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Knowledge is Key to Variability in Solving Algebraic Word Problems[1]

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In this paper I propose that teaching students the most efficient method of problem solving may curtail students' creativity. Instead it is important to arm students with a variety of problem solving heuristics. It is the students' responsibility to decide which heuristic will solve the problem. The chosen heuristic is the one which is meaningful to the students.

Keywords: problem solving heuristics, algebraic word problems, letter-symbolic algebra
MESC Classification: D73, H13
MSC2010 Classification: 97D70, 97H10

INTRODUCTION

Problem solving is at the heart of the Singapore mathematics curriculum. Although elementary students experience a wide variety of problems the more common type are those identified as structurally complex word problems (also known as start unknown problems or algebraic word problems). Normally solution to these problems requires knowledge of letter-symbolic algebra and the ability to construct and solve a system of linear equations up to 2 unknowns. Although such problems may be challenging, there are elementary students who are able to solve these algebraic word problems. One possible reason for this phenomenon is that the curriculum places a high premium on teaching elementary students problem solving heuristics which include working backwards, systematic listing, guess and check and the model method (Ministry of Education, 2000, 2006). This method involves drawing a diagram and because it can be used to solve

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arithmetic and algebraic type word problems, it holds centre stage in the list of problem solving heuristics. This method has become the common currency of problem solving that it is known locally as the model method.

Because it is not widely known outside Singapore, I first describe the model method and discuss how Singapore elementary students who are taught and hence have knowledge of this method can use it to solve arithmetic word problems and algebraic word problems. Because Singapore has a centralized education system, all elementary students would have been taught the model method and the other problem-solving heuristics. Hence besides elementary students, high school students (16+) and pre-university students (17+) could be said to have some knowledge of these problem solving heuristics. In this paper I use the solutions of students from these three age groups to demonstrate how they were able to use a variety of problem solving heuristics to solve eight algebraic word problems. Although solutions of these problems would normally require knowledge of letter-symbolic algebra, the work of these elementary students showed otherwise. Furthermore high school students and pre-university students who have knowledge of letter-symbolic algebra may not choose to use this method if they have knowledge of problem solving heuristics which are more meaningful to them.

In the concluding section I discuss how knowledge of different methods to solve algebraic word problems contributes to flexibility in problem solving. Although letter-symbolic algebra is a powerful problem solving tool, for various reasons to be discussed it may not be the most meaningful method for students to use. Problem solving heuristics such as the model method or making a systematic list may be less elegant than letter-symbolic algebra; nevertheless students seem to know what they were doing when they accessed such tools. This paper presents how new methodological development such as functional Magnetic Resonance Imaging is used to answer questions related to algebraic word problem solving.

THE MODEL METHOD AS A PROBLEM SOLVING HEURISTIC

The model method (Ministry of Education, 2009) has been part of the Singapore elementary school curriculum since its formal introduction in 1987 (Kho, 1987). In this section four examples are used to illustrate how some Grade 5 students successfully used the model method to solve algebraic word problems that normally require the construction of appropriate system of linear algebraic equations. These examples are taken from a study conducted in 2003 with 151 Grade 5 children. Construction of a model drawing for any given word problem is underpinned by children's knowledge of the part-whole relationship of numbers. The resulting schematic representation is like a pictorial equation that captures all the information in word problems as a cohesive whole. Instead of letters, rectangles are used to represent unknowns in algebra type word problems (Author, 2004). The value of the unknown can be found by undoing each operation. (Please see the article Ng and Lee (2009) for a detailed discussion of the model method.)

The problems in this paper are named to facilitate discussion. The Enrolment Problem is an example of an arithmetic word problem. The rest are algebraic word problems. Furniture Problem is a homogenous algebraic word problem as the same comparison 'more than' is used to relate the different variables. The *Age Problem* and the *Water Problem* require multiplicative reasoning and knowledge of fractions. Furthermore, the Age problem requires a 'before and after' drawing to represent temporal change.

Arithmetic word problem: The length of the rectangles indicate the size of the numbers, a longer rectangle is used to represent a bigger number. Figure 1 shows how the schematic representation captures all the relevant information presented in the text of the problem. First a rectangle is used to represent the enrolment of Dunearn Primary School. The enrolment of Sunshine Primary is represented by the second set of rectangles, one representing the base of comparison (the enrolment of Dunearn Primary) and the difference in the enrolment of the two schools. The same process is used to represent the enrolment of Excellent Primary. The total enrolment can be found once all the information presented in the word problem is captured in the model drawing.

