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SEVERAL INTEGRAL REPRESENTATIONS INVOLVING TRIPLE HYPERGEOMETRIC FUNCTIONS

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Abstract. A (presumably) new class of generalized triple hypergeometric functions is presented. We also give integral representations of Laplace type for certain special cases of the new class of functions.

1. Introduction and Preliminaries

Investigation of multiple hypergeometric functions is essentially motivated by the fact that the solutions of many applied problems, for example, the thermal conductivity and dynamics, electromagnetic oscillation and aerodynamics, quantum mechanics and potential theory, are obtainable with the help of such hypergeometric (higher and special or transcendent) functions [1, 4, 5, 6]. Functions of such kind are often referred to as special functions of mathematical physics. They mainly appear in the solution of partial differential equations which are dealt with harmonic analysis method (see [2]). In view of various applications, it is interesting in itself and seems to be very important to make a continuous research of multiple hypergeometric functions. For instance, in [10], a comprehensive list of hypergeometric functions of three variables as many as 205 is recorded, together with their regions of convergence. In the course of investigation of properties of several classes of triple hypergeometric functions of Exton type $X_1, \ldots, X_{20}$ [3] and Srivastava's hypergeometric functions H_A, H_B, H_C, and $F^{(3)}$ [7, 8, 10], we have encountered a number of generalized triple hypergeometric functions of the fourth order whose series representations involve such products as $(a_j)_{2m+2n+p}$, $(g_j)_{m+n}$ and $(e_j)_{m+n+p}$, and various combinations of

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indices. This investigation enables us to present the following (presumably) new class of generalizations of the Exton type functions:

$$\begin{aligned}&\mathcal{G}^{(3)}\left[x,y,z\right]\\&=\mathcal{G}^{(3)}\left[\begin{array}{l}(a);\left(a'\right);\left(a''\right):\ \ (b);\left(b'\right);\left(b''\right):\ \ (c);\left(c'\right);\left(c''\right);\\(e);\left(e'\right);\left(e''\right):\ \ (g);\left(g'\right);\left(g''\right):\ \ (h);\left(h'\right);\left(h''\right);\end{array}x,y,z\right]\\&=\sum_{m,n,p=0}^{\infty}\Lambda\left(m,n,p\right)\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!},\end{aligned}\tag{1.1}$$

where

$$\Lambda\left(m,n,p\right)=\frac{\prod\limits_{j=1}^{A}\left(a_j\right)_{2m+2n+p}\prod\limits_{j=1}^{A'}\left(a_j'\right)_{2m+n}\prod\limits_{j=1}^{A''}\left(a_j''\right)_{n+2p}}{\prod\limits_{j=1}^{E}\left(e_j\right)_{m+n+p}\prod\limits_{j=1}^{E'}\left(e_j'\right)_{2m+n}\prod\limits_{j=1}^{E''}\left(e_j''\right)_{n+2p}}$$

$$\cdot\frac{\prod\limits_{j=1}^{B}\left(b_j\right)_{m+n}\prod\limits_{j=1}^{B'}\left(b_j'\right)_{n+p}\prod\limits_{j=1}^{B''}\left(b_j''\right)_{m+p}}{\prod\limits_{j=1}^{G}\left(g_j\right)_{m+n}\prod\limits_{j=1}^{G'}\left(g_j'\right)_{n+p}\prod\limits_{j=1}^{G''}\left(g_j''\right)_{m+p}}\cdot\frac{\prod\limits_{j=1}^{C}\left(c_j\right)_{m}\prod\limits_{j=1}^{C'}\left(c_j'\right)_{n}\prod\limits_{j=1}^{C''}\left(c_j''\right)_{p}}{\prod\limits_{j=1}^{H}\left(h_j\right)_{m}\prod\limits_{j=1}^{H'}\left(h_j'\right)_{n}\prod\limits_{j=1}^{H''}\left(h_j''\right)_{p}},$$

where (a) abbreviates the array of A parameters $a_1,\ \ldots\ ,a_A$, with similar interpretations for $b,b',\ b''$, and so on. We present various representations for triple hypergeometric functions of fourth order of special cases of (1.1):

$$\begin{aligned}&\mathcal{G}_1^{(3)}\left[\begin{array}{c}a_1,a_2,c_1'',c_2''\\g_1',g_2',h_1,h_2,h_3,h_1',h_1''\end{array}x,y,z\right]\\&\quad=\sum_{i,j,k=0}^{\infty}\frac{\left(a_1\right)_{2i+2j+k}\left(a_2\right)_{2i+2j+k}\left(c_1''\right)_k\left(c_2''\right)_k}{\left(g_1'\right)_{j+k}\left(g_2'\right)_{j+k}\left(h_1\right)_i\left(h_2\right)_i\left(h_3\right)_i\left(h_1'\right)_j\left(h_1''\right)_k i!j!k!}x^iy^jz^k;\end{aligned}\tag{1.2}$$

$$\begin{aligned}&\mathcal{G}_2^{(3)}\left[\begin{array}{c}a_1,a_2,c_1'',c_2''\\h_1,h_2,h_3,h_1',h_2',h_3',h_1'',h_2'',h_3''\end{array}x,y,z\right]\\&=\sum_{i,j,k=0}^{\infty}\frac{\left(a_1\right)_{2i+2j+k}\left(a_2\right)_{2i+2j+k}\left(c_1''\right)_k\left(c_2''\right)_k}{\left(h_1\right)_i\left(h_2\right)_i\left(h_3\right)_i\left(h_1'\right)_j\left(h_2'\right)_j\left(h_3'\right)_j\left(h_1''\right)_k\left(h_2''\right)_k\left(h_3''\right)_k i!j!k!}x^iy^jz^k;\end{aligned}\tag{1.3}$$

