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Modified Scheme for Tsunami Propagation with Variable Water Depths

Ha, Taemin* / Seo, Kyu-Hak** / Kim, Jihun*** / Cho, Yong-Sik****

Abstract

In this study, a modified dispersion-correction scheme describing tsunami propagation on variable water depths is proposed by introducing additional terms to the previous numerical scheme. The governing equations used in previous tsunami propagation models are slightly modified to consider the effects of a bottom slope. The numerical dispersion of the proposed model replaces the physical dispersion of the governing equations. Then, the modified scheme is employed to simulate tsunami propagation on variable water depths and numerical results are compared with those of the previous tsunami propagation model.

Keywords : tsunami, dispersion effects, variable water depth, boussinesq equations, modified scheme

1. INTRODUCTION

The frequency and severity of damage from tsunamis has seriously increased in the past 20 years because of active movement of tectonic plates. For example, the 2004 Sumatra tsunami resulted in around 300 thousand casualties and enormous property damage. In 1983, the Central East Sea Tsunami occurred on the western coast of Japan propagated across the East Sea and caused loss of human lives and property damage on the eastern coast of the Korean Peninsula (Cho et al., 2004). Furthermore, recently ocean earthquakes near the Korean Peninsula have increased in number. A fundamental way to minimize casualty and property damage in the coastal area resulted from tsunami inundation is to establish tsunami hazard inundation maps for the coastal areas of the East Sea (National Emergency Management Agency, 2008).

When a tsunami propagates over a long distance, in general, the frequency dispersion may play an important role (Cho et al., 2007). Thus, the linear Boussinesq equations may be adequate to describe the propagation of distant tsunamis (Cho, 1995). However,

* 한양대학교 일반대학원 건설환경공학과 박사과정 (e-mail: kevin4324@hanyang.ac.kr)
PhD. Student, Dept. of Civil and Envir. Engrg., Hanyang Univ., Seoul 133-791, Korea
** 한양대학교 일반대학원 건설환경공학과 박사과정 (e-mail: seokh0919@dycons.co.kr)
PhD. Student, Dept. of Civil and Envir. Engrg., Hanyang Univ., Seoul 133-791, Korea
*** 한양대학교 일반대학원 건설환경공학과 석사과정 (e-mail: k8k8k8@hanyang.ac.kr)
Graduate Student, Dept. of Civil and Envir. Engrg., Hanyang Univ., Seoul 133-791, Korea
**** 교신저자, 한양대학교 공과대학 건설환경공학과 교수 (e-mail: ysc59@hanyang.ac.kr)
Corresponding Author, Professor, Dept. of Civil and Envir. Engrg., Hanyang Univ., Seoul 133-791, Korea

solving the linear Boussinesq equations directly may not be an economical approach for simulating the transoceanic propagation of tsunamis (Imamura et al., 1988; Cho et al., 2007). As a result, a finite difference numerical model which can consider the dispersion effects has been developed by using the linear shallow-water equations instead of the linear Boussinesq equations.

Imamura and Goto (1988) first used numerical dispersion of a leap-frog scheme to consider dispersion effects. Cho and Yoon (1998) advanced in considering proper dispersion effects. However, the application of numerical models developed by Imamura and Goto (1988) and Cho and Yoon (1998) is limited to the case of constant water depth when a uniform finite difference grid is employed. In other words, both numerical models have a limitation that spatial grid size and time step sizes should be changed continuously as the water depth changes. Recently, Cho et al. (2007) developed a practical numerical model to simulate the distant propagation of tsunamis in real topography. This model can release the limitation of spatial grid and temporal step sizes, however, the governing equations of this model are derived on constant water depth. That is, this model has to be cautious in being employed in real topography and some limitations should be included during numerical simulation.

In this study, a modified dispersion-correction scheme describing tsunami propagation on variable water depths is proposed by introducing additional terms to Cho et al. (2007)'s scheme. The governing equations used in previous tsunami propagation models are slightly modified to consider the effect of a bottom slope and the numerical dispersion of the proposed model replaces the physical dispersion of the governing equations.