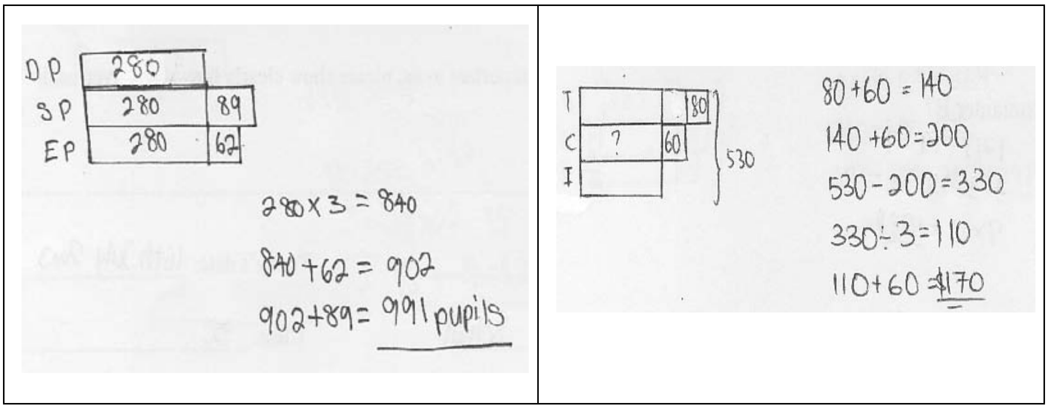

Figure 1. The model drawing on the left records accurately all the necessary information presented in the *Enrolment Problem*. Student's model drawing on the right is a schematic representation of the *Furniture Problem*. The cost of the table and chair are based on the cost of the iron, an unknown. The difference in cost of the two pieces of furniture is indicated clearly in the rectangles.

Enrolment Problem: Dunearn Primary School has 280 pupils. Sunshine Primary School has 89 pupils more than Dunearn Primary. Excellent Primary has 62 pupils more than

Dunearn Primary. What is the total enrolment of the three schools?

Algebraic word problem: The cost of any items could be used to start the model drawing. In this case the cost of the iron is used to generate the subsequent sets of rectangles representing the cost of each item. Hence the cost of the iron is the generator of the problem. The solution to the Furniture Problem is direct as homogenous comparison 'more than' relates the cost of the different items. The model drawing is clear, with the difference in cost clearly indicated in each of the difference rectangle. A question mark '?' is used to indicate the variable to be evaluated. The total cost is indicated to the right of the rectangles. The undoing/unwinding process is used to evaluate the unknown generator. If the total difference in cost of the two items ($200) is removed from the cost of the purchases ($530), then the total value of the three rectangles is the difference of these two totals ($530 – 200). The value of one rectangle or unit is equivalent to the quotient: ($330 ÷ 3 = 110). The cost of the chair is found by summing the value of one unit and the difference in cost of the chair and the iron ($110 + $60 = $170).

Furniture Problem: At a sale, Mrs. Tan spent $530 on a table, a chair and an iron. The chair cost $60 more than the iron. The table cost $80 more than the chair. How much did the chair cost?

Water Problem: A tank of water with 171 litres of water is divided into three containers, A, B and C. Container B has three times as much water as container A. Container C has 1/4 as much water as container B. How much water is there in container B?

Multiplicative reasoning is necessary to solve the Water Problem. For a more complete discussion of the solutions to this problem see Ng and Lee (2005). The left panel in Figure 2 illustrates how the solution was found by using *concepts* of, but not *operations* with, fractions. Concepts of fractions are necessary to transform the Water Problem to one where operations with whole numbers sufficed. A correct solution is found by translating the relationship that 'Container C has ¼ as much water as container B" to its equivalent "Container B has 4 times as much water as C", and then checking for the lowest common multiple for 4 and 3. The model drawing illustrates how important it was to be meticulous in the representations. The details in the model drawing showed the exact relationships between the three containers.

Age Problem: Mr. Raman is 45 years older than his son, Muthu. In 6 years time, Muthu will be 1/4 his father's age. How old is Muthu now?

The solution in the right panel of Figure 2 illustrates how the visual nature of the model drawings makes it possible to represent, despite a change of 6 years, the constancy in age difference between father and son. Knowledge of this fact is the key to the solution.

The ‘after model’ of the model drawing shows in detail the multiplicative relationships between father and son.