$$\mathcal{G}_3^{(3)}\left[\begin{array}{c} a_1, a_2, b_1^{'}, b_2^{'} \\ g_1, g_2, h_1, h_1^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{(a_1)_{2i+2j+k}\,(a_2)_{2i+2j+k}\left(b_1^{'}\right)_{j+k}\left(b_2^{'}\right)_{j+k}}{(g_1)_{i+j}\,(g_2)_{i+j}\,(h_1)_i\left(h_1^{'}\right)_j\left(h_1^{''}\right)_k\left(h_2^{''}\right)_k\left(h_3^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.4}$$

$$\mathcal{G}_4^{(3)}\left[\begin{array}{c} a_1, a_2, b_1^{'}, b_2^{'} \\ h_1, h_2, h_3, h_1^{'}, h_2^{'}, h_3^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{(a_1)_{2i+2j+k}\,(a_2)_{2i+2j+k}\left(b_1^{'}\right)_{j+k}\left(b_2^{'}\right)_{j+k}}{(h_1)_i\,(h_2)_i\,(h_3)_i\left(h_1^{'}\right)_j\left(h_2^{'}\right)_j\left(h_3^{'}\right)_j\left(h_1^{''}\right)_k\left(h_2^{''}\right)_k\left(h_3^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.5}$$

$$\mathcal{G}_5^{(3)}\left[\begin{array}{c} a_1, a_2, c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''} \\ e_1, e_2, h_1, h_1^{'}, h_1^{''} \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{(a_1)_{2i+2j+k}\,(a_2)_{2i+2j+k}\left(c_1^{'}\right)_j\left(c_2^{'}\right)_j\left(c_1^{''}\right)_k\left(c_2^{''}\right)_k}{(e_1)_{i+j+k}\,(e_2)_{i+j+k}\,(h_1)_i\left(h_1^{'}\right)_j\left(h_1^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.6}$$

$$\mathcal{G}_6^{(3)}\left[\begin{array}{c} a_1, a_2, c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''} \\ g_1, g_2, h_1, h_1^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{(a_1)_{2i+2j+k}\,(a_2)_{2i+2j+k}\left(c_1^{'}\right)_j\left(c_2^{'}\right)_j\left(c_1^{''}\right)_k\left(c_2^{''}\right)_k}{(g_1)_{i+j}\,(g_2)_{i+j}\,(h_1)_i\left(h_1^{'}\right)_j\left(h_1^{''}\right)_k\left(h_2^{''}\right)_k\left(h_3^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.7}$$

$$\mathcal{G}_7^{(3)}\left[\begin{array}{c} a_1, a_2, c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''} \\ g_1^{'}, g_2^{'}, h_1, h_2, h_3, h_1^{'}, h_2^{''} \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{(a_1)_{2i+2j+k}\,(a_2)_{2i+2j+k}\left(c_1^{'}\right)_j\left(c_2^{'}\right)_j\left(c_1^{''}\right)_k\left(c_2^{''}\right)_k}{\left(g_1^{'}\right)_{j+k}\left(g_2^{'}\right)_{j+k}(h_1)_i\,(h_2)_i\,(h_3)_i\left(h_1^{'}\right)_j\left(h_1^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.8}$$

$$\mathcal{G}_8^{(3)}\left[\begin{array}{c} a_1, a_2, c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''} \\ h_1, h_2, h_3, h_1^{'}, h_2^{'}, h_3^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{(a_1)_{2i+j+k}\,(a_2)_{2i+j+k}\left(c_1^{'}\right)_j\left(c_2^{'}\right)_j\left(c_1^{''}\right)_k\left(c_2^{''}\right)_k}{(h_1)_i\,(h_2)_i\,(h_3)_i\left(h_1^{'}\right)_j\left(h_2^{'}\right)_j\left(h_3^{'}\right)_j\left(h_1^{''}\right)_k\left(h_2^{''}\right)_k\left(h_3^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.9}$$

$$\mathcal{G}_9^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, a_1^{''}, a_2^{''} \\ e_1, e_2, h_1, h_1^{'}, h_1^{''} \end{array} x, y, z\right] = \sum_{i,j,k=0}^{\infty} \frac{\left(a_1^{'}\right)_{2i+j}\left(a_2^{'}\right)_{2i+j}\left(a_1^{''}\right)_{j+2k}\left(a_2^{''}\right)_{j+2k}}{(e_1)_{i+j+k}(e_2)_{i+j+k}(h_1)_i\left(h_1^{'}\right)_j\left(h_1^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.10}$$

$$\mathcal{G}_{10}^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, a_1^{''}, a_2^{''} \\ g_1, g_2, h_1, h_1^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right] = \sum_{i,j,k=0}^{\infty} \frac{\left(a_1^{'}\right)_{2i+j}\left(a_2^{'}\right)_{2i+j}\left(a_1^{''}\right)_{j+2k}\left(a_2^{''}\right)_{j+2k}}{(g_1)_{i+j}(g_2)_{i+j}(h_1)_i\left(h_1^{'}\right)_j\left(h_1^{''}\right)_k\left(h_2^{''}\right)_k\left(h_3^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.11}$$

$$\mathcal{G}_{11}^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, a_1^{''}, a_2^{''} \\ g_1^{''}, g_2^{''}, h_1, h_1^{'}, h_2^{'}, h_3^{'}, h_1^{''} \end{array} x, y, z\right] = \sum_{i,j,k=0}^{\infty} \frac{\left(a_1^{'}\right)_{2i+j}\left(a_2^{'}\right)_{2i+j}\left(a_1^{''}\right)_{j+2k}\left(a_2^{''}\right)_{j+2k}}{\left(g_1^{''}\right)_{i+k}\left(g_2^{''}\right)_{i+k}(h_1)_i\left(h_1^{'}\right)_j\left(h_2^{'}\right)_j\left(h_3^{'}\right)_j\left(h_1^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.12}$$

$$\mathcal{G}_{12}^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, a_1^{''}, a_2^{''} \\ h_1, h_2, h_3, h_1^{'}, h_2^{'}, h_3^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right] = \sum_{i,j,k=0}^{\infty} \frac{\left(a_1^{'}\right)_{2i+j}\left(a_2^{'}\right)_{2i+j}\left(a_1^{''}\right)_{j+2k}\left(a_2^{''}\right)_{j+2k}}{(h_1)_i(h_2)_i(h_3)_i\left(h_1^{'}\right)_j\left(h_2^{'}\right)_j\left(h_3^{'}\right)_j\left(h_1^{''}\right)_k\left(h_2^{''}\right)_k\left(h_3^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.13}$$