2. GOVERNING EQUATIONS

The wavelengths of tsunamis are much longer than those of wind-generated waves and shorter than those of tides. Thus, the dispersion effects of tsunamis are relatively strong and should be properly considered in numerical simulation of tsunami propagation to achieve better accuracy. Since the free surface displacements are small compared to a local water depth, the nonlinearity of waves can be neglected for the case of transoceanic propagation. As a result, following linear Boussinesq equations have been employed to simulate tsunami propagation in tsunami modelling (Cho et al., 2007; Yoon, 2002).

$$\frac{\partial \zeta}{\partial t}+\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}=0 \tag{1}$$

$$\frac{\partial P}{\partial t}+gh\frac{\partial \zeta}{\partial x}=\frac{h^2}{2}\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left(\frac{\partial P}{\partial t}\right)+\frac{\partial}{\partial y}\left(\frac{\partial Q}{\partial t}\right)\right]-\frac{h^3}{6}\frac{\partial}{\partial x}\left[\frac{\partial^2}{\partial t\partial x}\left(\frac{P}{h}\right)+\frac{\partial^2}{\partial t\partial y}\left(\frac{Q}{h}\right)\right] \tag{2}$$

$$\frac{\partial Q}{\partial t}+gh\frac{\partial \zeta}{\partial y}=\frac{h^2}{2}\frac{\partial}{\partial y}\left[\frac{\partial}{\partial x}\left(\frac{\partial P}{\partial t}\right)+\frac{\partial}{\partial y}\left(\frac{\partial Q}{\partial t}\right)\right]-\frac{h^3}{6}\frac{\partial}{\partial y}\left[\frac{\partial^2}{\partial t\partial x}\left(\frac{P}{h}\right)+\frac{\partial^2}{\partial t\partial y}\left(\frac{Q}{h}\right)\right] \tag{3}$$

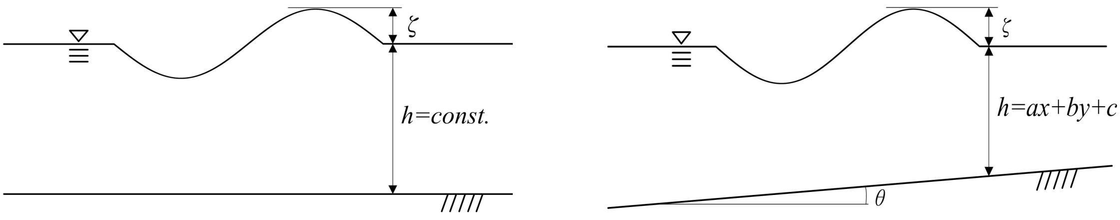
where, ζ is the free surface displacement, h is the still water depth, and P and Q are the depth-averaged volume fluxes in the $x-$ and $y-$axis directions, respectively, and g is the acceleration of gravity. It is remarked that the right-hand side terms of Eqs. (2) and (3) represent the frequency dispersion.

The linear Boussinesq Equations are reduced over a constant depth as the following equation (Mei, 1989).

$$\frac{\partial^2 \zeta}{\partial t^2}-gh\left(\frac{\partial^2 \zeta}{\partial x^2}+\frac{\partial^2 \zeta}{\partial y^2}\right)=\frac{gh^3}{3}\left(\frac{\partial^4 \zeta}{\partial x^4}+2\frac{\partial^4 \zeta}{\partial x^2\partial y^2}+\frac{\partial^4 \zeta}{\partial y^4}\right) \tag{4}$$

Recently, Cho et al. (2007) developed a practical numerical model to simulate the distant propagation of tsunamis by using above equations as governing equations. However, these equations are derived on a constant water depth and some errors should be included during numerical simulation in real topography. Therefore, governing equations which has been used in previous tsunami propagation models are required to be slightly modified to reduce this type of numerical errors.

In this study, bottom slope coefficients a and b are introduced to consider variable water depths. Bottom slopes are assumed to be the first-order equation and the coefficients are defined as $a=\partial h/\partial x$ and $b=\partial h/\partial y$, respectively (Fig. 1). With these coefficients, the linear Boussinesq equations can be reduced as following equation.



(a) Cho et al. (2007) (b) Present scheme

Fig. 1. Difference in Assumption between the Present Scheme and Cho et al.'s (2007)

$$\begin{aligned}&\frac{\partial^2\zeta}{\partial t^2}-g\left(a\frac{\partial\zeta}{\partial x}+b\frac{\partial\zeta}{\partial y}\right)-\\&\quad gh\left[\left(2a^2+b^2+1\right)\frac{\partial^2\zeta}{\partial x^2}+2ab\frac{\partial^2\zeta}{\partial x\partial y}+\left(a^2+2b^2+1\right)\frac{\partial^2\zeta}{\partial y^2}\right]\\&-gh^2\left[2a\left(\frac{\partial^3\zeta}{\partial x^3}+\frac{\partial^3\zeta}{\partial x\partial y^2}\right)+2b\left(\frac{\partial^3\zeta}{\partial x^2\partial y}+\frac{\partial^3\zeta}{\partial y^3}\right)\right]-\\&\frac{gh^3}{3}\left(\frac{\partial^4\zeta}{\partial x^4}+2\frac{\partial^4\zeta}{\partial x^2\partial y^2}+\frac{\partial^4\zeta}{\partial y^4}\right)=0\end{aligned}\qquad(5)$$

