응용시스템에 강건한 Wiener-Hopf 방정식

Wiener-Hopf Equation with Robustness to Application System

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요 약 본 논문에서 등가의 Wiener-Hopf 공식을 제안한다. 제안된 알고리듬은 입력신호들이 직교하는 경우 TDL 필터의 가중치 벡터와 오차를 동시에 가질 수 있게 된다. 등가의 Wiener-Hopf 방정식은 최소 평균 자승 오차 방식에 근거하여 이론적으로 분석이 되었다. 제안된 알고리듬의 성능 결과는 원래 Wiener-Hopf 방정식의 성능과 동일함을 확인할 수 있다. 결론적으로 제안된 방식은 격자 필터가 적용되는 경우 TDL 필터 계수를 가지게 된다. 게다가 새로운 비용함수가 제안되어 더욱 우수한 적응신호처리 분야에서의 발전을 보일 것으로 기대된다.

Abstract In this paper, we propose an equivalent Wiener-Hopf equation. The proposed algorithm can obtain the weight vector of a TDL(tapped-delay-line) filter and the error simultaneously if the inputs are orthogonal to each other. The equivalent Wiener-Hopf equation was analyzed theoretically based on the MMSE(minimum mean square error) method. The results present that the proposed algorithm is equivalent to original Wiener-Hopf equation. In conclusion, our method can find the coefficient of the TDL (tapped-delay-line) filter where a lattice filter is used, and also when the process of Gram-Schmidt orthogonalization is used. Furthermore, a new cost function is suggested which may facilitate research in the adaptive signal processing area.

Key Words : Wiener-Hopf equation, equivalent Wiener-Hopf equation, MMSE, Gram-Schmidt orthogonalization

Ⅰ. Introduction

The Wiener-Hopf equation is a core algorithm for developing new adaptation algorithms[1]-[3]. A lattice filter has generally been utilized for the modeling of linear time-varying systems[4],[5]. However, lattice filters have various constraints in these applications, since the coefficients of the lattice filter are not TDL filter coefficients[6],[7]. Generally, the lattice filter constitutes a prediction stage and a joint process estimation stage. The prediction stage produces

reflection coefficients and the joint process estimation stage produce regression coefficients, respectively[1]-[3]. Therefore, the weakness of the lattice filter is that the coefficient of the TDL filter cannot be obtained directly.

In this paper, we present an advanced Wiener-Hopf equation for producing coefficients of TDL filters directly in a lattice filter (or adaptation filter that uses the Gram-Schmidt orthogonalization process, etc.). In this paper, the new algorithm of the Wiener-Hopf equation will be referred to as an equivalent Wiener-Hopf equation.

This experiment showed similar results for both methods. Firstly, it provides a simple method of finding the coefficient of a TDL (tapped-delay-line) filter in the case where a lattice filter or the process of

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Gram–Schmidt orthogonalization is used. Secondly, a new cost function is defined. This cost function may facilitate research in the adaptive signal processing area.

II. Derivation of equivalent Wiener–Hopf equation

In Fig. 1, the input signal vector $x(n)$, optimal coefficient vector w_{opt} orthogonal input vector $b(n)$ and regression coefficient k are vectors. And Q is an $M \times M$ orthogonal transform matrix.
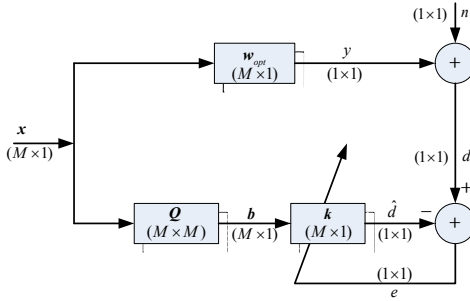

그림 1. 직교입력벡터를 이용한 적응 필터의 구조
Fig. 1. The structure of an adaptive filter using an orthogonal input vector.

The signal $d(n)$ is the desired response, $\hat{d}(n)$ is the estimated desired response and $v(n)$ is the measurement noise. As shown in Fig. 1, the system is excited by an input signal $x(n)$, and we are looking for the estimated values of the M, the unknown tap coefficients.

