# Mortality Forecasting for the Republic of Korea: the Coherent Lee-Carter Method 

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This paper examines the performance of the coherent Lee-Carter method for the mortality forecasting for the Republic of Korea combined with Japan and the Taiwan Province of China as a group by comparing it with the separately applied Lee-Carter method. It narrowed the gap of life expectancies between three countries from 6.8 years to 3.0 years in 2050, with higher life expectancy forecasts for the Taiwan Province of China and lower ones for Japan than with the separate forecast. This method did not affect the sex-combined life expectancy forecast for the Republic of Korea, but it accelerated the mortality decline for ages 65 and over and decelerated it for the younger age groups, diminishing sex differentials of life expectancy at a slower speed. It suggests that the integration of regional mortality information into mortality forecasting of one country gives several advantages in terms of short run fit within each country as well as long run convergence between countries, a modification of the age pattern of mortality decline, and a consistent application of the forecasting of subgroups within a country.

Key Words: Mortality forecasting, Lee-Carter method, Coherent Lee-Carter method, Common factor, Specific factor

## I. Introduction

The mortality rate in the Republic of Korea has decreased dramatically during the past few decades. Life expectancy at birth increased from 61.9 years in 1970 to 80.1 years in 2008. This rise in life expectancy at birth averages an increase of 0.5 years per year, which is unprecedentedly fast, compared to the world's increase of 0.3 years per year over the same period. As mortality advances differently by regions, the gap in life

[^0]expectancy at birth between more developed regions and less developed regions has decreased from 16 years in the period of 1970~1975 to 11 years in the period of 2005~2010(KOSTAT, 2010; United Nations, 2009). In particular, this reduction in the gap of mortality has been observed among countries that are in close proximity to each other within a close geographic region. In East Asia, Japan and the Taiwan Province of China had much higher life expectancies than the Republic of Korea in 1970. The range of life expectancy at birth among the three countries narrowed to four years in 2008 from ten years in 1970(Human Mortality Database; KOSTAT, 2010).

These changes are influenced by other countries with similar environments, as the world is becoming more closely connected by means of communication, biomedical research and medical practices(Wilson, 2001; Li and Lee, 2005). To reflect this close connection between members in group mortality, Li and Lee(2005) suggest the coherent mortality forecasting method for the mortality of a group as an extension of the Lee-Carter method. In the utility of this method, they foresee two advantages in that it could solve the divergence of mortality of a group, while improving mortality forecasting for a single population by exploiting additional information contained in the experience of similar populations.

This paper will apply the coherent Lee-Carter method(henceforth the coherent LC method) to forecast the mortality for the Republic of Korea. It will combine the mortalities of Japan and the Taiwan Province of China, which are geographically close to each other and possess similarity in diet, culture and a relatively small gap in economic classes in addition to having available historical data. This method could improve mortality forecasting for the Republic of Korea with a rather short historical mortality data series by supplementing data from other countries. The result by using the coherent LC method will be compared to the result of the separately applied Lee-Carter method(henceforth the separate LC method) to each country. This paper could be an early study to evaluate this method and apply it for a country, while the ordinary Lee-Carter method has been assessed widely and is among the best applications for mortality forecasting(Wilmoth, 1996; Tuljapurkar, Le and Boe, 2000; Lee and Miller, 2001; Renshaw and Haberman,

2003 a, b; Currie, Durban and Eilers, 2004; Li, Lee and Tuljapurkar, 2004; Bongaarts, 2005; Buettner and Zlotnik, 2005; Booth, Hyndman, Tickle and Jong, 2006; Girosi and King, 2006; Kim, 2006; Hwang and Jung, 2011).

