

## FIXED POINT THEOREMS FOR GENERALIZED NONEXPANSIVE SET-VALUED MAPPINGS IN CONE METRIC SPACES

SEUNG HYUN KIM AND BYUNG SOO LEE†

**ABSTRACT.** In 2007, Huang and Zhang [1] introduced a cone metric space with a cone metric generalizing the usual metric space by replacing the real numbers with Banach space ordered by the cone. They considered some fixed point theorems for contractive mappings in cone metric spaces. Since then, the fixed point theory for mappings in cone metric spaces has become a subject of interest in [1-6] and references therein. In this paper, we consider some fixed point theorems for generalized nonexpansive set-valued mappings under suitable conditions in sequentially compact cone metric spaces and complete cone metric spaces.

### 1. Introduction and Preliminaries

In 2007, Huang and Zhang [1] introduced a cone metric space with a cone metric generalizing the usual metric space by replacing the real numbers with Banach space ordered by the cone. They considered some fixed point theorems for contractive mappings in cone metric spaces. Since then, the fixed point theory for mappings in cone metric spaces has become a subject of interest in [1-6] and references therein. Especially, for single-valued mappings, Choudhury and Metiya [6] considered some fixed point theorems for weak contraction in cone metric spaces in 2010. In 2011, Wardowski [2] introduced  $H$ -cone metric in the collection of subsets of a given cone metric space. And he considered the concept of set-valued contraction of Nadler-type and proved a fixed point theorem for contractive set-valued mappings in  $H$ -cone metric spaces. Inspired and encouraged by the previous works, in this paper we consider some fixed point theorems for generalized nonexpansive set-valued mappings under suitable conditions in sequentially compact cone metric spaces and complete cone metric spaces.

Let  $E$  be a real Banach space and  $P$  be a subset of  $E$ .  
 $P$  is called a cone if and only if

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†Corresponding author.

- (P1)  $P$  is closed,  $P \neq \emptyset$ ,  $P \neq \{0\}$ ;  
 (P2)  $a, b \in \mathbb{R}$  with  $a, b \geq 0$ ,  $x, y \in P \Rightarrow ax + by \in P$ ;  
 (P3)  $x \in P$  and  $-x \in P \Rightarrow x = 0$ .

Let  $P \subset E$  be a cone; we define a partial ordering ' $\preceq$ ' with respect to  $P$ . For  $x, y \in E$ , we say that  $x \preceq y$  if and only if  $y - x \in P$ ,  $x \ll y$  if and only if  $y - x \in \text{int}P$ , where  $\text{int}P$  denotes the interior of  $P$ ,  $x \prec y$  if and only if  $x \preceq y$  and  $x \neq y$ . The cone  $P$  is called a normal cone if there is a number  $K > 0$  such that for all  $x, y \in E$ ,

$$0 \preceq x \preceq y \text{ implies } \|x\| \leq K\|y\|.$$

The least positive number  $K$  is called the normal constant of  $P$ .

**Definition 1.** ([1]) Let  $M$  be a nonempty set. Suppose that a mapping  $d : M \times M \rightarrow E$  satisfies the following;

- (d1)  $0 \preceq d(x, y)$  for all  $x, y \in M$  and  $d(x, y) = 0$  if and only if  $x = y$ ;  
 (d2)  $d(x, y) = d(y, x)$  for all  $x, y \in M$ ;  
 (d3)  $d(x, y) \preceq d(x, z) + d(y, z)$  for all  $x, y, z \in M$ .

Then  $d$  is called a cone metric on  $M$ , and  $(M, d)$  is called a cone metric space.

The following definitions and lemmas are considered in a cone metric space  $(M, d)$ .

**Definition 2.** ([1]) Let  $\{x_n\}$  be a sequence in  $M$  and  $x \in M$ .

- (i) If for every  $c \in E$  with  $c \gg 0$ , there is  $N$  such that for all  $n > N$ ,  $d(x_n, x) \ll c$ , then we say that  $\{x_n\}$  converges to  $x$  or  $\{x_n\}$  is convergent to  $x$ , and denote as  $\lim_{n \rightarrow \infty} x_n = x$ .  
 (ii) If for every  $c \in E$  with  $c \gg 0$ , there is  $N$  such that for all  $n, m > N$ ,  $d(x_n, x_m) \ll c$ , then  $\{x_n\}$  is called a Cauchy sequence in  $M$ .  
 (iii) For any sequence  $\{x_n\}$  in  $M$ , if there is a subsequence  $\{x_{n_k}\}$  of  $\{x_n\}$  such that  $\{x_{n_k}\}$  is convergent in  $M$ , then  $M$  is called a sequentially compact cone metric space.

