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ON THE CONVERGENCE OF HYBRID PROJECTION METHODS FOR ASYMPTOTICALLY PSEUDOCONTRACTIVE MAPPINGS IN THE INTERMEDIATE SENSE

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Abstract. In this paper, mappings which are asymptotically pseudocontractive in the intermediate sense are considered based on a hybrid projection method. Strong convergence theorems of fixed points are established in the framework of Hilbert spaces.

1. Introduction and preliminaries

In this paper, we always assume that H is a real Hilbert space whose inner product and norm are denoted by $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$, respectively. Let C be a nonempty closed convex subset of H and $T : C \to C$ a nonlinear mapping. In this paper, we use $F(T)$ to denote the fixed point set of T.

Recall that T is said to be *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \ \forall x, y \in C. \tag{1.1}$$

T is said to be *asymptotically nonexpansive* if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $k_n \to 1$ as $n \to \infty$ such that

$$\|T^n x - T^n y\| \leq k_n \|x - y\|, \ \forall n \geq 1, x, y \in C. \tag{1.2}$$

The class of asymptotically nonexpansive mappings was introduced by Goebel and Kirk [5] as a generalization of the class of nonexpansive mappings. They proved that if C is a nonempty closed convex and bounded subset of a real uniformly convex Banach space and T is an asymptotically nonexpansive mapping on C, then T has a fixed point.

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T is said to be *asymptotically nonexpansive in the intermediate sense* if it is continuous and the following inequality holds:

$$\limsup_{n\to\infty} \sup_{x,y\in C} \big(\|T^n x - T^n y\| - \|x-y\|\big) \le 0. \tag{1.3}$$

Observe that if we define

$$\tau_n = \max\big\{0, \sup_{x,y\in C} (\|T^n x - T^n y\| - \|x-y\|)\big\}, \tag{1.4}$$

then $\tau_n \to 0$ as $n \to \infty$. It follows that (1.3) is reduced to

$$\|T^n x - T^n y\| \le \|x-y\| + \tau_n, \quad \forall n \ge 1, x, y \in C. \tag{1.5}$$

The class of mappings which are asymptotically nonexpansive in the intermediate sense was introduced by Bruck, Kuczumow and Reich [3]. It is known [10] that if C is a nonempty closed convex bounded subset of a uniformly convex Banach space E and T is asymptotically nonexpansive in the intermediate sense, then T has a fixed point. It is worth mentioning that the class of mappings which are asymptotically nonexpansive in the intermediate sense contains properly the class of asymptotically nonexpansive mappings.

Recall that T is said to be *strictly pseudocontractive* if there exists a constant $\kappa \in [0, 1)$ such that

$$\|Tx - Ty\| \le \|x-y\|^2 + \kappa\|(I-T)x - (I-T)y\|^2, \quad \forall x, y \in C. \tag{1.6}$$

The class of strictpseudocontractions was introduced by Browder and Petryshyn [2] in a real Hilbert space.

Recall that T is said to be an *asymptotically strict pseudocontraction* if there exist a constant $\kappa \in [0, 1)$ and a sequence $\{k_n\} \subset [1, \infty)$ with $k_n \to 1$ as $n \to \infty$ such that

$$\|T^n x - T^n y\|^2 \le k_n\|x-y\|^2 + \kappa\|(I-T^n)x - (I-T^n)y\|^2, \quad \forall x, y \in C. \tag{1.7}$$

The class of asymptotically strict pseudocontractions was introduced by Qihou [17] in 1996.

Recently, Sahu, Xu and Yao [23] introduced a class of new mappings: *asymptotically strict pseudocontractive mappings in the intermediate sense.* Recall that T is said to be an *asymptotically strict pseudocontraction in the intermediate sense* if

$$\limsup_{n\to\infty} \sup_{x,y\in C} \big(\|T^n x - T^n y\|^2 - k_n\|x-y\|^2 - \kappa\|(I-T^n)x - (I-T^n)y\|^2\big) \le 0, \tag{1.8}$$

where $\kappa \in [0, 1)$ and $\{k_n\} \subset [1, \infty)$ such that $k_n \to 1$ as $n \to \infty$. Put

$$\xi_n = \max\big\{0, \sup_{x,y\in C} \big(\|T^n x - T^n y\|^2 - k_n\|x-y\|^2 - \kappa\|(I-T^n)x - (I-T^n)y\|^2\big)\big\}. \tag{1.9}$$