## II. The coherent Lee-Carter method

The coherent LC method is a modification to overcome the shortcomings of the separate LC method. The separate LC method is a method for the extrapolation of persistent long-term historical patterns and trends in mortality without any special additional assumptions. It involves a two-factor (age and time) model and uses matrix decomposition to extract a single time-varying index of the level of mortality and forecast it using a time series model. The strengths of the method are its simplicity and robustness in the context of linear trends in age-specific death rates. The model for the separate LC method is as follows.

$$
\log (\mathrm{m}(\mathrm{x}, \mathrm{t}))=\mathrm{a}(\mathrm{x})+\mathrm{b}(\mathrm{x}) \mathrm{k}(\mathrm{t})+\varepsilon(\mathrm{x}, \mathrm{t})
$$

$$
\begin{aligned}
& \text { where } \mathrm{a}(\mathrm{x}): \text { age pattern of mortality } \\
& \mathrm{b}(\mathrm{x}): \text { age pattern of mortality change } \\
& \mathrm{k}(\mathrm{t}): \text { time-varying index of the level of mortality }
\end{aligned}
$$

This is a significant departure from previous approaches, which have been made under the assumption that the advance in human mortality will decelerate in the near future by setting upper and lower limits to human life span or rationalizing the deceleration of gains in life expectancy(Lee and Carter, 1992; Booth et al., 2006).

However, the separate LC method has several fundamental weaknesses. The structural changes in historical patterns of mortality can be missed because it assumes a certain pattern of change in the age distribution of mortality and cannot take shifts in pattern into account. It has some limitations in mortality forecasting for a group; it could give implausible sex
differentials and geographic disaggregation in the long run(Lee and Nault, 1993; Lee, 2000; Li and Lee, 2005).

The coherent LC method which was suggested to improve these limitations in group mortality is composed of a common factor for group mortality and a specific factor of each member within the group. The model for the Coherent LC method is as follows.

$$
\begin{gathered}
\log (m(x, t, i))=a(x, i)+B(x) K(t)+b(x, i) k(t, i)+\varepsilon(x, t, i), 0 \leq t \leq T \\
\text { where } B(x) K(t): \text { common factor for a group } \\
b(x) k(t): \text { specific factor for each member }
\end{gathered}
$$

The procedure of estimating the parameters of the coherent LC method are as follows.

## 1. Estimating $\mathrm{a}(\mathrm{x}, \mathrm{i})$

The average age specific death rates, $\mathrm{a}(\mathrm{x}, \mathrm{i})$ for the coherent LC method is the same as the $\mathrm{a}(\mathrm{x}, \mathrm{i})$ for each country in the separate LC method, which is the average of $\log (\mathrm{m}(\mathrm{x}, \mathrm{t}, \mathrm{i}))$ for each country.

## 2. Estimating common factor for a group

The age pattern of the common mortality change, $\mathrm{B}(\mathrm{x})$ and the time index of the common mortality decline, $\mathrm{K}(\mathrm{t})$, which explains the common trend in mortality change within the three countries, were obtained from the combined mortality of the Republic of Korea, Japan, and the Taiwan Province of China by a singular value decomposition(SVD) method to find a least squares solution as an ordinary LC method(Wilmoth, 1993). Then, K(t) was modeled as a random walk with drift(RWD) to forecast the future mortality change within the group.
The common factor model is as follows.

$$
\log (\mathrm{m}(\mathrm{x}, \mathrm{t}, \mathrm{i}))=\mathrm{a}(\mathrm{x}, \mathrm{i})+\mathrm{B}(\mathrm{x}) \mathrm{K}(\mathrm{t})
$$

## 3. Estimating the specific factor for each country

The specific factors for each country $\mathrm{b}(\mathrm{x}, \mathrm{i})$ and $\mathrm{k}(\mathrm{t}, \mathrm{i})$, were obtained from the residual matrix of the common factor model, $[\log (\mathrm{m}(\mathrm{x}, \mathrm{t}, \mathrm{i}))-\mathrm{a}(\mathrm{x}, \mathrm{i})-\mathrm{B}(\mathrm{x}) \mathrm{K}(\mathrm{t})]$, by applying the SVD. To forecast the future $k(t, i), \operatorname{Ar}(1)$ and random walk without drift(RW) which were suggested by Li and Lee (2005) as a way of avoiding long-term divergence, were compared and RW was used to forecast $\mathrm{k}(\mathrm{t}, \mathrm{i})$ because it provided a more stable walk for the future trend.