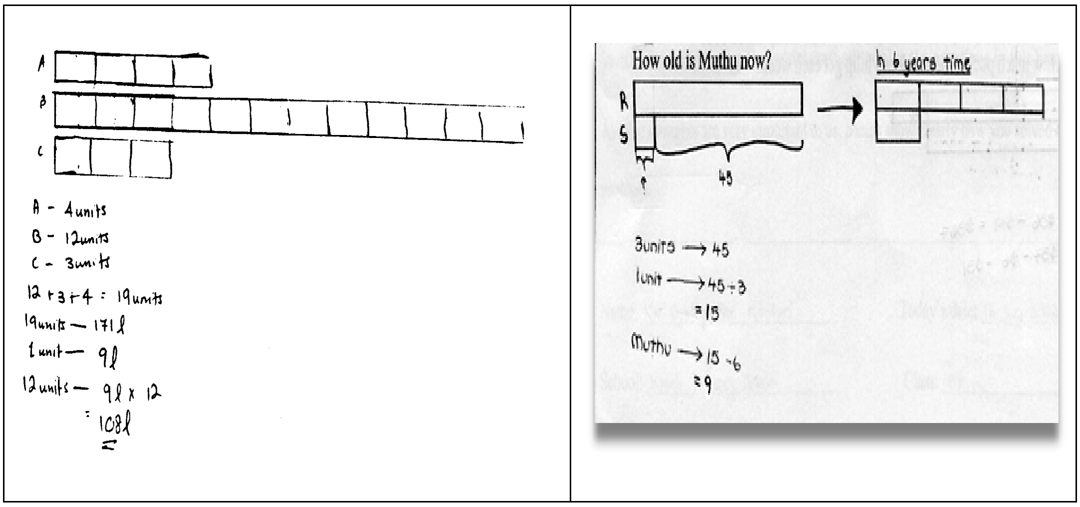

Figure 2. The model drawing on the left is the schematic representation to the *Water Problem*. The schematic representation on the right is student’s model drawing of the *Age Problem*. In this case two sets of drawing are necessary to represent all the information. The drawing on the left shows the relationship of the current age of father and son whereas the model drawing on the right shows the difference in age remains constant although six years have passed.

ONE PROBLEM, MANY SOLUTIONS

The Golden Apple Problem, an algebraic word problem was presented to fifth grade, high school and junior college students. They were told they could use any method to solve this problem. The analysis showed that letter-symbolic algebra was not the preferred method of solution. Instead the students used methods that made sense to them. In this section I will illustrate how students solved the Golden Apple problem.

Golden Apple Problem: A troll picked a basketful of golden apples in the enchanted orchard. On his way home, he was stopped by a troll who guarded the orchard. The troll demanded payment of one-half apples plus two more. The prince gave him the apples and set off again. A little further on, he was stopped by a second troll guard. This troll demanded payment of one-half of the apples the prince now had plus two more. The prince paid him and set off again. Just before leaving the enchanted orchard, a third troll stopped him and demanded one-half of his remaining apples plus two more. The prince paid him and sadly went home. He had only two golden apples left. How many apples had he picked?

The solution in the left in Figure 3 is an example of how a grade 5 student used the model method to solve the problem. A significant amount of processing is needed to construct this model drawing. First one has to decide how to begin drawing the model. Four important pieces of information were provided to help solve this problem:

i) How many apples were left after the third troll had collected the final payment,
ii) The proportion of apples demanded of the prince by each troll,
iii) The additional amount taken after half of the total was collected by the troll,
iv) The terms of payment are the same for each troll. The recursive nature of the problem made it possible to draw the model.

The construction of the model drawing begins with final two apples held by the prince as this is the only known value. Working backwards the number of apples held by the prince before the third troll took half of it could be found. If there were 2 apples left, this means that the prince had 4 (2 + 2 more) apples before the third troll took one-half away. Hence before the prince met the third troll he had 8 (2 × 4) apples. Repeating this process, the prince had 10 apples (8 + 2 more) before the second troll took half away. Therefore the prince had 20 (2 × 10) apples before he met the second troll. Before he gave one-half to the first troll, the prince had 22 (20 + 2 more) apples.

