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SOME NEW PROPERTIES ON THE q-GENOCCHI NUMBERS AND POLYNOMIALS ASSOCIATED WITH q-BERNSTEIN POLYNOMIALS

Serkan Araci, Dilek Erdal and Dong-Jin Kang

Abstract. The purpose of this study is to obtain some relations between q-Genocchi numbers and q-Bernstein polynomials by using fermionic p-adic q-integral on $\mathbb{Z}_p$.

1. Introduction, Definitions and Notations

Let p be a fixed odd prime number. Throughout this paper we use the following notations. By $\mathbb{Z}_p$ we denote the ring of p-adic rational integers, $\mathbb{Q}$ denotes the field of rational numbers, $\mathbb{Q}_p$ denotes the field of p-adic rational numbers, and $\mathbb{C}_p$ denotes the completion of algebraic closure of $\mathbb{Q}_p$. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{N}^* = \mathbb{N} \cup \{0\}$. The p-adic absolute value is defined by $|x|_p = \frac{1}{p}$. In this paper we assume $|q-1|_p < 1$ as an indeterminate. In [17]-[20], Kim defined the fermionic p-adic q-integral on $\mathbb{Z}_p$ as follows:

$$I_{-q}(g) = \int_{\mathbb{Z}_p} g(x)\, d\mu_{-q}(x) = \lim_{N\to\infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} g(x)(-q)^x \tag{1.1}$$

For $k, n \in \mathbb{N}^*$ and $x \in [0,1]$, Kim's q-Bernstein polynomials are given as follows ([8]):

$$B_{k,n}(x,q) = \binom{n}{k} [x]_q^k [1-x]_{q^{-1}}^{n-k}, \tag{1.2}$$

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where, $[x]_q$ is a q-extension of x defined by (see [1] − [22]) :

$$[x]_q = \frac{1-q^x}{1-q},$$

Note that $\lim_{q\to 1}[x]_q = x$.

For $n \in \mathbb{N}^*$, let us consider the q-Genocchi polynomials as follows:

$$G_{n+1,q}(x) = (n+1)\int_{\mathbb{Z}_p}[x+y]_q^n \, d\mu_{-q}(y). \tag{1.3}$$

In the special case, $x=0$, $G_{n,q}(0) = G_{n,q}$ are called the q-Genocchi numbers.

In this paper we obtain some relations between the q-Bernstein polynomials and the q-Genocchi numbers. From these relations, we derive some interesting identities on the q-Genocchi numbers and polynomials.

2. On the q-Genocchi numbers and q-Bernstein polynomials

By the definition of q-Genocchi polynomials, we easily get

$$\begin{aligned} G_{n+1,q}(x) &= (n+1)\int_{\mathbb{Z}_p}[x+y]_q^n \, d\mu_{-q}(y) \\ &= (n+1)\frac{[2]_q}{(1-q)^n}\sum_{k=0}^{n}\binom{n}{k}q^{kx}\frac{(-1)^k}{1+q^{k+1}} \\ &= (n+1)[2]_q\sum_{l=0}^{\infty}(-1)^l q^l [x+l]_q^n, \end{aligned}$$

and

$$\begin{aligned}\sum_{n=0}^{\infty} G_{n,q}(x)\frac{t^n}{n!} &= [2]_q\, t\sum_{n=0}^{\infty}(-1)^n q^n e^{[x+n]_q t}\\ &= [2]_q\, t\sum_{n=0}^{\infty}(-1)^n q^n e^{\left([x]_q+q^x[n]_q\right)t}\\ &= \left(\frac{e^{[x]_q t}}{q^x}\right)\left([2]_q\, q^x t\sum_{n=0}^{\infty}(-1)^n q^n e^{(q^x t)[n]_q}\right)\\ &= \left(\sum_{n=0}^{\infty}[x]_q^n\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty}q^{(n-1)x}G_{n,q}\frac{t^n}{n!}\right)\\ &= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\binom{n}{k}q^{(k-1)x}[x]_q^{n-k}G_{k,q}\right)\frac{t^n}{n!}\end{aligned}$$

Therefore we obtain the following theorem.