$$\mathcal{G}_{13}^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, b_1^{'}, b_2^{'}, c_1^{''}, c_2^{''} \\ e_1, e_2, h_1, h_1^{'}, h_1^{''} \end{array} x, y, z\right] = \sum_{i,j,k=0}^{\infty} \frac{\left(a_1^{'}\right)_{2i+j}\left(a_2^{'}\right)_{2i+j}\left(b_1^{'}\right)_{j+k}\left(b_2^{'}\right)_{j+k}\left(c_1^{''}\right)_k\left(c_2^{''}\right)_k}{(e_1)_{i+j+k}(e_2)_{i+j+k}(h_1)_i\left(h_1^{'}\right)_j\left(h_1^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.14}$$

$$\mathcal{G}_{14}^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, b_1^{'}, b_2^{'}, c_1^{''}, c_2^{''} \\ g_1, g_2, h_1, h_1^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right] = \sum_{i,j,k=0}^{\infty} \frac{\left(a_1^{'}\right)_{2i+j}\left(a_2^{'}\right)_{2i+j}\left(b_1^{'}\right)_{j+k}\left(b_2^{'}\right)_{j+k}\left(c_1^{''}\right)_k\left(c_2^{''}\right)_k}{(g_1)_{i+j}(g_2)_{i+j}(h_1)_i\left(h_1^{'}\right)_j\left(h_1^{''}\right)_k\left(h_2^{''}\right)_k\left(h_3^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{1.15}$$

$$\mathcal{G}_{15}^{(3)}\left[\begin{array}{c} a_1', a_2', b_1', b_2', c_1'', c_2'' \\ g_1', g_2', h_1, h_2, h_3, h_1', h_1'' \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{\left(a_1'\right)_{2i+j}\left(a_2'\right)_{2i+j}\left(b_1'\right)_{j+k}\left(b_2'\right)_{j+k}\left(c_1''\right)_k\left(c_2''\right)_k}{\left(g_1'\right)_{j+k}\left(g_2'\right)_{j+k}\left(h_1\right)_i\left(h_2\right)_i\left(h_3\right)_i\left(h_1'\right)_j\left(h_1''\right)_k i!j!k!} x^i y^j z^k; \tag{1.16}$$

$$\mathcal{G}_{16}^{(3)}\left[\begin{array}{c} a_1', a_2', b_1', b_2', c_1'', c_2'' \\ g_1'', g_2'', h_1, h_1', h_2', h_3', h_1'' \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{\left(a_1'\right)_{2i+j}\left(a_2'\right)_{2i+j}\left(b_1'\right)_{j+k}\left(b_2'\right)_{j+k}\left(c_1''\right)_k\left(c_2''\right)_k}{\left(g_1''\right)_{i+k}\left(g_2''\right)_{i+k}\left(h_1\right)_i\left(h_1'\right)_j\left(h_2'\right)_j\left(h_3'\right)_j\left(h_1''\right)_k i!j!k!} x^i y^j z^k; \tag{1.17}$$

$$\mathcal{G}_{17}^{(3)}\left[\begin{array}{c} a_1', a_2', b_1', b_2', c_1'', c_2'' \\ h_1, h_2, h_3, h_1', h_2', h_3', h_1'', h_2'', h_3'' \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{\left(a_1'\right)_{2i+j}\left(a_2'\right)_{2i+j}\left(b_1'\right)_{j+k}\left(b_2'\right)_{j+k}\left(c_1''\right)_k\left(c_2''\right)_k}{\left(h_1\right)_i\left(h_2\right)_i\left(h_3\right)_i\left(h_1'\right)_j\left(h_2'\right)_j\left(h_3'\right)_j\left(h_1''\right)_k\left(h_2''\right)_k\left(h_3''\right)_k i!j!k!} x^i y^j z^k; \tag{1.18}$$

$$\mathcal{G}_{18}^{(3)}\left[\begin{array}{c} a_1', a_2', c_1', c_2', c_1'', c_2'', c_3'', c_4'' \\ e_1, e_2, h_1, h_1', h_1'' \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{\left(a_1'\right)_{2i+j}\left(a_2'\right)_{2i+j}\left(c_1'\right)_j\left(c_2'\right)_j\left(c_1''\right)_k\left(c_2''\right)_k\left(c_3''\right)_k\left(c_4''\right)_k}{\left(e_1\right)_{i+j+k}\left(e_2\right)_{i+j+k}\left(h_1\right)_i\left(h_1'\right)_j\left(h_1''\right)_k i!j!k!} x^i y^j z^k; \tag{1.19}$$

$$\mathcal{G}_{19}^{(3)}\left[\begin{array}{c} a_1', a_2', c_1', c_2', c_1'', c_2'', c_3'', c_4'' \\ g_1', g_2', h_1, h_2, h_3, h_1', h_1'' \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{\left(a_1'\right)_{2i+j}\left(a_2'\right)_{2i+j}\left(c_1'\right)_j\left(c_2'\right)_j\left(c_1''\right)_k\left(c_2''\right)_k\left(c_3''\right)_k\left(c_4''\right)_k}{\left(g_1'\right)_{j+k}\left(g_2'\right)_{j+k}\left(h_1\right)_i\left(h_2\right)_i\left(h_3\right)_i\left(h_1'\right)_j\left(h_1''\right)_k i!j!k!} x^i y^j z^k; \tag{1.20}$$