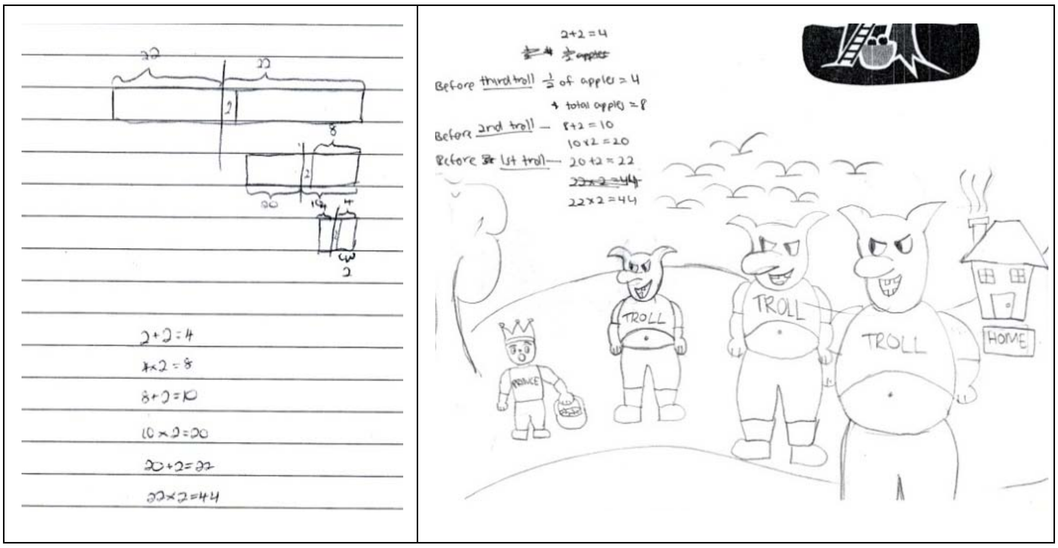

Figure 3. Grade 5 student constructed the model drawing on the left solve the Golden Apple Problem. The working backwards method without the aid of model drawing on the right panel is by a 16+ student. The two solutions showed that the same logical reasoning could be used to represent the information.

This meant that the prince had 44 (2 × 22) apples before he met the first troll. Once the number of apples held by the prince after the third troll has made its collection is correctly represented, the subsequent parts of the model can be drawn by repeating the process. The working backwards method without the aid of model drawing on the right panel by a 16+ student shows the same logical reasoning can also be used to represent the information. Of the two, the illustrations in the solution on the right panel suggest that the student seemed to have enjoyed solving this particular problem.

The solution in Figure 4 presented by a 16+ student shows how systematic listing could be used to solve the problem. Systematic listing assumes the actual number of apples picked by the prince as a given. In this example the student made every effort to write down the reasons for the choice of the starting number. The explanation showed the metacognitive activities exercised by the student. Thus although this numerical solution may not seem as mathematical or concise as the letter-symbolic algebra method, it nevertheless showed that the student was very insightful about the possible answer to the problem. The well organized table shows that the student kept track of each step of the problem encountered by the prince. This level of detail and metacognitive insightfulness may not be evident in the answers by students who used letter-symbolic algebra.

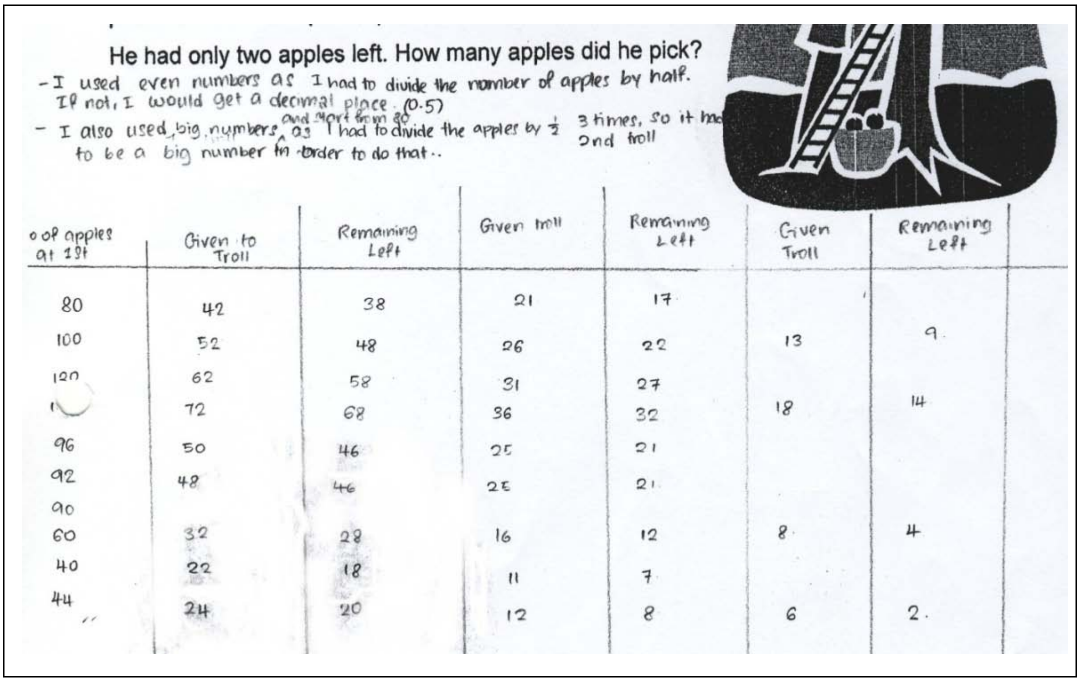

Figure 4. A 16+ student used systematic listing as a problem solving tool. Student recorded his metacognitive processes.