**Lemma 1.1.** ([1]) Let  $P$  be a normal cone with the normal constant  $K$ . Let  $\{x_n\}$  and  $\{y_n\}$  be sequences in  $M$ . Then;

- (i)  $\{x_n\}$  converges to  $x$  if and only if  $d(x_n, x) \rightarrow 0$  as  $n \rightarrow \infty$ ;  
 (ii)  $\{x_n\}$  is a Cauchy sequence if and only if  $d(x_n, x_m) \rightarrow 0$  as  $n, m \rightarrow \infty$ ;  
 (iii) If  $x_n \rightarrow x$  and  $y_n \rightarrow y$  as  $n \rightarrow \infty$ , then  $d(x_n, y_n) \rightarrow 0$  as  $n \rightarrow \infty$ .

**Definition 3.** ([2])  $(M, d)$  is said to be a complete cone metric space if every Cauchy sequence  $\{x_n\}$  in  $M$  is convergent to  $x \in M$ .

**Definition 4.** ([2]) Let  $\mathcal{A}$  be a collection of nonempty subsets of  $M$ . A mapping  $H : \mathcal{A} \times \mathcal{A} \rightarrow E$  is called an  $H$ -cone metric with respect to  $d$  if for any  $A_1, A_2 \in \mathcal{A}$ , the following conditions hold;

- (H1)  $H(A_1, A_2) = 0 \Rightarrow A_1 = A_2$ ;  
 (H2)  $H(A_1, A_2) = H(A_2, A_1)$ ;

- (H3)  $\forall \epsilon \in \text{int}P$  and  $\forall x \in A_1, \exists y \in A_2$ , such that  $H(A_1, A_2) + \epsilon - d(x, y) \in P$ ;
- (H4) one of the following is satisfied;
  - (1)  $\forall \epsilon \in \text{int}P, \exists x \in A_1$  such that  $\forall y \in A_2, d(x, y) + \epsilon - H(A_1, A_2) \in P$ ;
  - (2)  $\forall \epsilon \in \text{int}P, \exists x \in A_2$  such that  $\forall y \in A_1, d(x, y) + \epsilon - H(A_1, A_2) \in P$ .

For a cone metric space  $(M, d)$  and for an ordered Banach space  $(E, \preceq, \|\cdot\|)$  with an ordered complete cone  $P$ , a set  $A \subset M$  is called bounded if the set  $\{d(x, y) : x, y \in A\}$  is a norm bounded subset of  $E$ . From Definition 4., it is easily shown that  $H(\{a\}, \{b\}) = d(a, b)$  and  $H(A, \{b\}) = \inf_{a \in A} d(a, b)$  for a bounded set  $A \in \mathcal{A}$ . We denote  $d(A, b) = \inf_{a \in A} d(a, b)$ .

**Lemma 1.2.** ([2]) *The pair  $(\mathcal{A}, H)$  is also a cone metric space.*

### 2. Main Results

First of all, we consider a fixed point theorem for a  $\phi$ -nonexpansive set-valued mapping in sequentially compact cone metric spaces.

**Theorem 2.1.** *Let  $(M, d)$  be a sequentially compact cone metric space for a normal cone  $P$  with the normal constant  $K$ ,  $\mathcal{A}$  be a nonempty collection of nonempty closed subsets of  $M$ ,  $\phi : P \rightarrow P$  be a continuous mapping and let  $H : \mathcal{A} \times \mathcal{A} \rightarrow E$  be an  $H$ -cone metric with respect to  $d$ . If a mapping  $T : M \rightarrow \mathcal{A}$  satisfies the  $\phi$ -nonexpansive condition;*

$$H(Tx, Ty) \preceq \phi(d(x, y)), \text{ for all } x, y \in M \text{ with } x \neq y, \tag{2.1}$$

then  $\text{Fix}T \neq \emptyset$ .