$$\mathcal{G}_{20}^{(3)}\left[\begin{array}{c} a_1', a_2', c_1', c_2', c_1'', c_2'', c_3'', c_4'' \\ g_1'', g_2'', h_1, h_1', h_2', h_3', h_1'' \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{\left(a_1'\right)_{2i+j}\left(a_2'\right)_{2i+j}\left(c_1'\right)_j\left(c_2'\right)_j\left(c_1''\right)_k\left(c_2''\right)_k\left(c_3''\right)_k\left(c_4''\right)_k}{\left(g_1''\right)_{i+k}\left(g_2''\right)_{i+k}\left(h_1\right)_i\left(h_1'\right)_j\left(h_2'\right)_j\left(h_3'\right)_j\left(h_1''\right)_k i!j!k!} x^i y^j z^k, \tag{1.21}$$

where $(\lambda)_n$ is the Pochhammer symbol defined (for $\lambda \in \mathbb{C}$) by (see [9]):

$$(\lambda)_n := \begin{cases} 1 & (n = 0) \\ \lambda(\lambda+1)\ldots\lambda(\lambda+n-1) & (n \in \mathbb{N} := \{1, 2, 3, ...\}) \end{cases}$$
$$= \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} \qquad (\lambda \in \mathbb{C}\backslash\mathbb{Z}_0^-),$$

$\Gamma(z)$ being the well-known Gamma function, $\mathbb{C}$ and $\mathbb{Z}_0^-$ denoting the set of complex numbers and the set of nonpositive integers, respectively.

2. Integral Representations of Laplace Type

In this section by employing the Laplace integral representation formula [2]

$$(a)_m = \frac{1}{\Gamma(a)} \int_0^\infty e^{-t} t^{a+m-1} dt \qquad (\Re(a) > 0;\ m \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}) \tag{2.1}$$

to the series representations of aforementioned functions $\mathcal{G}_1^{(3)}, \ldots, \mathcal{G}_{20}^{(3)}$, we present certain integral representations of Laplace type. The regions of convergence of the series (1.2) to (1.21) may be investigated by means of Horn's theorem for triple series (see [10, Chapter 5, pp. 128–134]). For instance, the region of convergence for hypergeometric function $\mathcal{G}_2^{(3)}$ is given as follows:

$$\left\{(x,\,y,\,z)\ :\ |x| < \frac{1}{16},\, |y| < \frac{1}{16},\ |z| < 1,\ r := |x|,\ s := |y|,\ t := |z|,\ \sqrt[4]{r} + \sqrt[4]{s} + \sqrt{t} = 1\right\}.$$

By means of formula (2.1) we find the following integral representations of Laplace type:

(2.2)

$$\mathcal{G}_1^{(3)}\left[\begin{array}{c} a_1, a_2, c_1^{''}, c_2^{''} \\ g_1^{'}, g_2^{'}, h_1, h_2, h_3, h_1^{'}, h_1^{''} \end{array} x, y, z\right] = \frac{1}{\Gamma(a_1)\Gamma(a_2)}\int_0^\infty\int_0^\infty e^{-(\xi_1+\xi_2)}\xi_1^{a_1-1}\xi_2^{a_2-1}{}_0F_3\left(h_1, h_2, h_3; x\xi_1^2\xi_2^2\right) \cdot F_{2:1;1}^{0:0;2}\left[\begin{array}{ccc} -: & -; & c_1^{''}, c_2^{''}; \\ g_1^{'}, g_2^{'}: & h_1^{'}; & h_1^{''}; \end{array} y\xi_1^2\xi_2^2, z\xi_1\xi_2\right] d\xi_1 d\xi_2$$

$$(\Re(a_1) > 0,\ \Re(a_2) > 0);$$

(2.3)

$$\mathcal{G}_2^{(3)}\left[\begin{array}{c} a_1, a_2, c_1^{''}, c_2^{''} \\ h_1, h_2, h_3, h_1^{'}, h_2^{'}, h_3^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right] = \frac{1}{\Gamma(a_1)\Gamma(a_2)}\int_0^\infty\int_0^\infty e^{-(\xi_1+\xi_2)}\xi_1^{a_1-1}\xi_2^{a_2-1}\cdot{}_0F_3\left(h_1, h_2, h_3; x\xi_1^2\xi_2^2\right) {}_0F_3\left(h_1^{'}, h_2^{'}, h_3^{'}; y\xi_1^2\xi_2^2\right){}_2F_3\left(c_1^{''}, c_2^{''}; h_1^{''}, h_2^{''}, h_3^{''}; z\xi_1\xi_2\right) d\xi_1 d\xi_2\,(\Re(a_1) > 0,\ \Re(a_2) > 0);$$

(2.4)

$$\mathcal{G}_3^{(3)}\left[\begin{array}{c} a_1, a_2, b_1^{'}, b_2^{'} \\ g_1, g_2, h_1, h_1^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right] = \frac{1}{\Gamma(a_1)\Gamma(a_2)\Gamma\left(b_1^{'}\right)\Gamma\left(b_2^{'}\right)}\int_0^\infty\cdots\int_0^\infty e^{-(\xi_1+\xi_2+\xi_3+\xi_4)} \cdot\xi_1^{a_1-1}\xi_2^{a_2-1}\xi_3^{b_1^{'}-1}\xi_4^{b_2^{'}-1}F_{2:1;1}^{0:0;0}\left[\begin{array}{ccc} -: & -; & -; \\ g_1,\ g_2: & h_1; & h_1^{'}; \end{array} x\xi_1^2\xi_2^2, y\xi_1^2\xi_2^2\xi_3\xi_4\right] \cdot{}_0F_3\left(h_1^{''}, h_2^{''}, h_3^{''}; z\xi_1\xi_2\xi_3\xi_4\right) d\xi_1 d\xi_2 d\xi_3 d\xi_4$$

$$\left(\Re(a_1) > 0,\ \Re(a_2) > 0,\ \Re(b_1^{'}) > 0,\ \Re(b_2^{'}) > 0\right);$$