Metacognitive processes recorded by the student: I used even numbers as I had to divide the number of apples by half. If not, I will get a decimal place (0.5). I also used big numbers and start from 80 as I had to divide the apples by 3 times (2nd troll) so it had to be a big number to do that.

Figure 5 provides two solutions using the letter-symbolic algebra. In contrast with the model method which starts with the final number of apples in the possession of the prince, both the systematic listing and the letter-symbolic algebra begin with the actual number of apples picked by the prince. In the case of the letter-symbolic algebra this number is represented by the letter x. Although both algebraic methods are comparable, the solution on the left taken from a 16+ student is clearer than that presented on the right taken from a junior college student (17+). In the first solution, the expression representing the number of apples made as payment to each troll is identified. For example the expression represents the number of apples held by the prince after he had given the first troll his payment.

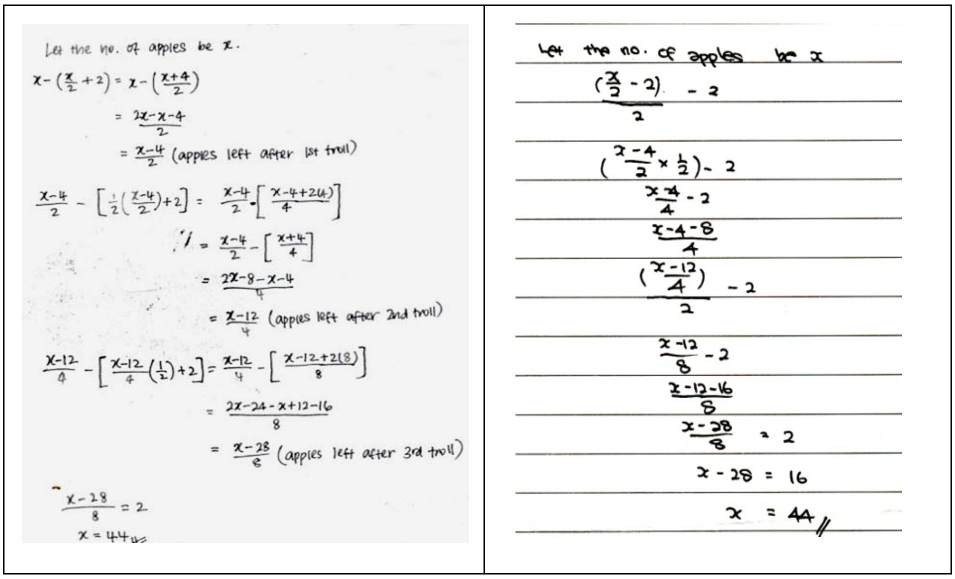

Figure 5. On the left is letter-symbolic algebra method by 16+ student. The solution on the right is by a junior college student (17+). 16+ and 17+ represent their ages.

The preceding discussion shows how different students at different stages of their mathematics education employed a variety of heuristics to solve the same problem. Of all the solutions, it could be said that letter-symbolic method is the most efficient. By using letters to represent the given information, a standard equation could then be constructed that would lead to the solution of any problem with a similar structure. Nevertheless the letter-symbolic method was less meaningful than the other heuristics as it hid many of the processing that went into solving of the problem that was afforded by the other heuristics.

In the following section we show how elementary students used problem solving heuristics to solve the *Egg and Fruit Tart* problem, a more challenging task than the *Golden Apple* problem. The Egg and Fruit Tart problem was one of the past Primary School Leaving Examination questions. Parents were concerned that this problem could not be solved because their children were not taught how to use letter-symbolic algebra as a problem solving tool as this was only developed in the first year of high school. Contrary to parents' belief, the following examples demonstrate that other alternative tools, however, could be used effectively.