*Proof.* Let  $x_0 \in M$  be arbitrary and fixed, and let  $\epsilon_n \gg 0$  with  $\lim_{n \rightarrow \infty} \epsilon_n = 0$  and let  $x_1 \in Tx_0$ . From (H3), there exists  $x_2 \in Tx_1$  such that  $d(x_1, x_2) \preceq H(Tx_0, Tx_1) + \epsilon_1$ . Inductively, for  $x_{n-1} (n > 1)$ , we have  $x_n \in Tx_{n-1}$  satisfying

$$d(x_{n-1}, x_n) \preceq H(Tx_{n-2}, Tx_{n-1}) + \epsilon_{n-1}.$$

Since  $(M, d)$  is a sequentially compact cone metric space, there is a convergent subsequence  $\{x_{n_k}\}$  of  $\{x_n\}$ . Say

$$\lim_{k \rightarrow \infty} x_{n_k} = x, \text{ where } x \in M. \tag{2.2}$$

From the  $\phi$ -nonexpansive condition (2.1),

$$H(Tx_{n_k}, Tx) \preceq \phi(d(x_{n_k}, x)) \text{ for } k \in \mathbb{N}.$$

By the normality of  $P$  with the normal constant  $K$ ,

$$\|H(Tx_{n_k}, Tx)\| \leq K \|\phi(d(x_{n_k}, x))\| \text{ for } k \in \mathbb{N}. \tag{2.3}$$

From (2.2) and (2.3), the sequence  $\{Tx_{n_k}\}$  converges to  $Tx$  with respect to the metric  $H$ , and since  $x_{n_k} \in Tx_{n_k-1}$  for any  $k \in \mathbb{N}$ , we obtain by (H3),

$$d(x_{n_k}, y_{n_k}) \preceq H(Tx_{n_k-1}, Tx) + \epsilon_{n_k},$$

where  $y_{n_k} \in Tx$  for all  $k \in \mathbb{N}$ . Thus  $\lim_{k \rightarrow \infty} d(x_{n_k}, y_{n_k}) = 0$ . By (2.2),  $\lim_{k \rightarrow \infty} y_{n_k} = x$  and since  $Tx$  is closed,  $x \in Tx$ .  $\square$

By putting  $\phi = I$ , where  $I$  is an identity mapping, in Theorem 2.1., we obtain the following Corollary 2.2., which is a fixed point theorem for a nonexpansive set-valued mapping in sequentially compact cone metric spaces.

**Corollary 2.2.** *Let  $(M, d)$  be a sequentially compact cone metric space for a normal cone  $P$  with the normal constant  $K$ ,  $\mathcal{A}$  be a nonempty collection of nonempty closed subsets of  $M$  and let  $H : \mathcal{A} \times \mathcal{A} \rightarrow E$  be an  $H$ -cone metric with respect to  $d$ . If a mapping  $T : M \rightarrow \mathcal{A}$  satisfies the nonexpansive condition;*

$$H(Tx, Ty) \preceq d(x, y), \text{ for all } x, y \in M \text{ with } x \neq y,$$

then  $FixT \neq \emptyset$ .

**Example 2.1.** Let  $M = [0, 1]$ ,  $E = \mathbb{R}^2$  be a Banach space with the standard norm  $P = \{(x, y) \in E; x, y \geq 0\}$  be a normal cone and let  $d : M \times M \rightarrow E$  be of the form  $d(x, y) = (|x - y|, \frac{1}{2}|x - y|)$ . Then the pair  $(M, d)$  is a sequentially compact cone metric space. Let  $\mathcal{A}$  be a family of subsets of  $M$  of the form

$$\mathcal{A} = \{[0, x]; x \in M\} \cup \{\{x\}; x \in M\}.$$

Define an  $H$ -cone metric  $H : \mathcal{A} \times \mathcal{A} \rightarrow E$  with respect to  $d$  by the formular

$$H(A, B) = \begin{cases} (|x - y|, \frac{1}{2}|x - y|) & \text{for } A = [0, x], B = [0, y] \\ (|x - y|, \frac{1}{2}|x - y|) & \text{for } A = \{x\}, B = \{y\} \\ (\max\{y, |x - y|\}, \frac{1}{2}\max\{y, |x - y|\}) & \text{for } A = [0, x], B = \{y\} \\ (\max\{x, |x - y|\}, \frac{1}{2}\max\{x, |x - y|\}) & \text{for } A = \{x\}, B = [0, y]. \end{cases}$$

Define the mapping  $T : M \rightarrow \mathcal{A}$  as follows;

$$Tx = \begin{cases} \{0\} & \text{for } x \in [0, \frac{1}{2}] \\ [0, (x - \frac{1}{2})^2] & \text{for } x \in (\frac{1}{2}, 1]. \end{cases}$$

Then  $T$  satisfies the nonexpansive condition and  $T$  has fixed point  $\{0\}$ .