(2.5)

$$\mathcal{G}_4^{(3)}\left[\begin{array}{c} a_1,a_2,b_1^{'},b_2^{'} \\ h_1,h_2,h_3,h_1^{'},h_2^{'},h_3^{'},h_1^{''},h_2^{''},h_3^{''} \end{array} x,y,z\right]$$
$$=\frac{1}{\Gamma(a_1)\Gamma(a_2)}\int_0^{\infty}\int_0^{\infty}e^{-(\xi_1+\xi_2)}\xi_1^{a_1-1}\xi_2^{a_2-1}\cdot {}_0F_3\left(h_1,h_2,h_1;x\xi_1^2\xi_2^2\right)$$
$$F_{0:3;3}^{2:0;0}\left[\begin{array}{lll} b_1^{'},b_2^{'}: & -; & -; \\ -: & h_1^{'},h_2^{'},h_3^{'}; & h_1^{''},h_2^{''},h_3^{''}; \end{array} y\xi_1^2\xi_2^2, z\xi_1\xi_2\right]d\xi_1 d\xi_2$$
$$(\Re(a_1)>0,\ \Re(a_2)>0);$$

(2.6)

$$\mathcal{G}_5^{(3)}\left[\begin{array}{c} a_1,a_2,c_1^{'},c_2^{'},c_1^{''},c_2^{''} \\ e_1,e_2,h_1,h_1^{'},h_1^{''} \end{array} x,y,z\right]$$
$$=\frac{1}{\Gamma(a_1)\Gamma(a_2)}\int_0^{\infty}\int_0^{\infty}e^{-(\xi_1+\xi_2)}\xi_1^{a_1-1}\xi_2^{a_2-1}\cdot F^{(3)}$$
$$\left[\begin{array}{c} c_1^{'},c_2^{'},c_1^{''},c_2^{''} \\ e_1,e_2,h_1,h_1^{'},h_1^{''} \end{array} x\xi_1^2\xi_2^2, y\xi_1^2\xi_2^2, z\xi_1\xi_2\right]d\xi_1 d\xi_2\,(\Re(a_1)>0,\ \Re(a_2)>0),$$

where $F^{(3)}$ is the generalized Srivastava's hypergeometric function:

$$F^{(3)}\left[\begin{array}{c} c_1^{'},c_2^{'},c_1^{''},c_2^{''} \\ e_1,e_2,h_1,h_1^{'},h_1^{''} \end{array} x,y,z\right]$$
$$=\sum_{i,j,k=0}^{\infty}\frac{\left(c_1^{'}\right)_j\left(c_2^{'}\right)_j\left(c_1^{''}\right)_k\left(c_2^{''}\right)_k}{(e_1)_{i+j+k}(e_2)_{i+j+k}(h_1)_i\left(h_1^{'}\right)_j\left(h_1^{''}\right)_k i!j!k!}x^i y^j z^k; \qquad (2.7)$$

(2.8)

$$\mathcal{G}_6^{(3)}\left[\begin{array}{c} a_1,a_2,c_1^{'},c_2^{'},c_1^{''},c_2^{''} \\ g_1,g_2,h_1,h_1^{'},h_1^{''},h_2^{''},h_3^{''} \end{array} x,y,z\right]$$
$$=\frac{1}{\Gamma(a_1)\Gamma(a_2)}\int_0^{\infty}\int_0^{\infty}e^{-(\xi_1+\xi_2)}\xi_1^{a_1-1}\xi_2^{a_2-1}$$
$$\cdot F^{(3)}\left[\begin{array}{c} c_1^{'},c_2^{'},c_1^{''},c_2^{''} \\ g_1,g_2,h_1,h_1^{'},h_1^{''},h_2^{''},h_3^{''} \end{array} x\xi_1^2\xi_2^2, y\xi_1^2\xi_2^2, z\xi_1\xi_2\right]d\xi_1 d\xi_2$$
$$(\Re(a_1)>0,\ \Re(a_2)>0),$$

where

$$F^{(3)}\left[\begin{array}{c} c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''} \\ g_1, g_2, h_1, h_1^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right] = \sum_{i,j,k=0}^{\infty} \frac{\left(c_1^{'}\right)_j \left(c_2^{'}\right)_j \left(c_1^{''}\right)_k \left(c_2^{''}\right)_k}{(g_1)_{i+j}(g_2)_{i+j}(h_1)_i \left(h_1^{'}\right)_j \left(h_1^{''}\right)_k \left(h_2^{''}\right)_k \left(h_3^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{2.9}$$

(2.10)
$$\mathcal{G}_7^{(3)}\left[\begin{array}{c} a_1, a_2, c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''} \\ g_1^{'}, g_2^{'}, h_1, h_2, h_3, h_1^{'}, h_2^{''} \end{array} x, y, z\right] = \frac{1}{\Gamma(a_1)\Gamma(a_2)} \int_0^{\infty}\int_0^{\infty} e^{-(\xi_1+\xi_2)} \xi_1^{a_1-1}\xi_2^{a_2-1} \cdot F^{(3)}\left[\begin{array}{c} c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''} \\ g_1^{'}, g_2^{'}, h_1, h_2, h_3, h_1^{'}, h_1^{''} \end{array} x\xi_1^2\xi_2^2, y\xi_1^2\xi_2^2, z\xi_1\xi_2\right] d\xi_1 d\xi_2$$
$$(\Re(a_1) > 0,\ \Re(a_2) > 0),$$

where

$$F^{(3)}\left[\begin{array}{c} c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''} \\ g_1^{'}, g_2^{'}, h_1, h_2, h_3, h_1^{'}, h_1^{''} \end{array} x, y, z\right] = \sum_{i,j,k=0}^{\infty} \frac{\left(c_1^{'}\right)_j \left(c_2^{'}\right)_j \left(c_1^{''}\right)_k \left(c_2^{''}\right)_k}{\left(g_1^{'}\right)_{j+k}\left(g_2^{'}\right)_{j+k}(h_1)_i (h_2)_i (h_3)_i \left(h_1^{'}\right)_j \left(h_1^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{2.11}$$