It follows that $\xi_n \to 0$ as $n \to \infty$. Then, (1.8) is reduced to the following:

$$\|T^n x - T^n y\|^2 \le k_n\|x-y\|^2 + \kappa\|(I-T^n)x - (I-T^n)y\|^2 + \xi_n, \quad \forall x, y \in C. \tag{1.10}$$

Recall that T is said to be an *asymptotically pseudocontraction* if there exists a sequence $\{k_n\} \subset [1,\infty)$ with $k_n \to 1$ as $n \to \infty$ such that

$$\langle T^n x - T^n y, x-y\rangle \le k_n \|x-y\|^2, \quad \forall x,y \in C. \tag{1.11}$$

It is clear that (1.11) is equivalent to

$$\|T^n x - T^n y\|^2 \le k_n \|x-y\|^2 + \|(I-T^n)x - (I-T^n)y\|^2, \quad \forall x,y \in C. \tag{1.12}$$

The class of asymptotically pseudocontractive mapping was introduced by Schu [24]. In [22], Rhoades gave an example to show that the class of asymptotically pseudocontractive mappings contains properly the class of asymptotically nonexpansive mappings; see [22] for more details. In 2009, Zhou [27] showed that every uniformly Lipschitz and asymptotically pseudocontractive mapping which is also uniformly asymptotically regular has a fixed point.

Recently, Qin, Cho and Kim [21] introduced the class of mappings which are *asymptotically pseudocontractive mappings in the intermediate sense.*

Recall that T is said to be an *asymptotically pseudocontractive mapping in the intermediate sense* if

$$\limsup_{n\to\infty} \sup_{x,y\in C} \left(\langle T^n x - T^n y, x-y\rangle - k_n\|x-y\|^2\right) \le 0, \tag{1.13}$$

where $\{k_n\}$ is a sequence in $[1,\infty)$ such that $k_n \to 1$ as $n \to \infty$. Put

$$\nu_n = \max\big\{0, \sup_{x,y\in C} (\langle T^n x - T^n y, x-y\rangle - k_n\|x-y\|^2)\big\}. \tag{1.14}$$

It follows that $\nu_n \to 0$ as $n \to \infty$. Then, (1.13) is reduced to the following:

$$\langle T^n x - T^n y, x-y\rangle \le k_n\|x-y\|^2 + \nu_n, \quad \forall n \ge 1, x,y \in C. \tag{1.15}$$

It is easy to see that (1.15) is equivalent to

$$\begin{aligned}\|T^n x - T^n y\|^2 \le (2k_n - 1)\|x-y\|^2 &+ \|(I-T^n)x - (I-T^n)y\|^2 \\ &+ 2\nu_n, \ \forall n \ge 1, x,y \in C.\end{aligned} \tag{1.16}$$

We remark that if $\nu_n = 0$ for each $n \ge 1$, then the class of asymptotically pseudocontractive mappings in the intermediate sense is reduced to the class of asymptotically pseudocontractive mappings.

Recall that the normal Mann iteration was introduced by Mann [11] in 1953. Since then, the constructions of fixed points for nonexpansive mappings via the Mann iteration have been extensively investigated by many authors. The Mann iteration generates a sequence $\{x_n\}$ in the following manner:

$$\begin{cases} x_1 \in C \quad \textit{chosen arbitrarily}, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n, \quad \forall n \ge 1, \end{cases} \tag{1.17}$$

where $\{\alpha_n\}$ is a sequence in the interval $(0,1)$.

If T is a nonexpansive mapping with a fixed point and the control sequence $\{\alpha_n\}$ is chosen so that $\sum_{n=1}^{\infty} \alpha_n(1-\alpha_n) = \infty$, then the sequence $\{x_n\}$ generated by the Mann iteration (1.17) converges weakly to a fixed point of T.