The model for the $\log$ ASDR for each country was driven by adjusting the launch time mortality in 2008.

$$
\log (\mathrm{m}(\mathrm{x}, \mathrm{t}, \mathrm{i}))=\log (\mathrm{m}(\mathrm{x}, \mathrm{~T}, \mathrm{i}))+\mathrm{B}(\mathrm{x})[\mathrm{K}(\mathrm{t})-\mathrm{K}(\mathrm{~T})]+\mathrm{b}(\mathrm{x}, \mathrm{i})[\mathrm{k}(\mathrm{t}, \mathrm{i})-\mathrm{k}(\mathrm{~T}, \mathrm{i})], \mathrm{t}\rangle \mathrm{T}, \mathrm{~T}=2008
$$

According to this model, $\mathrm{B}(\mathrm{x}) \mathrm{K}(\mathrm{t})$ accounts for the long term trend and random variation of mortality for the whole group of the Republic of Korea, Japan and the Taiwan Province of China, while the $b(x, i)$ and $k(t, i)$ accounts for the special short-term trend for each country.

## 4. Estimating the specific factor for subgroups of each country

As an application of mortality forecasting for subgroups such as sex and provinces within each country, which is consistent with the country coherent LC method, the specific factor for subgroups within a country can be obtained by the same procedure as the specific factors for each country, although Li and Lee(2005) proposed the separate application of subgroups, which may need another adjustment to get a consistent result with the country coherent LC method. The common factor of the combined country mortality and residual of subgroups from the common factor can be used for the common and specific factors of subgroups within each country, because the ASDR for the subgroups of each country are included in the common factor of the combined country mortality and the residual from common mortality explain the specific factor for the subgroups.

The final model for the $\log$ ASDR for a subgroup for each country could be driven as follows.

$$
\log (\mathrm{m}(\mathrm{x}, \mathrm{t}, \mathrm{i}, \mathrm{j}))=\log (\mathrm{m}(\mathrm{x}, \mathrm{~T}, \mathrm{i}, \mathrm{j}))+\mathrm{B}(\mathrm{x})[\mathrm{K}(\mathrm{t})-\mathrm{K}(\mathrm{~T})]+\mathrm{b}(\mathrm{x}, \mathrm{i}, \mathrm{j})[\mathrm{k}(\mathrm{t}, \mathrm{i}, \mathrm{j})-\mathrm{k}(\mathrm{~T}, \mathrm{i}, \mathrm{j})], \mathrm{t}\rangle \mathrm{T}, \mathrm{~T}=2008
$$

where $\mathrm{b}(\mathrm{x}, \mathrm{i}, \mathrm{j})$ and $\mathrm{k}(\mathrm{t}, \mathrm{i}, \mathrm{j})$ are the specific factors for the subgroups within a country and are obtained from the residual of the country common factor model $[\log (m(x, t, i, j))-a(x, i, j)-B(x) K(t)]$

## III. The Historical Data

The annual age-specific death rates(ASDR) for the Republic of Korea, Japan and the Taiwan Province of China from 1970 to 2008 were used to forecast the ASDR for the period from 2009 to 2050. Although the mortality data for Japan was available since 1947, data from 1970 was used because the only available data for the Republic of Korea and the Taiwan Province of China were from 1970. ASDR for Japan and the Taiwan Province of China were from the Human Mortality Database and ASDR for the Republic of Korea were converted from the age-specific probability of death from the annual period life tables from 1970 to 2008, which were the only source of mortality data from 1970 to 1982.

Before fitting model, ASDR for the Republic of Korea was extended to age 99 because the historical data for death rates was available up to from ages 80 to 94 before 2000. To extend ASDR to age 99, the Brass-Logit relational model ${ }^{1)}$ was applied using the life tables for the period from 2004 to 2006 for the Republic of Korea as a standard life table, which was constructed up to age 100 and over²)(Wachter, 2009). Extending the life

[^1]tables in this way slightly altered(about less than 0.2 years) the observed life expectancies over the sample period and such differences were regarded as negligible.