Now, we consider more generalized fixed point theorem for a  $(\phi_1, \phi_2, \phi_3)$ -nonexpansive set-valued mapping in complete cone metric spaces.

**Theorem 2.3.** *Let  $(M, d)$  be a complete cone metric space for a normal cone  $P$  with the normal constant  $K$ ,  $\mathcal{A}$  be a nonempty collection of nonempty bounded closed subsets of  $M$ ,  $H : \mathcal{A} \times \mathcal{A} \rightarrow E$  be an  $H$ -cone metric with respect to  $d$  and let  $\phi_1, \phi_2 : [0, 1] \rightarrow [0, 1]$  and  $\phi_3 : [0, 1) \rightarrow [0, 1)$  are mappings with  $\phi_1(\lambda_1) + \phi_2(\lambda_2) + \phi_3(\lambda_3) \leq 1$ . If a mapping  $T : M \rightarrow \mathcal{A}$  satisfies the generalized  $(\phi_1, \phi_2, \phi_3)$ -nonexpansive condition;*

$$H(Tx, Ty) \preceq \phi_1(\lambda_1)d(x, y) + \phi_2(\lambda_2)d(Tx, x) + \phi_3(\lambda_3)d(Ty, y),$$

for all  $x, y \in M$  with  $x \neq y$ ,

(2.4)

where  $\lambda_1, \lambda_2, \lambda_3$  are nonnegative constants with  $\lambda_1 + \lambda_2 + \lambda_3 \leq 1$  and  $\lambda_3 \neq 1$ , then  $\text{Fix}T \neq \emptyset$ .

*Proof.* Let  $\{\epsilon_n\}$  be a sequence in  $\text{int}P$  such that  $\|\epsilon_n\| \leq \phi^{n+1}(\lambda_i) < 1$  for some  $i \in \{1, 2, 3\} (n \in \mathbb{N})$ . Let  $x_0 \in M$  be an arbitrary fixed element and take  $x_1 \in Tx_0$ . From (H3), there exists  $x_2 \in Tx_1$  such that

$$d(x_1, x_2) \preceq H(Tx_0, Tx_1) + \epsilon_1.$$

Inductively, for  $x_{n-1} (n > 1)$ , we have  $x_n \in Tx_{n-1}$  satisfying

$$d(x_{n-1}, x_n) \preceq H(Tx_{n-2}, Tx_{n-1}) + \epsilon_{n-1}. \tag{2.5}$$

From the generalized  $(\phi_1, \phi_2, \phi_3)$ -nonexpansive condition (2.4) and (2.5), for any  $n \in \mathbb{N}$ ,

$$\begin{aligned} d(x_{n-1}, x_n) &\preceq H(Tx_{n-2}, Tx_{n-1}) + \epsilon_{n-1} \\ &\preceq \phi_1(\lambda_1)d(x_{n-2}, x_{n-1}) + \phi_2(\lambda_2)d(Tx_{n-2}, x_{n-2}) \\ &\quad + \phi_3(\lambda_3)d(Tx_{n-1}, x_{n-1}) + \epsilon_{n-1} \\ &\preceq \phi_1(\lambda_1)d(x_{n-2}, x_{n-1}) + \phi_2(\lambda_2)d(x_{n-1}, x_{n-2}) \\ &\quad + \phi_3(\lambda_3)d(x_n, x_{n-1}) + \epsilon_{n-1}. \end{aligned}$$

Hence,

$$(1 - \phi_3(\lambda_3))d(x_{n-1}, x_n) \preceq (\phi_1(\lambda_1) + \phi_2(\lambda_2))d(x_{n-2}, x_{n-1}) + \epsilon_{n-1}.$$

So, we have

$$\begin{aligned} d(x_{n-1}, x_n) &\preceq \frac{\phi_1(\lambda_1) + \phi_2(\lambda_2)}{1 - \phi_3(\lambda_3)}d(x_{n-2}, x_{n-1}) + \frac{1}{1 - \phi_3(\lambda_3)}\epsilon_{n-1} \\ &= \rho d(x_{n-1}, x_{n-2}) + \gamma \epsilon_{n-1}, \end{aligned}$$

where  $\rho = \frac{\phi_1(\lambda_1) + \phi_2(\lambda_2)}{1 - \phi_3(\lambda_3)}$ ,  $\gamma = \frac{1}{1 - \phi_3(\lambda_3)}$ .