It is well known that, in the infinite-dimensional Hilbert space, only weak convergence theorems for the normal Mann iteration were established even for nonexpansive mappings, see [4]. Attempts to modify the normal Mann iteration for nonexpansive mapping by hybrid projection methods have recently been made so that strong convergence theorems are obtained; see, for example, [1], [6]-[9], [12]-[16], [18]-[21], [25]-[27] and the references therein.

In 2008, Kim and Xu [7] considered the class of asymptotically strict pseudocontractions. To be more precise, they obtained the following results.

Theorem KX. *Let C be a closed convex subset of a Hilbert space H and let $T : C \to C$ be an asymptotically κ-strict pseudocontraction for some $0 \le \kappa < 1$. Assume that the fixed point set $F(T)$ of T is nonempty and bounded. Let $\{x_n\}$ be the sequence generated by the following (CQ) algorithm*

$$
\begin{cases}
x_0 \in C \quad \text{chosen arbitrarily}, \\
y_n = \alpha_n x_n + (1-\alpha_n)T^n x_n, \\
C_n = \{z \in C : \|y_n - z\|^2 \le \|x_n - z\|^2 \\
\qquad\qquad + [\kappa - \alpha_n(1-\alpha_n)]\|x_n - T^n x_n\|^2 + \theta_n\}, \\
Q_n = \{z \in C : \langle x_0 - x_n, x_n - z\rangle \ge 0\}, \\
x_{n+1} = P_{C_n \cap Q_n} x_0,
\end{cases}
\tag{1.18}
$$

where

$$
\theta_n = \Delta_n^2(1-\alpha_n)(k_n - 1) \to 0 \ (n \to \infty), \ \Delta_n = \sup\{\|x_n - z\| : z \in F(T)\} < \infty.
$$

Assume that the control sequence $\{\alpha_n\}$ is chosen so that $\limsup_{n\to\infty} \alpha_n < 1-\kappa$. Then $\{x_n\}$ converges strongly to $P_{F(T)}x_0$.

Recently, Sahu, Xu and Yao [23] extended Theorem KX by considering the class of asymptotically strict pseudocontractions in the intermediate sense. To be more precise, they obtained the following results.

Theorem SXY. *Let C be a nonempty closed convex subset of a real Hilbert space H and $T : C \to C$ a uniformly continuous asymptotically κ-strict pseudocontractive mapping in the intermediate sense with sequence $\{k_n\}$ such that $F(T)$ is nonempty and bounded. Let $\{\alpha_n\}$ be a sequence in $[0, 1]$ such that $0 < \delta \le \alpha_n \le 1 - \kappa$ for all n. Let $\{x_n\}$ be the sequence in C generated by the following (CQ) algorithm*

$$
\begin{cases}
u = x_1 \in C \quad \text{chosen arbitrarily}, \\
y_n = (1-\alpha_n)x_n + \alpha_n T^n x_n, \\
C_n = \{z \in C : \|y_n - z\|^2 \le \|x_n - z\|^2 + \theta_n\}, \\
Q_n = \{z \in C : \langle u - x_n, x_n - z\rangle \ge 0\}, \\
x_{n+1} = P_{C_n \cap Q_n} u,
\end{cases}
\tag{1.19}
$$

where

$$\theta_n = \max\left\{0, \sup_{x,y\in C}\left(\|T^nx - T^ny\|^2 - k_n\|x-y\|^2 - \kappa\|(I-T^n)x - (I-T^n)y\|^2\right)\right\} \\ + (k_n-1)\Delta_n$$

and

$$\Delta_n = \sup\{\|x_n - z\| : z \in F(T)\} < \infty.$$

Then $\{x_n\}$ *converges strongly to* $P_{F(T)}u$.

For the class of asymptotically pseudocontractive mappings, Zhou [27] obtained the following results.