## IV. Results

〈Figure 1〉shows the historical life expectancies at birth for the Republic of Korea, Japan and the Taiwan Province of China from 1970 to 2008. The life expectancy at birth in the Republic of Korea started from the low sixties in 1970, with a disadvantage of ten years from Japan and seven years from the Taiwan Province of China. However, the Republic of Korea is continuously catching up with Japan and since 2001, has caught up with the Taiwan Province of China.
<Figure 1> Life Expectancy at Birth for the Republic of Korea, Japan, and the Taiwan Province of China, 1970~2008


## 1. Applying the separate LC method

To apply the LC method separately, the average age specific death rate, $a(x, i)$, the age pattern of mortality change, $b(x, i)$ and the time index for the mortality decline, $\mathrm{k}(\mathrm{t}, \mathrm{i})$ for each country were driven using the ordinary LC
method．〈Figure 2〉 shows the average age specific death rate，$a(x, i)$ ，for the period from 1970 to 2008 for the Republic of Korea，Japan and the Taiwan Province of China．The mortality for the Republic of Korea is highest among the three countries over all age groups，with a larger disadvantage from Japan and a slight disadvantage from the Taiwan Province of China．The gap between the Republic of Korea and Japan is large over all age groups and the gap between the Republic of Korea and the Taiwan Province of China is relatively larger in younger age groups and a little difference is existent after age 70．The crossover in the mortality after age 95 may be from the difference of the extending method of old age mortality．
＜Figure 2＞Average Age Specific Death Rates for the Republic of Korea，Japan， and the Taiwan Province of China，1970～2008


〈Figure 3〉 shows the age pattern of mortality change，$b(x)$ for each country．The mortality change for the Republic of Korea is much greater than Japan and the Taiwan Province of China in younger ages and much smaller than Japan and the Taiwan Province of China in older ages．The Republic of Korea＇s greater mortality change in younger ages implies that mortality decline in the Republic of Korea has been concentrated in the younger ages during the past four decades，when compared to Japan and the Taiwan Province of China．The Republic of Korea＇s much lower life expectancy at birth in 1970 than Japan and the Taiwan Province of China’s
(Figure 1) can be explained for the environment of fast improvement for mortality for younger age groups.
<Figure 3> Age Pattern of Mortality Change for the Republic of Korea, Japan, and the Taiwan Province of China, 1970~2008


〈Figure 4〉 shows the time index for the mortality decline, $k(t)$ for the Republic of Korea, Japan and the Taiwan Province of China. The $k(t)$ for the Republic of Korea has a steepest slope over the entire period, with a slightly steeper slope for Japan in the earlier period.
<Figure 4> Mortality Time Index for the Republic of Korea, Japan, and the Taiwan Province of China, 1970~2008


Again, to compare the change in the slope of mortality time index, $k(t)$ over period, the mean slopes of $k(t)$ were presented for the entire period from 1970 to 2008 and for each decade (Table 1). For the entire period, the Republic of Korea had the steepest slope followed by Japan and then the Taiwan Province of China. The slope of $k(t)$ for the Republic of Korea started with a middle speed of decline among the three countries and it had been increased by the 1990s and began to slow down in the 2000s. Japan had the steepest slope in the 1970s, decreased until the 1990s and slightly increased in the 2000s. The Taiwan Province of China had the slowest slope for the overall period but it started to increase since the 1990s and had a steeper slope than Japan since the 2000s. The changing slope of each country's $k(t)$ over time implies that for Japan, the separate LC method using the mean slope as a drift may give a steeper decline in mortality than a recent decline in observed mortality. For the Taiwan Province of China, it may have opposite results than Japan. It implies that combining the mortality from other countries could improve mortality forecasting for each country by altering the mean slope for each country by the ordinary LC method.
<Table 1> The Slope of Mortality Time Index (kt) for the Republic of Korea, Japan, and the Taiwan Province of China

|  | $1970 \sim 2008$ | 1970 s | 1980 s | 1990 s | 2000 s |
| :--- | :---: | :---: | :---: | :---: | :---: |
| The Republic of Korea | 0.20 | 0.14 | 0.22 | 0.23 | 0.22 |
| Japan | 0.13 | 0.18 | 0.12 | 0.10 | 0.11 |
| The Taiwan Province of China | 0.11 | 0.11 | 0.09 | 0.11 | 0.17 |