For  $n \geq m$ ,

$$\begin{aligned} d(x_n, x_m) &\preceq d(x_n, x_{n-1}) + d(x_{n-1}, x_{n-2}) + \dots + d(x_{m+1}, x_m) \\ &\preceq \rho d(x_{n-1}, x_{n-2}) + \gamma \epsilon_{n-1} + \rho d(x_{n-2}, x_{n-3}) + \gamma \epsilon_{n-2} + \dots \\ &\quad + \rho d(x_m, x_{m-1}) + \gamma \epsilon_m \\ &\preceq \rho^2 d(x_{n-2}, x_{n-3}) + \rho \gamma \epsilon_{n-2} + \gamma \epsilon_{n-1} + \rho^2 d(x_{n-3}, x_{n-4}) \\ &\quad + \rho \gamma \epsilon_{n-3} + \gamma \epsilon_{n-2} + \dots + \rho^2 d(x_{m-1}, x_{m-2}) + \rho \gamma \epsilon_{m-1} + \gamma \epsilon_m \\ &\preceq \dots \end{aligned}$$

$$\begin{aligned}
&\preceq \rho^{n-1}d(x_1, x_0) + \gamma(\rho^{n-2}\epsilon_1 + \rho^{n-3}\epsilon_2 + \cdots + \epsilon_{n-1}) \\
&\quad + \rho^{n-2}d(x_1, x_0) + \gamma(\rho^{n-3}\epsilon_1 + \rho^{n-4}\epsilon_2 + \cdots + \epsilon_{n-2}) + \cdots \\
&\quad + \rho^m d(x_1, x_0) + \gamma(\rho^{m-1}\epsilon_1 + \rho^{m-2}\epsilon_2 + \cdots + \epsilon_m) \\
&= (\rho^{n-1} + \rho^{n-2} + \cdots + \rho^m)d(x_1, x_0) \\
&\quad + \gamma[(\rho^{n-2} + \rho^{n-3} + \cdots + \rho^{m-1})\epsilon_1 \\
&\quad + (\rho^{n-3} + \rho^{n-4} + \cdots + \rho^{m-2})\epsilon_2 \\
&\quad + \cdots + (\rho^{n-m} + \cdots + \rho)\epsilon_{m+1} + (\epsilon_{n-1} + \epsilon_{n-2} + \cdots + \epsilon_m)] \\
&\preceq \frac{\rho^m}{1-\rho}d(x_1, x_0) + \gamma\left[\frac{\rho^{m-1}}{1-\rho}\epsilon_1 + \frac{\rho^{m-2}}{1-\rho}\epsilon_2 + \cdots + \frac{\rho}{1-\rho}\epsilon_{m+1}\right. \\
&\quad \left. + \sum_{j=m}^{n-1} \epsilon_j\right].
\end{aligned}$$

By the normality of  $P$  with the normal constant  $K$ ,

$$\begin{aligned}
\|d(x_n, x_m)\| &\leq K\left\|\frac{\rho^m}{1-\rho}d(x_1, x_0) + \gamma\left[\frac{\rho^{m-1}}{1-\rho}\epsilon_1 + \frac{\rho^{m-2}}{1-\rho}\epsilon_2 + \cdots\right.\right. \\
&\quad \left.\left. + \frac{\rho}{1-\rho}\epsilon_{m+1} + \sum_{j=m}^{n-1} \epsilon_j\right]\right\| \\
&\leq \frac{\rho^m}{1-\rho}K\|d(x_1, x_0)\| + \gamma K\left[\frac{\rho^{m-1}}{1-\rho}\|\epsilon_1\| + \frac{\rho^{m-2}}{1-\rho}\|\epsilon_2\| + \cdots\right. \\
&\quad \left. + \frac{\rho}{1-\rho}\|\epsilon_{m+1}\| + \sum_{j=m}^{n-1} \|\epsilon_j\|\right] \\
&\leq \frac{\rho^m}{1-\rho}K\|d(x_1, x_0)\| + \gamma K\left[\frac{\rho^{m-1}}{1-\rho}\lambda_i^2 + \frac{\rho^{m-2}}{1-\rho}\lambda_i^3 + \cdots\right. \\
&\quad \left. + \frac{\rho}{1-\rho}\lambda_i^{m+2} + \sum_{j=m}^{n-1} \lambda_i^{j+1}\right],
\end{aligned}$$