(2.12)
$$\mathcal{G}_8^{(3)}\left[\begin{array}{c} a_1, a_2, c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''} \\ h_1, h_2, h_3, h_1^{'}, h_2^{'}, h_3^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right] = \frac{1}{\Gamma(a_1)\Gamma(a_2)} \int_0^{\infty}\int_0^{\infty} e^{-(\xi_1+\xi_2)} \xi_1^{a_1-1}\xi_2^{a_2-1} {}_0F_3\left(h_1, h_2, h_3; x\xi_1^2\xi_2^2\right) \cdot {}_2F_3\left(c_1^{'}, c_2^{'}; h_1^{'}, h_2^{'}, h_3^{'}; y\xi_1^2\xi_2^2\right) {}_2F_3\left(c_1^{''}, c_2^{''}; h_1^{''}, h_2^{''}, h_3^{''}; z\xi_1\xi_2\right) d\xi_1 d\xi_2$$
$$(\Re(a_1) > 0,\ \Re(a_2) > 0);$$

(2.13)

$$\mathcal{G}_9^{(3)}\left[\begin{matrix} a_1^{'}, a_2^{'}, a_1^{''}, a_2^{''} \\ e_1, e_2, h_1, h_1^{'}, h_1^{''} \end{matrix} x, y, z\right] = \frac{1}{\Gamma\left(a_1^{'}\right)\Gamma\left(a_2^{'}\right)\Gamma\left(a_1^{''}\right)\Gamma\left(a_2^{''}\right)}\int_0^\infty \cdots \int_0^\infty e^{-(\xi_1+\xi_2+\xi_3+\xi_4)}$$
$$\xi_1^{a_1^{'}-1}\xi_2^{a_2^{'}-1}\xi_3^{a_1^{''}-1}\xi_4^{a_2^{''}-1}\cdot F^{(3)}\left[\begin{matrix} - \\ e_1, e_2, h_1, h_1^{'}, h_1^{''} \end{matrix} x\xi_1^2\xi_2^2, y\xi_1\xi_2\xi_3\xi_4, z\xi_3^2\xi_4^2\right]$$
$$d\xi_1\cdots d\xi_4\left(\Re(a_1^{'})>0,\ \Re(a_2^{'})>0,\ \Re(a_1^{''})>0,\ \Re(a_2^{''})>0\right),$$

where

$$F^{(3)}\left[\begin{matrix} - \\ e_1, e_2, h_1, h_1^{'}, h_1^{''} \end{matrix} x, y, z\right] = \sum_{i,j,k=0}^{\infty}\frac{1}{(e_1)_{i+j+k}(e_2)_{i+j+k}(h_1)_i\left(h_1^{'}\right)_j\left(h_1^{''}\right)_k i!j!k!}x^iy^jz^k; \tag{2.14}$$

(2.15)

$$\mathcal{G}_{10}^{(3)}\left[\begin{matrix} a_1^{'}, a_2^{'}, a_1^{''}, a_2^{''} \\ g_1, g_2, h_1, h_1^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{matrix} x, y, z\right] = \frac{1}{\Gamma\left(a_1^{'}\right)\Gamma\left(a_2^{'}\right)\Gamma\left(a_1^{''}\right)\Gamma\left(a_2^{''}\right)}\int_0^\infty \cdots \int_0^\infty e^{-(\xi_1+\xi_2+\xi_3+\xi_4)}$$
$$\xi_1^{a_1^{'}-1}\xi_2^{a_2^{'}-1}\xi_3^{a_1^{''}-1}\xi_4^{a_2^{''}-1}\cdot F_{2:1;1}^{0:0;0}\left[\begin{matrix} -: & -; & -; \\ g_1, g_2: & h_1; & h_1^{'}; \end{matrix} x\xi_1^2\xi_2^2, y\xi_1\xi_2\xi_3\xi_4\right]$$
$${}_0F_3\left(h_1^{''}, h_2^{''}, h_3^{''}; z\xi_3^2\xi_4^2\right)$$
$$d\xi_1\cdots d\xi_4\left(\Re(a_1^{'})>0,\ \Re(a_2^{'})>0,\ \Re(a_1^{''})>0,\ \Re(a_2^{''})>0\right);$$

(2.16)

$$\mathcal{G}_{11}^{(3)}\left[\begin{matrix} a_1^{'}, a_2^{'}, a_1^{''}, a_2^{''} \\ g_1^{''}, g_2^{''}, h_1, h_1^{'}, h_2^{'}, h_3^{'}, h_1^{''} \end{matrix} x, y, z\right] = \frac{1}{\Gamma\left(a_1^{'}\right)\Gamma\left(a_2^{'}\right)\Gamma\left(a_1^{''}\right)\Gamma\left(a_2^{''}\right)}$$
$$\cdot\int_0^\infty \cdots \int_0^\infty e^{-(\xi_1+\xi_2+\xi_3+\xi_4)}\xi_1^{a_1^{'}-1}\xi_2^{a_2^{'}-1}\xi_3^{a_1^{''}-1}\xi_4^{a_2^{''}-1}$$
$$\cdot F_{2:1;1}^{0:0;0}\left[\begin{matrix} -: & -; & -; \\ g_1^{''}, g_2^{''}: & h_1; & h_1^{''}; \end{matrix} x\xi_1^2\xi_2^2, z\xi_3^2\xi_4^2\right]{}_0F_3\left(h_1^{'}, h_2^{'}, h_3^{'}; y\xi_1\xi_2\xi_3\xi_4\right)$$
$$d\xi_1\cdots d\xi_4\left(\Re(a_1^{'})>0,\ \Re(a_2^{'})>0,\ \Re(a_1^{''})>0,\ \Re(a_2^{''})>0\right);$$