## 2. Applying the coherent LC method

To apply the coherent LC method, the common factor for the three countries, $\mathrm{B}(\mathrm{x})$ and $\mathrm{K}(\mathrm{t})$, and separate factor for each country, $\mathrm{b}(\mathrm{x}, \mathrm{i})$ and $\mathrm{k}(\mathrm{t}, \mathrm{i})$ were estimated using the combined mortality and the residual matrix of the common factor model. 〈Figure 5〉 shows the common trend of mortality decline, $\mathrm{K}(\mathrm{t})$ for the combined mortality of the Republic of Korea, Japan and the Taiwan Province of China. $\mathrm{K}(\mathrm{t})$ for the combined mortality of
the three countries is almost a straight line，which implies the exponential rates of mortality decline during the past four decades and the expectation of continuing trend in the future within the Republic of Korea，Japan and the Taiwan Province of China．
＜Figure 5＞Mortality Time Index for Combined Mortality（Kt），1970～2008


## 3．Comparing the separate LC method and the Coherent LC method

Before comparing the performance of the separate LC method and the coherent LC method over the forecasting period，the quality of the fit of the two methods over the sample period was examined．〈Figure 6〉 shows the actual and fitted deaths rates for selected age groups for the period from 1970 to 2008 by the separated LC method and the coherent LC method． Both two methods perform well on within－sample forecasts．

To compare the separate LC method and the coherent LC method over the forecasting period，the median forecast of life expectancy at birth for the Republic of Korea，Japan，and the Taiwan Province of China by the separate LC method and the coherent LC method were presented in 〈Figure 7〉．By the separate LC method，the forecast of life expectancy at birth for Japan has increased quickly and the forecast of life expectancy at birth for the Taiwan Province of China has increased slowly，while the life expectancy at birth for the Republic of Korea has increased at an average speed．The life expectancy at births for three countries diverges over time，which is the
opposite from the historical trend of the life expectancy at births of the three countries.
<Figure 6> Actual and Fitted Log Death Rates for the Republic of Korea, Japan, and the Taiwan Province of China, 1970~2008


However, by the coherent LC method, the increase in life expectancy at birth for Japan has been slowing down and the increase in life expectancy at birth for the Taiwan Province of China is being accelerated. These changes have been influenced by the common trend of speed of life expectancy at birth increase within the three countries and less influenced by the drift of its own trend in the long term, although the coherent LC method does not make much of a difference for the Republic of Korea which has an average speed of increase among the three countries. Japan's slower slope in the 2000s and the Taiwan Province of China's steeper slope in the 2000s than the mean slope of kt for the entire period are reflected in the coherent LC method by the common trends of combined mortality and bring consistency with the recent trends of life expectancy at birth increase in the short run. The coherent LC model narrows the gap of life expectancy at birth between these three countries and brings the convergence of life expectancy at birth in the long run(Table 1, Figure 7).

〈Table 2〉 presents the median forecasts and ranges by the separate LC method and the coherent LC method. The median forecast for the Republic of Korea by the coherent LC method gives slightly lower forecast of life expectancy at birth ( 0.2 years) until 2030 and a slightly higher forecast of life expectancy at birth( 0.4 years) in 2050. For the Taiwan Province of China, the coherent LC method gives $0.1 \sim 1.5$ years higher forecast of life expectancy at birth over the entire forecasting period, while for Japan, the coherent LC method gives 0.1~2.2 years lower forecast over the whole forecasting period. The range of life expectancy at birth in 2050 by the coherent LC method has decreased from 4.82 years to 1.89 years for Japan and from 3.81 years to 2.08 years for the Taiwan Province of China. For the Republic of Korea, the range has increased by nearly one and half times from 1.04 to 1.52 , which was too narrow partly due to the data limitation of using smoothed data from life tables rather than primary data. It was improved when combined with a relatively wide range from Japan and the Taiwan Province of China.
<Figure 7> Life Expectancy at Birth for the Republic of Korea, Japan, and the Taiwan Province of China, 1970~2050

<Table 2> Forecast of Life Expectancy at Birth, the Separate LC method vs. the Coherent LC method

|  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Forecast | 2015 |  | 2030 |  | 2050 |  |
|  |  | Median | Range | Median | Range | Median | Range |
| The Republic of Korea | Separate | 81.95 | 0.66 | 85.33 | 0.93 | 88.81 | 1.04 |
|  | Coherent | 81.79 | 0.93 | 85.18 | 1.29 | 89.16 | 1.52 |
| The Tapan | Separate | 84.63 | 2.02 | 88.52 | 3.52 | 93.62 | 4.82 |
|  | Coherent | 84.45 | 0.78 | 87.67 | 1.30 | 91.39 | 1.89 |
|  | Separate | 79.95 | 1.97 | 83.05 | 3.19 | 86.85 | 3.81 |
|  | Coherent | 80.32 | 1.22 | 84.09 | 1.80 | 88.37 | 2.08 |