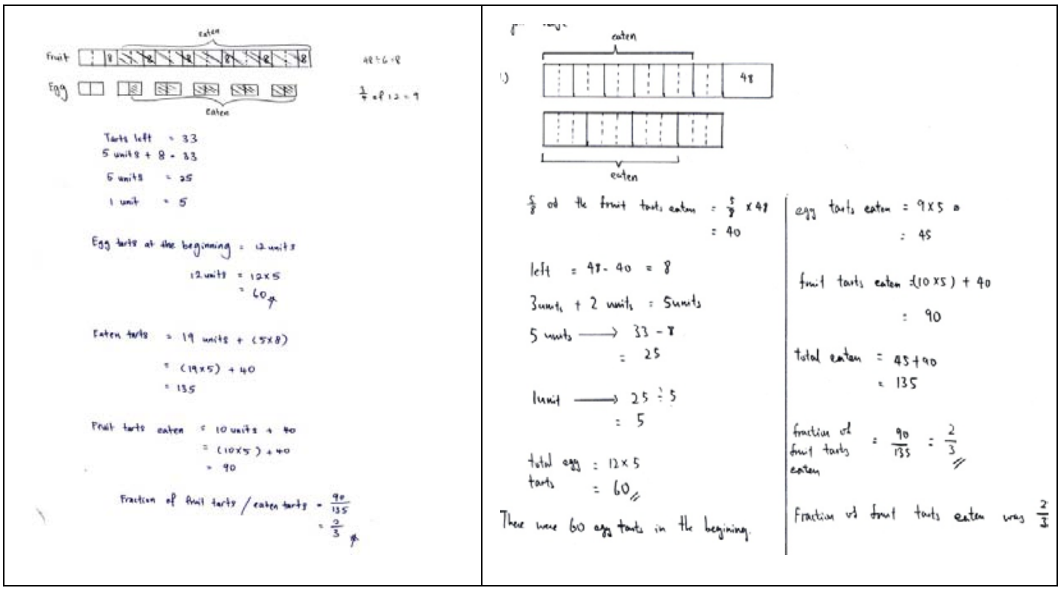

Figure 6. Two solutions using the model method used to solve the *Egg and Fruit Tart* Problem.

The examples in Figure 6 demonstrate how students could use the model method to solve this problem. Although the two model drawings are different, the same set of mathematical constructs is used to construct each model drawing. The students reasoned that there should be 12 equal parts to the fruit and egg tarts because 12 is the lowest common multiple to the two fractions 5/6 and ¾. But because there are 48 more fruit tarts than egg tarts, the model drawing on the left shows how the 48 more fruit tarts are equally distributed among the 6 equal parts. The solution on the right, however, instead of interspersing equal 8 parts in between the fruit tarts places the extra 48 more fruit tarts at the end of the 12 equal parts. Once the number of eaten tarts is removed the value represented by each rectangle could be evaluated.

Egg and Fruit Tart Problem: There are 48 more fruit tarts than egg tarts. After 5/6 of the fruit tarts and ¾ of the egg tarts were eaten, there were 33 tarts left. How many egg tarts were there in the beginning, and what fraction of the eaten tarts were fruit tarts?

The solution in Table 1 shows how systematic listing is another possible tool to solve the Egg and Fruit Tart problem. In this case the solution can be started by exploring with two numbers gives a difference of 48. By eliminating the assumed number of egg and fruit tarts that did not result in a remainder of 33 tarts and by repeating the process the answer is found when the resulting remainder is a total of 33 tarts.

Table 1. Systematic listing as a possible problem solving tool for the *Egg and Fruit Tart* Problem

Difference in the two sets of tarts must always be 48	Fruit tarts left 1/6	Egg tarts left 1/4
168 – 120 = 48	Total = 168 1/6 × 168 = 28	Total = 120 ¼ × 120 = 30
108 – 60 = 48	Total = 108 1/6 × 108 =18	Total 60 1/4 × 60 =15

Number of fruit tarts = 108 Number of fruit tarts eaten = 108 – 18 = 90
Number of egg tarts = 60 Total number of tarts left = 18 + 15 = 33
Number of tarts eaten = 168 – 33= 135
Fraction of eaten tarts were fruit tarts = 90/135 = 2/3

LETTER-SYMBOLIC ALGEBRA VERSUS PROBLEM SOLVING APPRO ACH: WHAT RESEARCH SAYS

Many parents of elementary students may not be aware that other problem solving tools such as the model method or systematic listing could be used to solve such problems.

Because they lacked such knowledge they used the only tool they know – letter-symbolic algebra to coach their children to solve algebraic word problems. The examples in Figure 7 show how some fifth grade elementary students used letter-symbolic algebra to solve algebraic word problems. Many elementary teachers advocate this route of problem solving because some students coached by their parents demonstrated that they were able to use letter-symbolic algebra to solve algebraic word problems. Although the numbers of such students may be small some elementary teachers believe that teaching letter-symbolic algebra to all elementary students may mean that many more could use this method to solve word problems

Sticker problem: The ratio of the number of Joe's stickers to the number of Nick's stickers is 3 : 7. After Joe bought another 9 stickers and Nick gave away 4 stickers, the ratio of the number of Joe's stickers to the number of Nick's stickers became 2 : 3. Find the total number of stickers the two boys had at first.