(2.17)

$$\mathcal{G}_{12}^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, a_1^{''}, a_2^{''} \\ h_1, h_2, h_3, h_1^{'}, h_2^{'}, h_3^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right]$$
$$= \frac{1}{\Gamma\left(a_1^{'}\right)\Gamma\left(a_2^{'}\right)\Gamma\left(a_1^{''}\right)\Gamma\left(a_2^{''}\right)} \int_0^{\infty}\cdots\int_0^{\infty} e^{-(\xi_1+\xi_2+\xi_3+\xi_4)}$$
$$\xi_1^{a_1^{'}-1}\xi_2^{a_2^{'}-1}\xi_3^{a_1^{''}-1}\xi_4^{a_2^{''}-1} \cdot {}_0F_3\left(h_1, h_2, h_3; x\xi_1^2\xi_2^2\right)$$
$${}_0F_3\left(h_1^{'}, h_2^{'}, h_3^{'}; y\xi_1\xi_2\xi_3\xi_4\right) {}_0F_3\left(h_1^{''}, h_2^{''}, h_3^{''}; z\xi_3^2\xi_4^2\right) d\xi_1\cdots d\xi_4$$
$$\left(\Re(a_1^{'})>0,\ \Re(a_2^{'})>0,\ \Re(a_1^{''})>0,\ \Re(a_2^{''})>0\right);$$

(2.18)

$$\mathcal{G}_{13}^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, b_1^{'}, b_2^{'}, c_1^{''}, c_2^{''} \\ e_1, e_2, h_1, h_1^{'}, h_1^{''} \end{array} x, y, z\right]$$
$$= \frac{1}{\Gamma\left(a_1^{'}\right)\Gamma\left(a_2^{'}\right)\Gamma\left(b_1^{'}\right)\Gamma\left(b_2^{'}\right)} \int_0^{\infty}\cdots\int_0^{\infty} e^{-(\xi_1+\xi_2+\xi_3+\xi_4)}$$
$$\cdot\,\xi_1^{a_1^{'}-1}\xi_2^{a_2^{'}-1}\xi_3^{b_1^{'}-1}\xi_4^{b_2^{'}-1} F^{(3)}\left[\begin{array}{c} c_1^{''}, c_2^{''} \\ e_1, e_2, h_1, h_1^{'}, h_1^{''} \end{array} x\xi_1^2\xi_2^2, y\xi_1\xi_2\xi_3\xi_4, z\xi_3\xi_4\right]$$
$$d\xi_1\cdots d\xi_4\left(\Re(a_1^{'})>0,\ \Re(a_2^{'})>0,\ \Re(b_1^{'})>0,\ \Re(b_2^{'})>0\right);$$

(2.19)

$$\mathcal{G}_{14}^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, b_1^{'}, b_2^{'}, c_1^{''}, c_2^{''} \\ g_1, g_2, h_1, h_1^{'}, h_1^{''}, h_2^{''}, h_3^{''} \end{array} x, y, z\right]$$
$$= \frac{1}{\Gamma\left(a_1^{'}\right)\Gamma\left(a_2^{'}\right)\Gamma\left(b_1^{'}\right)\Gamma\left(b_2^{'}\right)} \int_0^{\infty}\cdots\int_0^{\infty} e^{-(\xi_1+\xi_2+\xi_3+\xi_4)}$$
$$\xi_1^{a_1^{'}-1}\xi_2^{a_2^{'}-1}\xi_3^{b_1^{'}-1}\xi_4^{b_2^{'}-1} \cdot F_{2:1;1}^{0:0;0}\left[\begin{array}{ccc} - : & -; & -; \\ g_1, g_2 : & h_1; & h_1^{'}; \end{array} x\xi_1^2\xi_2^2, y\xi_1\xi_2\xi_3\xi_4\right]$$
$${}_2F_3\left(c_1^{''}, c_2^{''}; h_1^{''}, h_2^{''}, h_3^{''}; z\xi_3\xi_4\right) d\xi_1\cdots d\xi_4$$
$$\left(\Re(a_1^{'})>0,\ \Re(a_2^{'})>0,\ \Re(b_1^{'})>0,\ \Re(b_2^{'})>0\right);$$

(2.20)
$$\mathcal{G}_{15}^{(3)}\left[\begin{array}{c} a_1', a_2', b_1', b_2', c_1'', c_2'' \\ g_1', g_2', h_1, h_2, h_3, h_1', h_1'' \end{array} x, y, z\right] = \frac{1}{\Gamma\left(a_1'\right)\Gamma\left(a_2'\right)\Gamma\left(b_1'\right)\Gamma\left(b_2'\right)}$$
$$\cdot \int_0^\infty \cdots \int_0^\infty e^{-(\xi_1+\xi_2+\xi_3+\xi_4)}\, \xi_1^{a_1'-1} \xi_2^{a_2'-1} \xi_3^{b_1'-1} \xi_4^{b_2'-1}$$
$$\cdot\, {}_0F_3\left(h_1, h_2, h_3; x\xi_1^2\xi_2^2\right) F_{2:1;1}^{0:0;2}\left[\begin{array}{ccc} -: & -; & c_1'', c_2''; \\ g_1', g_2': & h_1'; & h_1''; \end{array} y\xi_1\xi_2\xi_3\xi_4, z\xi_3\xi_4\right]$$
$$d\xi_1 \cdots d\xi_4 \left(\Re(a_1') > 0,\ \Re(a_2') > 0,\ \Re(b_1') > 0,\ \Re(b_2') > 0\right);$$

(2.21)
$$\mathcal{G}_{16}^{(3)}\left[\begin{array}{c} a_1', a_2', b_1', b_2', c_1'', c_2'' \\ g_1'', g_2'', h_1, h_1', h_2', h_3', h_1'' \end{array} x, y, z\right]$$
$$= \frac{1}{\Gamma\left(a_1'\right)\Gamma\left(a_2'\right)\Gamma\left(b_1'\right)\Gamma\left(b_2'\right)} \int_0^\infty \cdots \int_0^\infty e^{-(\xi_1+\xi_2+\xi_3+\xi_4)}$$
$$\xi_1^{a_1'-1} \xi_2^{a_2'-1} \xi_3^{b_1'-1} \xi_4^{b_2'-1} \cdot F_{2:1;1}^{0:0;2}\left[\begin{array}{ccc} -: & -; & c_1'', c_2''; \\ g_1'', g_2'': & h_1; & h_1''; \end{array} x\xi_1^2\xi_2^2, z\xi_3\xi_4\right]$$
$${}_0F_3\left(h_1', h_2', h_3'; y\xi_1\xi_2\xi_3\xi_4\right) d\xi_1 \cdots d\xi_4$$
$$\left(\Re(a_1') > 0,\ \Re(a_2') > 0,\ \Re(b_1') > 0,\ \Re(b_2') > 0\right);$$