<Figure 8> The Probability Bands of Life Expectancy at Birth for the Republic of Korea, Japan, and the Taiwan Province of China, 1970~2050


To examine the influence of the coherent LC method on the age specific mortality forecasting, the log of age specific death rates for the Republic of Korea by the separate LC method and the coherent LC method were compared in 〈Figure 9〉. When compared to the separate LC method, the log death rates by the coherent LC method were higher for the age groups before age 65 and lower for the age groups after age 65. Because they are the log-transformed rates, little difference in older ages contribute similarly to the life expectancy at birth with big differences in younger ages. It implies that the coherent LC method decelerates the declining mortality in younger age groups and accelerates the declining mortality in older age
groups in the Republic of Korea. The coherent LC method alters the age pattern of the mortality decline in the Republic of Korea, which has a more rapid decline in the mortality of younger age groups than of older age groups during the past four decades. Although the opposite influence on the mortality by overall age offsets the differences in life expectancies by the two methods(Figure 7), the coherent LC method seems to reflect the transitional mortality decline pattern which starts in younger age groups and widens to older age groups by using the common pattern of declining mortality from countries in different stage of the declining process.
<Figure 9> Log Age Specific Death Rates for the Republic of Korea, 1970~2050


To examine how this country coherent LC method works for sex specific mortality forecasting within a country, the forecast of life expectancy at birth by the coherent LC method using country-combined mortality as a common factor, the sex coherent LC method using sex-combined mortality within the Republic of Korea as a common factor, and the separate LC method by sex were presented in 〈Figure 10〉. Although life expectancy at birth by sex converges in all three methods, the gap of life expectancy at
birth between females and males in the separate LC method declined much faster than in the coherent LC method because of the faster decline of mortality for males than females(Table 3; KOSTAT, 2010). By the separate LC method, the gap of life expectancy at birth between females and males in the Republic of Korea was forecasted to be 2.7 years in 2050, which decreased more than a half from the 6.8 years in 2008. The separate LC method using only the specific change pattern of mortality decline in each sex seemed to bring the crossover of life expectancy at birth between males and females in a few decades, which is difficult to explain in terms of the existence of the natural biological sex differential in life expectancy at birth.
<Table 3> The Increase of Life Expectancy at Birth by Sex, the Republic of Korea, 1970~2008

|  | $1970 \sim 2008$ | $1970 \sim 1979$ | $1980 \sim 1989$ | $1990 \sim 1999$ | $2000 \sim 2008$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Male | 17.87 | 2.61 | 5.06 | 4.42 | 4.29 |
| Female | 17.72 | 3.94 | 5.04 | 3.71 | 3.69 |

The sex coherent LC method using the sex-combined mortality within the Republic of Korea gave a slightly smaller gap in life expectancy at birth between males and females than the country coherent LC method using the country-combined mortality, which was 4.1 years and 4.7 years of difference in 2050, respectively. Although it may be hard to predict how the sex differential of life expectancy at birth will change in the future, the country coherent LC method using the same common factor model for forecasting of the subgroups within a country seems to be a more reasonable application rather than applying the coherent LC method by subgroups. It uses more information and does not require an additional adjustment, which is needed to fit the original group when the coherent LC method applied by subgroups.
<Figure 10> Life Expectancy at Birth by Sex for the Republic of Korea, 1970~2050


## V. Summary and Conclusion

To forecast the mortality for the Republic of Korea, this paper applies the coherent LC method using the combined mortality of Japan and the Taiwan Province of China. By comparing the coherent LC method to the separate LC method, it found that the integration of regional mortality information into mortality forecasting of one country gives several advantages in terms of a short run fit within each country as well as a long run convergence between countries. A modification of the age pattern of mortality decline and a consistent application of forecasting of subgroups within a country were found.