which gives that  $\{x_n\}$  is a Cauchy sequence. Let  $x^* \in M$  be such that  $\lim_{n \rightarrow \infty} x_n = x^*$ , then, by Lemma 1.1.,

$$\lim_{n \rightarrow \infty} d(x_n, x^*) = 0. \quad (2.6)$$

Now, we show that a sequence  $\{Tx_n\}$  converges to  $Tx^*$  with respect to the  $H$ -cone metric. From the generalized nonexpansive condition (2.4), we have

$$\begin{aligned}
H(Tx_n, Tx^*) &\preceq \phi_1(\lambda_1)d(x_n, x^*) + \phi_2(\lambda_2)d(Tx_n, x_n) + \phi_3(\lambda_3)d(Tx^*, x^*) \\
&\preceq \phi_1(\lambda_1)d(x_n, x^*) + \phi_2(\lambda_2)d(Tx_n, x_n) \\
&\quad + \phi_3(\lambda_3)\{H(Tx^*, Tx_n) + d(Tx_n, x^*)\},
\end{aligned}$$

provided that  $x^* \notin Tx^*$ .

Thus,

$$H(Tx_n, Tx^*) \preceq \frac{1}{1 - \phi_3(\lambda_3)} \{ \phi_1(\lambda_1)d(x_n, x^*) + \phi_2(\lambda_2)d(x_{n+1}, x_n) + \phi_3(\lambda_3)d(x_{n+1}, x^*) \}$$

by the fact that  $x_{n+1} \in Tx_n (n > 1)$ . Hence  $H(Tx_n, Tx^*) \rightarrow 0$  as  $n \rightarrow \infty$ .

On the other hand, since  $x_n \in Tx_{n-1} (n > 1)$ , for arbitrary  $\delta_n \in \text{int}P (n > 1)$ , there exists  $y_n \in Tx^* (n > 1)$  satisfying

$$H(Tx_{n-1}, Tx^*) + \delta_n - d(x_n, y_n) \in P.$$

Thus

$$H(Tx_{n-1}, Tx^*) - d(x_n, y_n) \in P - \text{int}P,$$

which implies that  $\lim_{n \rightarrow \infty} d(x_n, y_n) = 0$  from the fact that  $\lim_{n \rightarrow \infty} H(Tx_{n-1}, Tx^*) = 0$ . Using (2.6), we have

$$\lim_{n \rightarrow \infty} d(y_n, x^*) \preceq \lim_{n \rightarrow \infty} (d(x_n, y_n) + d(x_n, x^*)) = 0,$$

which shows that  $y_n \rightarrow x^*$  as  $n \rightarrow \infty$ . Since  $Tx^*$  is closed,  $x^* \in Tx^*$ .  $\square$

By putting  $\phi_i = I (i = 1, 2, 3)$ , in Theorem 2.3., we obtain the following Corollary 2.4., which is a fixed point theorem for a generalized nonexpansive set-valued mapping in complete cone metric spaces.

**Corollary 2.4.** *Let  $(M, d)$  be a complete cone metric space for a normal cone  $P$  with the normal constant  $K$ ,  $\mathcal{A}$  be a nonempty collection of nonempty bounded closed subsets of  $M$  and let  $H : \mathcal{A} \times \mathcal{A} \rightarrow E$  be an  $H$ -cone metric with respect to  $d$ . If a mapping  $T : M \rightarrow \mathcal{A}$  satisfies the generalized nonexpansive condition;*

$$H(Tx, Ty) \preceq \lambda_1 d(x, y) + \lambda_2 d(Tx, x) + \lambda_3 d(Ty, y), \text{ for all } x, y \in M \text{ with } x \neq y,$$

where  $\lambda_1, \lambda_2, \lambda_3$  are nonnegative constants with  $\lambda_1 + \lambda_2 + \lambda_3 \leq 1$  and  $\lambda_3 \neq 1$ , then  $\text{Fix}T \neq \emptyset$ .

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SEUNG HYUN KIM  
DEPARTMENT OF MATHEMATICS, KYUNGSUNG UNIVERSITY, BUSAN 608-736, REPUBLIC OF  
KOREA  
*E-mail address:* `jiny0610@hotmail.com`

BYUNG SOO LEE  
DEPARTMENT OF MATHEMATICS, KYUNGSUNG UNIVERSITY, BUSAN 608-736, REPUBLIC OF  
KOREA  
*E-mail address:* `bslee@ks.ac.kr`