Theorem Z. *Let* C *be a closed convex bounded subset of a real Hilbert space* H. *Let* $T : C \to C$ *be a uniformly* L*-Lipschitzian and asymptotically pseudocontractive mapping with a fixed point. Assume that the control sequence* $\{\alpha_n\}$ *is chosen so that* $\alpha_n \in [a, b]$ *for some* $a, b \in (0, \frac{1}{1+L})$. *Let a sequence* $\{x_n\}$ *be generated in the following manner*:

$$(1.20)\qquad \begin{cases} x_0 \in C \quad \text{chosen arbitrarily}, \\ y_n = (1-\alpha_n)x_n + \alpha_nT^nx_n, \\ C_n = \{z \in C : \alpha_n[1-(1+L)\alpha_n]\|x_n - T^nx_n\|^2 \\ \qquad\qquad \leq \langle x_n - z, (I-T^n)y_n\rangle + (k_n-1)(\text{diam C})^2\}, \\ Q_n = \{z \in C : \langle z - x_n, x_n - z\rangle \geq 0\}, \\ x_{n+1} = P_{C_n\cap Q_n}x_0. \end{cases}$$

Then $\{x_n\}$ *converges strongly to* $P_{F(T)}x_0$.

In this paper, motivated Theorem KX, Theorem SXY and Theorem Z, we consider mappings which are asymptotically pseudocontractive mappings in the intermediate sense based on a shrinking projection method. Strong convergence theorems of fixed points are established in the framework of real Hilbert spaces.

In order to prove our main results, we also need the following lemmas.

Lemma 1.1. *Let* C *be a nonempty closed convex subset of real Hilbert space* H. *Given* $x \in H$ *and* $z = P_Cx$, *then if and only if there holds the relation*

$$\langle x - z, z - y\rangle \geq 0, \quad \forall y \in C.$$

Lemma 1.2 ([21, Qin, Cho and Kim])**.** *Let* C *be a nonempty closed convex bounded subset of* H *and* T *a uniformly* L*-Lipschitzian and asymptotically pseudocontractive mapping in the intermediate sense. Then* $F(T)$ *is a closed convex subset of* C.

2. Main results

Theorem 2.1. *Let* C *be a nonempty closed convex bounded subset of a real Hilbert space* H, P_C *the metric projection from* H *onto* C *and* $T : C \to C$ *a mapping which is uniformly* L*-Lipschitz and asymptotically pseudocontractive* *in the intermediate sense with the sequence* $\{k_n\} \subset [1, \infty)$*, where* $k_n \to 1$ *as* $n \to \infty$*. Assume that* $F(T)$ *is nonempty. Let* $\{x_n\}$ *be a sequence generated in the following manner*:

$$\tag{2.1} \begin{cases} x_0 \in H, \quad \textit{chosen arbitrarily}, \\ C_1 = C, \\ x_1 = P_{C_1} x_0, \\ y_n = (1-\alpha_n) x_n + \alpha_n T^n x_n, \\ C_{n+1} = \{v \in C_n : \alpha_n [1-(1+L)\alpha_n] \|x_n - T^n x_n\|^2 \\ \qquad\qquad \leq \langle x_n - v, y_n - T^n y_n \rangle + (k_n - 1)(\operatorname{diam} C)^2 + \nu_n\}, \\ x_{n+1} = P_{C_{n+1}} x_0, \end{cases}$$

where

$$\nu_n = \max\bigl\{0, \sup_{y_n, p \in C} (\langle p - T^n y_n, p - y_n \rangle - k_n \|p - y_n\|^2)\bigr\}, \quad \forall p \in F(T).$$

Assume that the control sequence $\{\alpha_n\}$ *is chosen such that* $\alpha_n \in [a, b]$ *for some* $a, b \in (0, \frac{1}{1+L})$*. Then the sequence* $\{x_n\}$ *generated in* (2.1) *converges strongly to* $P_{F(T)} x_0$*.*

Proof. From Lemma 1.2, we see that $F(T)$ is closed and convex. This shows that $P_{F(T)} x_0$ is well defined for each $x_0 \in H$. Next, we show that C_n is closed and convex for all $n \geq 1$. This can be proved by induction on n. Indeed, we know that $C_1 = C$ is closed and convex. Assume that C_m is closed and convex for some $m \geq 1$. Next, we show C_{m+1} is also closed and convex for the same m. It is obvious C_{m+1} is closed. Take v_1 and v_2 in C_{m+1}. Then we get that