That is,

$$J = E[e(n)e^*(n)] = E[|e(n)|^2] \tag{1}$$

is a minimum.

The error is defined as following

$$e(n) = d(n) - k^H b(n) \tag{2}$$

where, (H)is a Hermitian transpose. $\hat{d}(n)$ and $b(n)$ are expressed as follows

$$\hat{d}(n) = k^H b(n) = k^H Q x(n) \tag{3}$$

$$b(n) = Q x(n) \tag{4}$$

And we define the desired response $d(n)$ as

$$d(n) = w_{opt}^H x(n) \tag{5}$$

The error $e(n)$ reaches zero if $d(n)$ is equal to $\hat{d}(n)$. That is,

$$d(n) - \hat{d}(n) = [w_{opt}^H - k^H Q]x(n) = 0 \tag{6}$$

If $x(n) \neq 0$, the solution of the above equation becomes

$$w_{opt} = Q^H k \tag{7}$$

Generally, if the orthogonal input vector is used, the regression coefficient k and the coefficient of the TDL filter w_{opt} are related to each other, as presented in (7).

Using (1), the gradient of the mean–squared error can be obtained as following

$$\frac{\partial J}{\partial k} = 2E[(d(n) - k^H b(n))(-b^H(n))] \tag{8}$$

From the above equation, the vector k can be represented as

$$k = E[b(n)b^H(n)]^{-1} E[b(n)d^*(n)] \tag{9}$$

Rewriting the above equation using a matrix form, we obtain.

$$k = R_{bb}^{-1} p_{bd} \tag{10}$$

$$R_{bb} = E[b(n)b^H(n)] \tag{11}$$

$$p_{bd} = E[b(n)d^*(n)] \tag{12}$$

Where R_{bb} is an autocorrelation matrix of $b(n)$, We have to assume that R_{bb} is a nonsingular matrix. In this paper, we proposed the theorem that can determine w_{opt} directly without any calculation of k.

Theorem 1. If the desired response $d(n)$ and the estimated desired response $\hat{d}(n)$ have so similar values, the relationship $R_{bx} w_{opt} = p_{bd}$ is defined.

proof)

Substituting (4) and (7) into (8), we get

$$\frac{\partial J}{\partial \boldsymbol{k}} = 2E[(d(n) - \boldsymbol{w}_{opt}^{H}\boldsymbol{Q}^{-1}\boldsymbol{Q}\boldsymbol{x}(n))(-\boldsymbol{b}^{H}(n))] \tag{13}$$

The optimal coefficient $\boldsymbol{w}_{opt}$ can be obtained from (13) using $\boldsymbol{Q}^{-1}\boldsymbol{Q} = I$.

$$E[\boldsymbol{b}(n)\boldsymbol{x}^{H}(n)]\boldsymbol{w}_{opt} = E[\boldsymbol{b}(n)d^{*}(n)] \tag{14}$$

The equation (14) can be simplified as

$$\boldsymbol{R}_{bx}\boldsymbol{w}_{opt} = \boldsymbol{p}_{bd} \tag{15}$$

where

$$\boldsymbol{R}_{bx} = E[\boldsymbol{b}(n)\boldsymbol{x}^{H}(n)] \tag{16}$$

Q.E.D.

The matrix $\boldsymbol{R}_{bx}$ in (16) is the correlation matrix of the orthogonal input vector $\boldsymbol{b}(n)$ and nonorthogonal input vector $\boldsymbol{x}(n)$, and $\boldsymbol{R}_{bx}$ is nonsingular[1].

Ⅲ. Computer Simulation

To evaluate the performance of the proposed algorithm, the identification of an unknown FIR system is performed. The input signal $x(n)$ to the adaptive filter was obtained from the output of a low pass filter which transfer function is as follows;

$$H(z) = \frac{1}{1 - 0.98z^{-1} + 0.693z^{-2} - 0.22z^{-3} + 0.309z^{-4} - 0.177z^{-5}} \tag{17}$$

where, its input is a white, zero mean and pseudorandom Gaussian noise. The response signal $y(n)$ was obtained from the FIR system, as shown in (18).