First, combining the common trend of the group mortality and specific trend of each country gave a more plausible mortality forecasting in the short run as well as in the long run. Using a common trend changed the mean level of mortality decline in each country and gave a more plausible result in the short run. In the long run, it gave the convergence of life expectancy by removing the long term specific drift for each member. For Japan and the Taiwan Province of China, this short run consistency with the recent life expectancy at birth increase and the long run convergence is
distinct. The Republic of Korea which placed in the middle level of mortality among the three countries and had similar patterns of mortality decline with common trends seemed to be less influenced by the coherent LC method.

Second, the coherent LC method modified the age pattern of mortality decline by decelerating the declining mortality in younger age groups and accelerating the declining mortality in older age groups in the Republic of Korea. It implies that the coherent LC method reflects the transitional mortality decline pattern by using the common pattern of declining mortality from countries in different stages of the declining process.

Finally, applying the coherent LC method to the subgroups within a country by using the same common factor model can be a more reasonable application rather than applying the coherent LC method by subgroups. It enables the use of more information and to avoid additional adjustments which may needed to fit the original group when it is applied by subgroups. The result of this application satisfied the long direction of convergence.

The coherent LC method worked well for the Republic of Korea, Japan and the Taiwan Province of China as a group. It gave a reasonable mortality forecast in the context of a plausible age pattern of mortality decline and sex differentials. Even though the coherent LC method did not make much of a difference for the Republic of Korea in terms of sex-combined life expectancies at birth, comparing the separate LC method, it still improved the mortality forecasting by age and sex and was comparable to other populations of East Asia with precedent and similar experiences.

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## 한국의 사망력 추계 : 통합 Lee-Carter 방법

김수영

이 연구에서는 한국의 사망력 추계를 위해 Li 와 Lee가 그룹 인구의 일관성 있는 사망력 추 계를 위해 제안한 Coherent Lee-Carter방법을 일본, 타이완의 자료를 결합하여 적용하고, 이 방 법의 적합을 검증하였으며, Le-Carter방법을 각각 적용했을 때와의 결괴를 비교하였다. 세 국가 를 하나의 그룹으로 작성한 이 방법은 세 국가에 각각 Lee-Carter방법을 적용했을 때에 비해 타 이완의 기대수명 증가를 가속시키고, 일본의 기대수명증가를 감속시키면서, 2050년 세 국가의 기대수명 범위를 6.8 세에서 3.0 세로 감소시켰다. 한국의 경우는 남녀전체의 기대수명은 크게 변 화시키지 않았으나, 65세 이상의 기대수명 증기를 가속화시키고, 65 세 이하의 기대수명증가를 감속화 시켰으며, 남녀 기대수명 차이를 서서히 감소시켰다. 이 방법은 한 국가의 사망력 추계 를 위해 사망환경이 유사한 국가의 사망력을 결합하는 것은 장기간의 사망력 수렴뿐만 아니라, 단기간의 사망력 추계의 적합도 향상, 연령별 사망력 감소 패턴의 보정 및 한 국가 내 세부 그 룹 인구의 일관성 있는 사망력 추계에의 응용 등 여러 가지 장점이 있음을 시사한다.

핵심단어: 사망력 추계, Lee-Carter 방법, 통합 Lee-Carter 방법, 공통요인, 개별요인


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[^1]:    1) The parameter values of $\alpha$ and $\beta$ to match each of the pervious life tables for the periods from 1970 to 2000 were estimated. The Brass lx up to age 99 which were calculated from the parameter values for each year, were used to calculate ${ }_{n} q_{x}$ after age 85 and ${ }_{n} q_{x}$ was converted to ${ }_{n} \mathrm{M}_{\mathrm{x}}$ to apply Lee-Carter model by the ordinary life table function.
    2) The observed value ${ }_{n} q_{x}$ after age 85 was fitted to the model, ${ }_{n} q_{x+n} /{ }_{n} q_{x}=e^{a+b x}$, on an assumption of an exponential increase of the ratio of ${ }_{n} \mathrm{q}_{\mathrm{x}}$ to ${ }_{\mathrm{n}} \mathrm{q}_{\mathrm{x}+\mathrm{n}}$ (KOSTAT, 2007).