Theorem 1. *For $n\in\mathbb{N}^*$, we have*

$$\begin{aligned}G_{n+1,q}(x) &= (n+1)[2]_q\sum_{l=0}^{\infty}(-1)^l q^l [x+l]_q^n\\ &= \sum_{k=0}^{n+1}\binom{n+1}{k}q^{(k-1)x}[x]_q^{n+1-k}G_{k,q}\\ &= q^{-x}\left([x]_q+q^x G_q\right)^{n+1},\end{aligned}$$

with usual convention about replacing $(G_q)^n$ by $G_{n,q}$.

By Theorem 1, we have

$$\sum_{n=0}^{\infty} G_{n,q}(x)\frac{t^n}{n!} = [2]_q\, t\sum_{n=0}^{\infty}(-1)^n q^n e^{[x+n]_q t}. \tag{2.1}$$

By (2.1), we get

$$\begin{aligned} G_{n+1,q^{-1}}(1-x) &= (n+1)\int_{\mathbb{Z}_p}[1-x+y]_{q^{-1}}^n\,d\mu_{-q^{-1}}(y) \\ &= (n+1)\frac{1+q^{-1}}{(1-q^{-1})^n}\sum_{k=0}^{n}\binom{n}{k}q^{-k(1-x)}\frac{(-1)^k}{1+q^{-k-1}} \\ &= (n+1)(-1)^n q^n\frac{[2]_q}{(1-q)^n}\sum_{k=0}^{n}\binom{n}{k}q^{kx}\frac{(-1)^k}{1+q^{k+1}} \\ &= (-1)^n q^n G_{n+1,q}(x). \end{aligned}$$

Therefore, we obtain the following theorem.

Theorem 2. *For $n\in\mathbb{N}^*$, we have*

$$G_{n,q^{-1}}(1-x) = (-1)^{n-1}q^{n-1}G_{n,q}(x).$$

From (2.1) we note that

$$q\sum_{n=0}^{\infty}G_{n,q}(1)\frac{t^n}{n!}+\sum_{n=0}^{\infty}G_{n,q}\frac{t^n}{n!} = [2]_q\,t. \tag{2.2}$$

By (2.2), we get the following recurrence formula:

$$G_{0,q}=0,\ qG_{n,q}(1)+G_{n,q} = \begin{cases}[2]_q, & n=1\\ 0, & n>1\end{cases} \tag{2.3}$$

From (2.3) and Theorem 1, we have the following theorem.

Theorem 3. *For $n\in\mathbb{N}^*$, we have*

$$G_{0,q}=0,\ \text{and } q\,(qG_q+1)^n+G_{n,q} = \begin{cases}[2]_q, & n=1\\ 0, & n>1\end{cases}$$

with usual convention about replacing $(G_q)^n$ by $G_{n,q}$.

For $n\in\mathbb{N}$, by Theorem 3, we note that

$$\begin{aligned} q^{-2}G_{n,q}(2) &= q^{-2}\sum_{l=0}^{n}\binom{n}{l}q^l(qG_q+1)^l \\ &= nq^{-2}\left([2]_q-G_{1,q}\right)+q^{-2}\sum_{l=0}^{n}\binom{n}{l}q^lG_{l,q}(1) \\ &= nq^{-1}-q^{-3}\sum_{l=0}^{n}\binom{n}{l}q^lG_{l,q} \\ &= nq^{-1}+q^{-4}G_{n,q}\ \text{, if } n>1 \end{aligned}$$

Therefore, we have the following theorem.

Theorem 4. *For $n \in \mathbb{N}$, we have*

$$G_{n+1,q}(2) = q^{-2} G_{n+1,q} + (n+1) q.$$

From Theorem 2, we see that

$$\begin{aligned} (2.4) \qquad & (n+1) \int_{\mathbb{Z}_p} [1-x]_{q^{-1}}^{n} \, d\mu_{-q}(x) \\ = \; & (-q)^n (n+1) \int_{\mathbb{Z}_p} [x-1]_q^n \, d\mu_{-q}(x) \\ = \; & (-1)^n q^n G_{n+1,q}(-1) = G_{n+1,q^{-1}}(2). \end{aligned}$$

Therefore, we have the following theorem.