$$H(z) = 2.65 - 3.31z^{-1} + 2.24z^{-2} - 0.7z^{-3} \tag{18}$$

The measurement noise signal was an additive white, zero mean and pseudorandom Gaussian and was uncorrelated with the input signal. The computer simulation is designed to determine the similarities of the coefficient of the filters by solving the Wiener-Hopf equation and the equivalent Wiener-Hopf equation. All results presented in this paper are ensemble averages of 100 independent runs with 9000 data. Fig. 2 shows that the curves of the $(\boldsymbol{w}_{xx}^{(i)} - \boldsymbol{w}_{opt}^{(i)}) / \boldsymbol{w}_{opt}^{(i)}$, $(\boldsymbol{w}_{bx}^{(i)} - \boldsymbol{w}_{opt}^{(i)}) / \boldsymbol{w}_{opt}^{(i)}$ and $(\boldsymbol{w}_{bs}^{(i)} - \boldsymbol{w}_{opt}^{(i)}) / \boldsymbol{w}_{opt}^{(i)}$ obtained from the solutions of the Wiener-Hopf and equivalent Wiener-Hopf equation for an SNR of -90dB, -30dB, -10dB and 10dB, respectively. $\boldsymbol{w}_{opt}$ is the i-th optimal coefficient, $\boldsymbol{w}_{xx}$ is the i-th solution of original Wiener-Hopf equation and $\boldsymbol{w}_{bx}^{(i)}$ is the i-th solution of the proposed equivalent Wiener-Hopf equation.
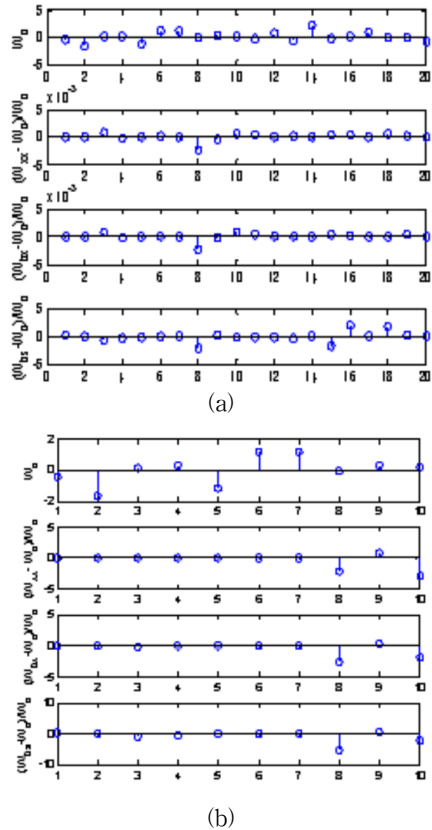

(a)

(b)

그림 2. 상관행렬의 변화에 따른 계수 비교
(a) 미지시스템 차수 : 20, 시스템식별차수 : 20
(b) 미지시스템 차수 : 20, 시스템식별차수 : 10
Fig. 2. Comparison of coefficients according to variation of correlation matrix.
(a) order of unknown system; 20, order of system identification; 20.
(b) order of unknown system;20, order of system identification; 10.

The following simulation is made when the rank is deficient. Fig. 2 illustrates the results obtained by varying the rank of autocorrelation.

In Fig. 2, we set an order of unknown system to 20 and one of computer simulation to 10 under SNR of -30dB, respectively.

Fig. 2 shows the results of the coefficients for various ranks of the correlation matrix. The matrix is full rank. The figure shows that both the existing method and the proposed method have similar performance. Fig. 2 (b) is the case of a rank deficiency.

This figure shows the simulation results of an unknown system of order 20, while the order of the simulated adaptive filter is 10. The figure shows that the proposed method and existing method produce similar results.

Ⅳ. Conclusion

In this paper, an equivalent Wiener-Hopf equation is proposed. The proposed algorithm can determine the coefficient of a TDL filter directly when the adaptive filter has an orthogonal input signal. Also, a theoretical analysis was performed for the MMSE using a solution to the equivalent Wiener-Hopf equation. Furthermore, the proposed algorithm shows similar performance to that of the original Wiener-Hopf equation.

The proposed algorithm has the advantage of allowing the application area of adaptation filters which have an orthogonal input signal to be expanded.

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