Refreshment problem: Jeff went to the snacks stall at the canteen to buy cakes and puffs for a class party. He spent $21 altogether to buy 40 cakes and puffs. Each cake cost 60 cents and each puff cost 50 cents. How many cakes and puffs did he buy?

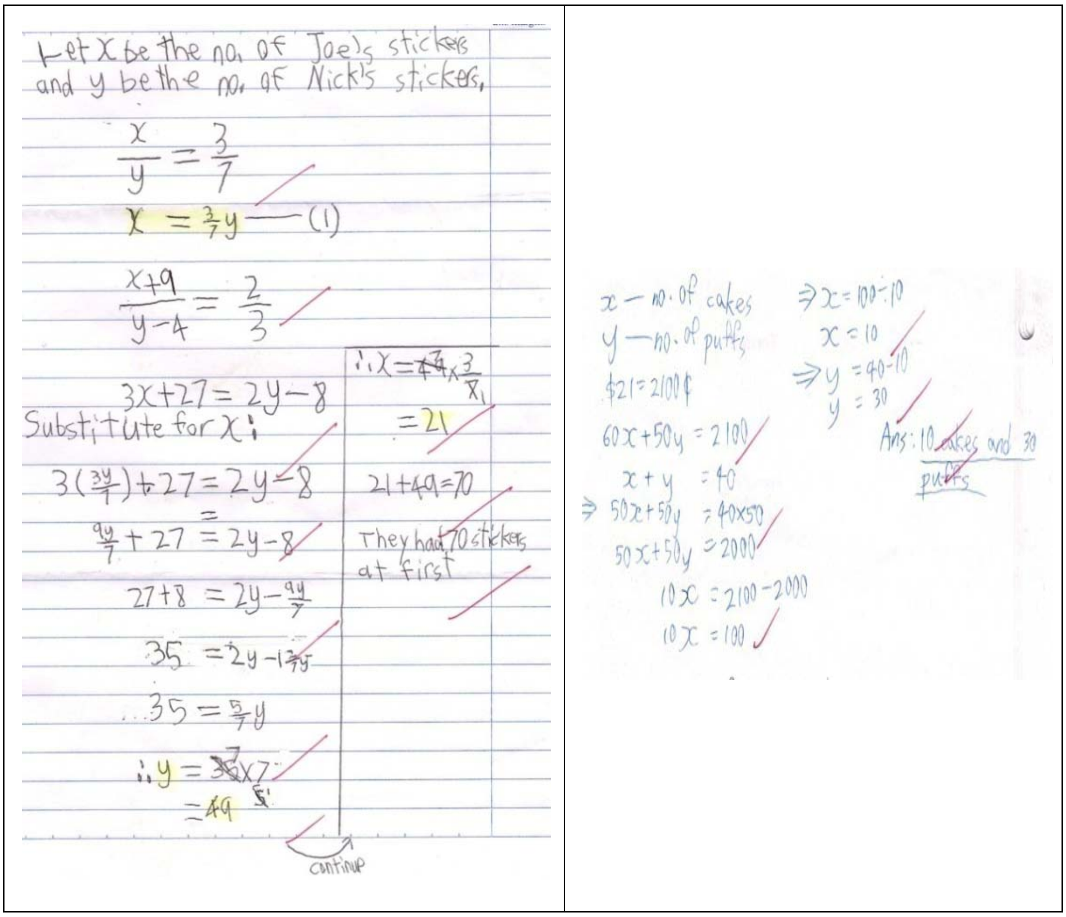

Figure 7. The solution on the left shows how a fifth grade student used letter-symbolic algebra to solve the *Sticker* problem. The example on the right is the letter-symbolic solution by another student to the refreshment problem.

Both these problems require the construction of two algebraic equations and the solution of two simultaneous equations. Such an approach challenges high school students, what more elementary students. The students who gave the solutions in Figure 7 could be described as very competent with letter-symbolic algebra. This situation could not be generalised across all elementary students. The research literature based on behavioural studies demonstrates that many students are challenged by letter-symbolic algebra. The difficulties range from, but are not limited to, students difficulties interpreting the meaning of letters (Harper, 1987; Kuchemann, 1981), interpreting algebraic notations (Koedinger & Nathan, 2004; Pierce, Stacey & Bardini, 2010); understanding structures (Kieran, 1989; Sfard, 1994); identifying equivalent equations (Steinberg, Sleeman & Ktorza, 1991); translating word problems into equations (Clement, 1982; Stacey & MacGregor, 2000; Wollman, 1983).