Theorem 5. *For $n \in \mathbb{N}^*$, we have*

$$(n+1) \int_{\mathbb{Z}_p} [1-x]_{q^{-1}}^{n} \, d\mu_{-q}(x) = G_{n+1,q^{-1}}(2).$$

Let $n \in \mathbb{N}$. By Theorem 4 and Theorem 5, we get

$$\begin{aligned} (2.5) \qquad & (n+1) \int_{\mathbb{Z}_p} [1-x]_{q^{-1}}^{n} \, d\mu_{-q}(x) \\ = \; & q^2 G_{n+1,q^{-1}} + \frac{(n+1)}{q} \end{aligned}$$

For (2.5), we obtain

$$(n+1) \int_{\mathbb{Z}_p} [1-x]_{q^{-1}}^{n} \, d\mu_{-q}(x) = q^2 G_{n+1,q^{-1}} + \frac{(n+1)}{q}.$$

Corollary 1. *For $n \in \mathbb{N}^*$, we have*

$$\int_{\mathbb{Z}_p} [1-x]_{q^{-1}}^{n} \, d\mu_{-q}(x) = \frac{q^2}{n+1} G_{n+1,q^{-1}} + \frac{1}{q}.$$

3. New identities on the q-Genocchi numbers and q-Genocchi polynomials

In this section, we give some interesting relationship between the q-Genocchi numbers and q-Bernstein polynomials.

For $x \in \mathbb{Z}_p$, the p-adic analogues of q-Bernstein polynomials are given by

$$(3.1) \qquad B_{k,n}(x,q) = \binom{n}{k} [x]_q^k [1-x]_{q^{-1}}^{n-k}, \text{ where } n,k \in \mathbb{N}^*.$$

By (3.1), Kim [10] gave the symmetry of q-Bernstein polynomials as follows:

$$B_{k,n}(x,q) = B_{n-k,n}\left(1-x,q^{-1}\right), \tag{3.2}$$

Thus, from Corollary 6, (3.1) and (3.2), we see that

$$\begin{aligned}\int_{\mathbb{Z}_p} B_{k,n}(x,q)\,d\mu_{-q}(x) &= \int_{\mathbb{Z}_p} B_{n-k,n}\left(1-x,q^{-1}\right)d\mu_{-q}(x)\\ &= \binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}\int_{\mathbb{Z}_p}[1-x]_{q^{-1}}^{n-l}\,d\mu_{-q}(x)\\ &= \binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}\left(\frac{q^2}{n-l+1}G_{n-l+1,q^{-1}}+\frac{1}{q}\right).\end{aligned}$$

For $n,k\in\mathbb{N}^*$ with $n>k$, we obtain

$$\begin{aligned}&\int_{\mathbb{Z}_p} B_{k,n}(x,q)\,d\mu_{-q}(x)\\ (3.3)\quad &= \binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}\left(\frac{q^2}{n-l+1}G_{n-l+1,q^{-1}}+\frac{1}{q}\right)\\ &= \begin{cases}\frac{q^2}{n+1}G_{n+1,q^{-1}}+\frac{1}{q}, & \text{if } k=0,\\ \binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}\left(\frac{q^2}{n-l+1}G_{n-l+1,q^{-1}}+\frac{1}{q}\right), & \text{if } k>0.\end{cases}\end{aligned}$$

Let us take the fermionic q-integral on $\mathbb{Z}_p$ for the q-Bernstein polynomials of degree n as follows:

$$\begin{aligned}(3.4)\int_{\mathbb{Z}_p} B_{k,n}(x,q)\,d\mu_{-q}(x) &= \binom{n}{k}\int_{\mathbb{Z}_p}[x]_q^k[1-x]_{q^{-1}}^{n-k}\,d\mu_{-q}(x)\\ &= \binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\frac{G_{l+k+1,q}}{l+k+1}.\end{aligned}$$

Therefore, by (3.3) and (3.4), we obtain the following theorem.