$$\tag{2.2} \begin{aligned} &\alpha_m [1-(1+L)\alpha_m] \|x_m - T^m x_m\|^2 \\ &\leq \langle x_m - v_1, y_m - T^m y_m \rangle + (k_m - 1)(\operatorname{diam} C)^2 + \nu_m \end{aligned}$$

and

$$\tag{2.3} \begin{aligned} &\alpha_m [1-(1+L)\alpha_m] \|x_m - T^m x_m\|^2 \\ &\leq \langle x_m - v_2, y_m - T^m y_m \rangle + (k_m - 1)(\operatorname{diam} C)^2 + \nu_m, \end{aligned}$$

where $v_1, v_2 \in C_m$. Let $v = t v_1 + (1-t) v_2$, where $t \in (0,1)$. Multiply (2.2) by t, (2.3) by $(1-t)$ and add their both sides respectively. Then we obtain that

$$\begin{aligned} &\alpha_m [1-(1+L)\alpha_m] \|x_m - T^m x_m\|^2 \\ &\leq \langle x_m - v, y_m - T^m y_m \rangle + (k_m - 1)(\operatorname{diam} C)^2 + \nu_m. \end{aligned}$$

This means $v \in C_{m+1}$. This implies that C_n is closed and convex for all $n \geq 1$. Now, we are in a position to show that $F(T) \subset C_n$ for all $n \geq 1$. From the assumption, we see $F(T) \subset C_1 = C$. Suppose that $F(T) \subset C_j$ for some $j \geq 1$. Next, we show that $F(T) \subset C_{j+1}$ for the same j, which completes the proof.

For any $p \in F(T) \subset C_j$, we calculate that

$$\begin{aligned}
&\|x_j - T^j x_j\|^2 \\
&= \langle x_j - T^j x_j, x_j - T^j x_j \rangle \\
&= \frac{1}{\alpha_j} \langle x_j - y_j, x_j - T^j x_j \rangle \\
&= \frac{1}{\alpha_j} \langle x_j - y_j, x_j - T^j x_j - (y_j - T^j y_j) \rangle + \frac{1}{\alpha_j} \langle x_j - y_j, y_j - T^j y_j \rangle \\
&\le \frac{1+L}{\alpha_j} \|x_j - y_j\|^2 + \frac{1}{\alpha_j} \langle x_j - p + p - y_j, y_j - T^j y_j \rangle \\
&= \frac{1+L}{\alpha_j} \|x_j - y_j\|^2 + \frac{1}{\alpha_j} \langle x_j - p, y_j - T^j y_j \rangle + \frac{1}{\alpha_j} \langle p - y_j, y_j - T^j y_j \rangle \\
&= \frac{1+L}{\alpha_j} \|x_j - y_j\|^2 + \frac{1}{\alpha_j} \langle x_j - p, y_j - T^j y_j \rangle \\
&\quad + \frac{1}{\alpha_j} (\langle p - y_j, y_j - p \rangle + \langle p - y_j, p - T^j y_j \rangle) \\
&\le (1+L)\alpha_j \|x_j - T^j x_j\|^2 + \frac{1}{\alpha_j} \langle x_j - p, y_j - T^j y_j \rangle \\
&\quad + \frac{1}{\alpha_j} \big((k_j - 1)\|p - y_j\|^2 + \nu_j \big),
\end{aligned}$$

where

$$\nu_j = \max\big\{0, \sup_{y_j, p \in C} (\langle p - T^j y_j, p - y_j \rangle - k_j \|p - y_j\|^2)\big\}, \quad \forall p \in F(T).$$

It follows that

$$\begin{aligned}
&\alpha_j[1 - (1+L)\alpha_j]\|x_j - T^j x_j\|^2 \\
&\le \langle x_j - p, y_j - T^j y_j \rangle + (k_j - 1)\|p - y_j\|^2 + \nu_j.
\end{aligned}$$

