 Pittsburgh International

Problem B

Problem: Tsunami ("Wipe Out!")

Recent events have reminded us about the devastating effects of distant or underwater earthquakes. Build a model that compares the devastation of various-sized earthquakes and their resulting Tsunamis on the following cities: San Francisco, CA; Hilo, HI; New Orleans, LA; Charleston, SC; New York, NY; Boston, MA; and any city of your choice. Prepare an article for the local newspaper that explains what you discovered in your model about one of these cities.