(2.22)
$$\mathcal{G}_{17}^{(3)}\left[\begin{array}{c} a_1', a_2', b_1', b_2', c_1'', c_2'' \\ h_1, h_2, h_3, h_1', h_2', h_3', h_1'', h_2'', h_3'' \end{array} x, y, z\right]$$
$$= \frac{1}{\Gamma\left(a_1'\right)\Gamma\left(a_2'\right)\Gamma\left(b_1'\right)\Gamma\left(b_2'\right)} \int_0^\infty \cdots \int_0^\infty e^{-(\xi_1+\xi_2+\xi_3+\xi_4)}$$
$$\xi_1^{a_1'-1} \xi_2^{a_2'-1} \xi_3^{b_1'-1} \xi_4^{b_2'-1} \cdot {}_0F_3\left(h_1, h_2, h_3; x\xi_1^2\xi_2^2\right) {}_0F_3\left(h_1', h_2', h_3'; y\xi_1\xi_2\xi_3\xi_4\right)$$
$${}_2F_3\left(c_1'', c_2''; h_1'', h_2'', h_3''; z\xi_3\xi_4\right) d\xi_1 \cdots d\xi_4$$
$$\left(\Re(a_1') > 0,\ \Re(a_2') > 0,\ \Re(b_1') > 0,\ \Re(b_2') > 0\right);$$

(2.23)
$$\mathcal{G}_{18}^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''}, c_3^{''}, c_4^{''} \\ e_1, e_2, h_1, h_1^{'}, h_1^{''} \end{array} x, y, z\right] = \frac{1}{\Gamma\left(a_1^{'}\right)\Gamma\left(a_2^{'}\right)}\int_0^\infty\int_0^\infty e^{-(\xi_1+\xi_2)}$$
$$\xi_1^{a_1^{'}-1}\xi_2^{a_2^{'}-1}\cdot F^{(3)}\left[\begin{array}{c} c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''}, c_3^{''}, c_4^{''} \\ e_1, e_2, h_1, h_1^{'}, h_1^{''} \end{array} x\xi_1^2\xi_2^2,\ y\xi_1\xi_2,\ z\right] d\xi_1 d\xi_2$$
$$\left(\Re(a_1^{'}) > 0,\ \Re(a_2^{'}) > 0\right),$$

where
$$F^{(3)}\left[\begin{array}{c} c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''}, c_3^{''}, c_4^{''} \\ e_1, e_2, h_1, h_1^{'}, h_1^{''} \end{array} x, y, z\right]$$
$$= \sum_{i,j,k=0}^{\infty} \frac{\left(c_1^{'}\right)_j\left(c_2^{'}\right)_j\left(c_1^{''}\right)_k\left(c_2^{''}\right)_k\left(c_3^{''}\right)_k\left(c_4^{''}\right)_k}{(e_1)_{i+j+k}(e_2)_{i+j+k}(h_1)_i\left(h_1^{'}\right)_j\left(h_1^{''}\right)_k i!j!k!} x^i y^j z^k; \tag{2.24}$$

(2.25)
$$\mathcal{G}_{19}^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''}, c_3^{''}, c_4^{''} \\ g_1^{'}, g_2^{'}, h_1, h_2, h_3, h_1^{'}, h_1^{''} \end{array} x, y, z\right]$$
$$= \frac{1}{\Gamma\left(a_1^{'}\right)\Gamma\left(a_2^{'}\right)}\int_0^\infty\int_0^\infty e^{-(\xi_1+\xi_2)}\xi_1^{a_1^{'}-1}\xi_2^{a_2^{'}-1}{}_0F_3\left(h_1, h_2, h_3; x\xi_1^2\xi_2^2\right)$$
$$\cdot F_{2:1;1}^{0:2;4}\left[\begin{array}{cccc} - : & c_1^{'}, c_2^{'}; & c_1^{''}, c_2^{''}, c_3^{''}, c_4^{''}; \\ g_1^{'}, g_2^{'} : & h_1^{'}; & h_1^{''}; \end{array} y\xi_1\xi_2, z\right] d\xi_1 d\xi_2$$
$$\left(\Re(a_1^{'}) > 0,\ \Re(a_2^{'}) > 0\right);$$

(2.26)
$$\mathcal{G}_{20}^{(3)}\left[\begin{array}{c} a_1^{'}, a_2^{'}, c_1^{'}, c_2^{'}, c_1^{''}, c_2^{''}, c_3^{''}, c_4^{''} \\ g_1^{''}, g_2^{''}, h_1, h_1^{'}, h_2^{'}, h_3^{'}, h_1^{''} \end{array} x, y, z\right] = \frac{1}{\Gamma\left(a_1^{'}\right)\Gamma\left(a_2^{'}\right)}\int_0^\infty\int_0^\infty e^{-(\xi_1+\xi_2)}$$
$$\xi_1^{a_1^{'}-1}\xi_2^{a_2^{'}-1}\cdot F_{2:1;1}^{0:0;4}\left[\begin{array}{cccc} - : & -; & c_1^{''}, c_2^{''}, c_3^{''}, c_4^{''}; \\ g_1^{''}, g_2^{''} : & h_1; & h_1^{''}; \end{array} x\xi_1^2\xi_2^2, z\right]$$
$${}_2F_3\left(c_1^{'}, c_2^{'}; h_1^{'}, h_2^{'}, h_3^{'}; y\xi_1\xi_2\right) d\xi_1 d\xi_2 \left(\Re(a_1^{'}) > 0,\ \Re(a_2^{'}) > 0\right),$$

where ${}_pF_q$ is the generalized hypergeometric function and $F_{p:q;k}^{m:n;r}$ is the Kampé de Fériet hypergeometric function.

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