This means $p \in C_{j+1}$. Hence, we get that $F(T) \subset C_n$ for all $n \ge 1$. Notice that the sequence $\{x_n\}$ is a Cauchy sequence. Indeed, we have the following. Since $x_{n+1} = P_{C_{n+1}} x_0 \in C_{n+1} \subset C_n$, we get that

$$\begin{aligned}
0 &\le \langle x_0 - x_n, x_n - x_{n+1} \rangle \\
&= \langle x_0 - x_n, x_n - x_0 + x_0 - x_{n+1} \rangle \\
&\le -\|x_0 - x_n\|^2 + \|x_n - x_0\|\|x_{n+1} - x_0\|,
\end{aligned}$$

which yields that

$$\|x_n - x_0\| \le \|x_{n+1} - x_0\|.$$

From the boundness of $\{x_n\}$ we see that $\lim_{n\to\infty} \|x_n - x_0\|$ exists. For any positive integer $m \ge 1$, we know that $x_{n+m} = P_{C_{n+m}} x_0 \in C_{n+m} \subset C_n$. From $x_n = P_{C_n} x_0$, we obtain that

$$\langle x_0 - x_n, x_n - x_{n+m} \rangle \ge 0.$$

On the other hand, we have

$$\begin{aligned}\|x_n - x_{m+n}\|^2 &= \|x_n - x_0 + x_0 - x_{m+n}\|^2\\ &= \|x_n - x_0\|^2 + \|x_0 - x_{m+n}\|^2 - 2\langle x_0 - x_n, x_0 - x_{n+m}\rangle\\ &= \|x_0 - x_{m+n}\|^2 - \|x_n - x_0\|^2 - 2\langle x_0 - x_n, x_n - x_{n+m}\rangle\\ &\le \|x_0 - x_{m+n}\|^2 - \|x_n - x_0\|^2.\end{aligned}$$

It follows that $\|x_n - x_{m+n}\| \to 0$ as $n \to \infty$. This implies that $\{x_n\}$ is a Cauchy sequence. Since H is a Hilbert space and C is closed convex, we may assume that

$$x_n \to \omega \in C \quad \text{as } n \to \infty.$$

Since $x_{n+1} \in C_{n+1}$ and T is uniformly Lipschitz, we can easily conclude that $\omega \in F(T)$.

Next, we show that $\omega = P_{F(T)}x_0$. In view of Lemma 1.1, we obtain from $x_n = P_{C_n}x_0$ that

$$\langle x_0 - x_n, x_n - p\rangle \ge 0, \quad \forall p \in F(T) \subset C_n.$$

Hence, we have

$$\langle x_0 - \omega, \omega - p\rangle \ge 0, \quad \forall p \in F(T).$$

From Lemma 1.1, we see that $\omega = P_{F(T)}x_0$. This proof is completed. □

Remark 2.2. Comparing with Theorem Z, we have the following:

(a) the class of mappings is extended from the class of asymptotically pseudocontractive mappings to the class of asymptotically pseudocontractive mappings in the intermediate sense;
(b) the iterative set Q_n is removed;
(c) the mapping $I - T$ is demi-closed at zero which is removed.

Corollary 2.3. *Let C be a nonempty closed convex bounded subset of a real Hilbert space H, P_C the metric projection from H onto C and $T : C \to C$ a uniformly L-Lipschitz and asymptotically pseudocontractive mapping with the sequence $\{k_n\} \subset [1, \infty)$, where $k_n \to 1$ as $n \to \infty$. Assume that $F(T)$ is nonempty. Let $\{x_n\}$ be a sequence generated in the following manner:*

$$\begin{cases} x_0 \in H, \quad \textit{chosen arbitrarily},\\ C_1 = C,\\ x_1 = P_{C_1}x_0,\\ y_n = (1-\alpha_n)x_n + \alpha_n T^n x_n,\\ C_{n+1} = \{v \in C_n : \alpha_n[1-(1+L)\alpha_n]\|x_n - T^n x_n\|^2\\ \qquad\qquad \le \langle x_n - v, y_n - T^n y_n\rangle + (k_n - 1)(\operatorname{diam} C)^2\},\\ x_{n+1} = P_{C_{n+1}}x_0.\end{cases}$$

Assume that the control sequence $\{\alpha_n\}$ is chosen such that $\alpha_n \in [a, b]$ for some $a, b \in (0, \frac{1}{1+L})$. Then the sequence $\{x_n\}$ converges strongly to $P_{F(T)}x_0$.

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