Contrary to popular belief and research investigating the difficulties students face in solving word problems, Koedinger & Nathan (2004) showed that students found solving algebraic equations more challenging than using other methods to solve algebraic word problems which shared the same structure as the algebraic equations. They argued that the abstract nature of the algebraic equations meant that students had more difficulties interpreting the meaning of these equations than interpreting the meaning of word problems set in context. Furthermore, new methodologies such as using functional Magnetic Resonance Imaging (fMRI) found that letter-symbolic algebra can be challenging to students for other reasons. In Singapore because we have access to students who could use the model method and letter-symbolic algebra to solve the same problem two studies using fMRI methodology were conducted to examine whether the two methods drew on different cognitive processes and imposed different cognitive demands on those using these methods to solve problems.

The first study (Lee, Lim, Yeong, Ng, Venkatraman & Chee, 2007) focused on the initial stages of problem solving: translating word problems into either model drawing or letter-symbolic representations. Eighteen adults, matched on academic proficiency and competency in the model method and letter-symbolic method, were asked to transform algebraic word problems into letter-symbolic equations or model drawings, and validated and presented solutions. Both strategies were associated with activation of areas linked to working memory and quantitative processing. These findings suggest that the two strategies are effected using similar processes but imposed different attentional demands with the letter-symbolic method being more working memory intensive than the model method.

In the second study, Lee, Yeong, Ng, Venkatraman, Graham & Chee (2010) focused on the later stages of problem solving, namely computing numeric solutions from presented model drawings or letter-symbolic representations. Seventeen participants who were equally proficient with the two methods were asked to solve simple algebraic word problems presented in either format. The findings suggest that generating and computing solutions from letter-symbolic equations required greater general cognitive and numeric processing resources than do processes involving model representations. These two studies showed that constructing and solving letter-symbolic equations were more working memory intensive of the two methods. These findings were based on work with adults who were equally competent with two methods. If this is the case, then perhaps elementary students may find letter-symbolic algebra even more demanding to learn than other methods.

In summary, findings from behavioural studies and neuroimgaing research suggest that its abstract nature and the demands it make on students' cognition may mean that students find letter-symbolic algebra challenging and cognitively demanding to learn. Hence although there are elementary students who seem capable of using letter-symbolic algebra to solve algebraic word problems, it is best to leave the teaching of letter-symbolic until students are more mature to cope with tools that require more attentional resources.

EFFICIENCY, MEANINGFULNESS OR CREATIVITY: A CHOICE TO BE MADE

In this paper I have presented examples of how students of different ages chose to solve algebraic word problems. Although letter-symbolic algebra could be used as a problem solving tool, it might not be the best tool. Other tools, albeit less efficient than letter-symbolic algebra were used. These alternative tools seemed to be more meaningful than the abstract letter-symbolic algebra. Although the alternative tools such as the model method and systematic listing may lack the sophistication and the efficiency of the letter-symbolic algebra, the use of these heuristics nevertheless require a creative way of thinking about a problem. Solution of these problems using these alternative heuristics suggests clarity of thought and a deep understanding of the problem. The evidence presented in this paper shows that when students have a variety of heuristics to choose from then the routes to finding the solution to word problems are many. Therefore rather than teaching students the most efficient tool – letter-symbolic algebra – a better alternative is to teach students a wide variety of tools to solve problems. The work of the older group of students (16+) demonstrated how they were able to use a variety of tools to solve the same problem. The elementary students had limited choices because their knowledge of such heuristics is not so wide. The choice of tools is then dependent upon students' understanding of the demands of the task and the meaning the problems have for students and how students make meanings of the tools available.

Of course it could be argued that the problems presented in this paper could be solved using alternative strategies rather than letter-symbolic algebra because whole numbers rather than rational numbers were used. If the latter were used then, compared to letter-symbolic algebra, model drawing and systematic listing or guess and check method may not be that efficient. It would be difficult to construct models for problems where decimals are used. Also the iterative nature of systematic listing and the guess and check method may mean students giving up before a solution could be found. Nevertheless there are ways to get round such problems if students used an estimate of the value rather than the exact value to arrive at an approximate solution. Again this calls for creative ways to evaluate how to solve given problems.

In conclusion problem solving requires knowledge and flexibility of mind. Students need to know what tools are available and when to use which tool to solve a given problem.

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