Theorem 6. *For $n,k\in\mathbb{N}^*$ with $n>k$. we have*

$$\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\frac{G_{l+k+1,q}}{l+k+1} = \begin{cases}\frac{q^2}{n+1}G_{n+1,q^{-1}}+\frac{1}{q}, & \text{if } k=0,\\ \sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}\left(\frac{q^2}{n-l+1}G_{n-l+1,q^{-1}}+\frac{1}{q}\right), & \text{if } k>0.\end{cases}$$

Let $n_1, n_2, k \in \mathbb{N}^*$ with $n_1 + n_2 > 2k$.Then we get

$$\begin{aligned}
&\int_{\mathbb{Z}_p} B_{k,n_1}(x,q)\, B_{k,n_2}(x,q)\, d\mu_{-q}(x) \\
=& \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k+l}\int_{\mathbb{Z}_p}[1-x]_{q^{-1}}^{n_1+n_2-l}\, d\mu_{-q}(x) \\
=& \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k+l}\left(\frac{q^2}{n_1+n_2-l+1}G_{n_1+n_2-l+1,q^{-1}}+\frac{1}{q}\right) \\
=& \begin{cases} \frac{q^2}{n_1+n_2+1}G_{n_1+n_2+1,q^{-1}}+\frac{1}{q}, & \text{if } k=0, \\ \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k+l}\left(\frac{q^2}{n_1+n_2-l+1}G_{n_1+n_2-l+1,q^{-1}}+\frac{1}{q}\right), & \text{if } k\neq 0. \end{cases}
\end{aligned}$$

Therefore, we obtain the following theorem.

Theorem 7. *For $n_1, n_2, k \in \mathbb{N}^*$ with $n_1 + n_2 > 2k$, we have*

$$\begin{aligned}
&\int_{\mathbb{Z}_p} B_{k,n_1}(x,q)\, B_{k,n_2}(x,q)\, d\mu_{-q}(x) \\
=& \begin{cases} \frac{q^2}{n_1+n_2+1}G_{n_1+n_2+1,q^{-1}}+\frac{1}{q}, & \text{if } k=0, \\ \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k+l}\left(\frac{q^2}{n_1+n_2-l+1}G_{n_1+n_2-l+1,q^{-1}}+\frac{1}{q}\right), & \text{if } k\neq 0. \end{cases}
\end{aligned}$$

From the binomial theorem, we can derive the following equation.

$$\begin{aligned}
&\int_{\mathbb{Z}_p} B_{k,n_1}(x,q)\, B_{k,n_2}(x,q)\, d\mu_{-q}(x) \\
(3.5) =& \prod_{i=1}^{2}\binom{n_i}{k}\sum_{l=0}^{n_1+n_2-2k}\binom{n_1+n_2-2k}{l}(-1)^l\int_{\mathbb{Z}_p}[x]_q^{2k+l}\, d\mu_{-q}(x) \\
=& \prod_{i=1}^{2}\binom{n_i}{k}\sum_{l=0}^{n_1+n_2-2k}\binom{n_1+n_2-2k}{l}(-1)^l\frac{G_{l+2k+1,q}}{l+2k+1}.
\end{aligned}$$

Thus, for theorem 8 and (3.6) , we obtain the following corollary.

Corollary 2. *Let $n_1, n_2, k \in \mathbb{N}^*$ with $n_1 + n_2 > 2k$, we have*

$$\begin{aligned}
&\sum_{l=0}^{n_1+n_2-2k}\binom{n_1+n_2-2k}{l}(-1)^l\frac{G_{l+2k+1,q}}{l+2k+1} \\
=& \begin{cases} \frac{q^2}{n_1+n_2+1}G_{n_1+n_2+1,q^{-1}}+\frac{1}{q}, & \text{if } k=0 \\ \sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k+l}\left(\frac{q^2}{n_1+n_2-l+1}G_{n_1+n_2-l+1,q^{-1}}+\frac{1}{q}\right), & \text{if } k\neq 0. \end{cases}
\end{aligned}$$

For $x \in \mathbb{Z}_p$ and $s \in \mathbb{N}$ with $s \geq 2$, let $n_1, n_2, ..., n_s, k \in \mathbb{N}^*$ with $\sum_{l=1}^{s} n_l > sk$. Then we take the fermionic p-adic q-integral on $\mathbb{Z}_p$ for the q-Bernstein polynomials of degree n as follows:

$$
\begin{aligned}
& \int_{\mathbb{Z}_p} \underbrace{B_{k,n_1}(x,q) B_{k,n_2}(x,q) ... B_{k,n_s}(x,q)}_{s-times} d\mu_{-q}(x) \\
= & \prod_{i=1}^{s} \binom{n_i}{k} \int_{\mathbb{Z}_p} [x]_q^{sk} [1-x]_{q^{-1}}^{n_1+n_2+\ldots+n_s-sk} d\mu_{-q}(x) \\
= & \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{l+sk} \int_{\mathbb{Z}_p} [1-x]_{q^{-1}}^{n_1+n_2+\ldots+n_s-sk} d\mu_{-q}(x) \\
= & \begin{cases} \frac{q^2}{n_1+\ldots n_s+1} G_{n_1+\ldots+n_s+1,q^{-1}} + \frac{1}{q}, & \text{if } k=0, \\ \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} \left(\frac{q^2}{n_1+\ldots+n_s-l+1} G_{n_1+\ldots+n_s-l+1,q^{-1}} + \frac{1}{q} \right), & \text{if } k \neq 0. \end{cases}
\end{aligned}
$$

Therefore, we obtain the following theorem.

Theorem 8. *For $s \in \mathbb{N}$ with $s \geq 2$, let $n_1, n_2, ..., n_s, k \in \mathbb{N}^*$ with $\sum_{l=1}^{s} n_l > sk$. Then we have*

$$
\begin{aligned}
& \int_{\mathbb{Z}_p} \prod_{i=1}^{s} B_{k,n_i}(x) d\mu_{-q}(x) \\
= & \begin{cases} \frac{q^2}{n_1+\ldots n_s+1} G_{n_1+\ldots n_s+1,q^{-1}} + \frac{1}{q}, & \textit{if } k=0, \\ \prod_{i=1}^{s} \binom{n_i}{s} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk+l} \left(\frac{q^2}{n_1+\ldots+n_s-l+1} G_{n_1+\ldots+n_s-l+1,q^{-1}} + \frac{1}{q} \right), & \textit{if } k \neq 0. \end{cases}
\end{aligned}
$$

From the definition of q-Bernstein polynomials and the binomial theorem, we easily get

$$
\begin{aligned}
& \int_{\mathbb{Z}_p} \underbrace{B_{k,n_1}(x,q) B_{k,n_2}(x,q) ... B_{k,n_s}(x,q)}_{s-times} d\mu_{-q}(x) \\
= & \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{n_1+\ldots+n_s-sk} \binom{\sum_{d=1}^{s}(n_d-k)}{l} (-1)^l \int_{\mathbb{Z}_p} [x]_q^{sk+l} d\mu_{-q}(x) \\
(3.6) = & \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=1}^{n_1+\ldots+n_s-sk} \binom{\sum_{d=1}^{s}(n_d-k)}{l} (-1)^l \frac{G_{l+sk+1,q}}{l+sk+1}.
\end{aligned}
$$

Therefore, from (3.6) and Theorem 10. we have the following corollary.

Corollary 3. *For $s \in \mathbb{N}$ with $s \geq 2$, let $n_1, n_2, ..., n_s, k \in \mathbb{N}^*$ with $\sum_{l=1}^{s} n_l > sk$. we have*

$$
\begin{aligned}
& \sum_{l=1}^{n_1+\ldots+n_s-sk} \binom{\sum_{d=1}^{s}(n_d-k)}{l} (-1)^l \frac{G_{l+sk+1,q}}{l+sk+1} \\
= & \begin{cases} \frac{q^2}{n_1+\ldots+n_s+1} G_{n_1+\ldots+n_s+1,q^{-1}} + \frac{1}{q}, & \textit{if } k=0, \\ \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk+l} \left(\frac{q^2}{n_1+\ldots n_s-l+1} G_{n_1+\ldots+n_s-l+1,q^{-1}} + \frac{1}{q} \right), & \textit{if } k \neq 0. \end{cases}
\end{aligned}
$$

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Serkan Aracı
Department of Mathematics, University of Gaziantep, Faculty of Science and Arts,
Gaziantep 27310, Turkey.
E-mail: mtsrkn@hotmail.com

Dilek Erdal
Department of Mathematics, University of Gaziantep, Faculty of Science and Arts,
Gaziantep 27310, Turkey.
E-mail: dilekvecii@mynet.com

Dong-Jin Kang
Information Technology Service, Kyungpook National University,
Taegu 702-701, Korea.
E-mail: djkang@knu.ac.kr