The equations of motion, (2) and (3), include dispersion terms on the right hand side to take into account the physical dispersion of the waves resulting from the non-hydrostatic pressure forces. These dispersion terms introduce numerical difficulties in practice due to the presence of mixed type of differentiations with respect to time and space at the same time. Because of these difficulties, the following shallow water equations are generally employed instead. The shallow water equations can be solved using a relatively simple numerical scheme such as the leapfrog finite difference technique and the physical dispersion of waves is replaced by the numerical dispersion arising from the explicit scheme (Cho and Yoon, 1998; Yoon, 2002).

$$\frac{\partial\zeta}{\partial t}+\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}=0\qquad(6)$$

$$\frac{\partial P}{\partial t}+gh\frac{\partial\zeta}{\partial x}=0\qquad(7)$$

$$\frac{\partial Q}{\partial t}+gh\frac{\partial\zeta}{\partial y}=0\qquad(8)$$

3. NUMERICAL SCHEME

In this study, a staggered grid system has been used to discretize the governing equations. As shown in Fig. 2, indices (i,j) and n denote spatial nodes and the time level, respectively. The spatial grid sizes in the $x-$ and $y-$axis directions are represented by Δx and Δy, respectively, and the time step size is symbolized by Δt.

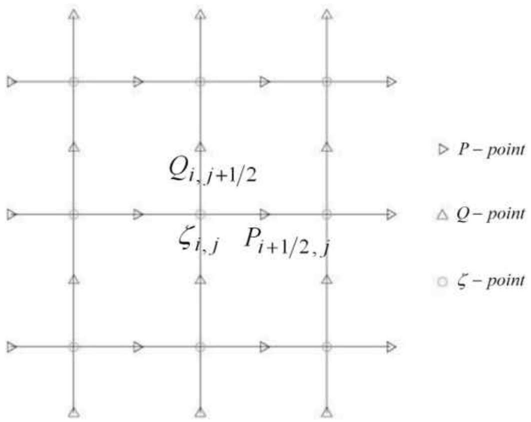


Fig. 2. Staggered Grid System

First, we start from a modified equation of the practical dispersion-correction scheme (Cho et al., 2007). Eq. (9) is a modified equation of the practical dispersion-correction scheme.

$$\begin{aligned}&\frac{\partial^2\zeta}{\partial t^2}-g\left(\frac{\partial\zeta}{\partial x}+\frac{\partial\zeta}{\partial y}\right)-gh\left(\frac{\partial^2\zeta}{\partial x^2}+\frac{\partial^2\zeta}{\partial y^2}\right)\\&\quad-gh\frac{(\Delta x)^2}{12}\left(1+\alpha-gh\frac{(\Delta t)^2}{(\Delta x)^2}\right)\left(\frac{\partial^4\zeta}{\partial x^4}+2\frac{\partial^4\zeta}{\partial x^2\partial y^2}+\frac{\partial^4\zeta}{\partial x^4}\right)\\&\quad+(1+\alpha-\gamma)\frac{(\Delta x)^2}{6}gh\frac{\partial^4\zeta}{\partial x^2\partial y^2}\end{aligned}\qquad(9a)$$

$$\begin{aligned}&+g^2\frac{(\Delta t)^2}{12}\left[\left(2a^2+b^2\right)\frac{\partial^2\zeta}{\partial x^2}+2ab\frac{\partial^2\zeta}{\partial x\partial y}+\left(a^2+2b^2\right)\frac{\partial^2\zeta}{\partial y^2}\right]\\&+g^2h\frac{(\Delta t)^2}{6}\left[2a\left(\frac{\partial^3\zeta}{\partial x^3}+\frac{\partial^3\zeta}{\partial x\partial y^2}\right)+2b\left(\frac{\partial^3\zeta}{\partial x^2\partial y}+\frac{\partial^3\zeta}{\partial y^3}\right)\right]\end{aligned}\qquad(9b)$$

$$-g\frac{(\Delta x)^2}{12}\left[(\alpha+2)\left(a\frac{\partial^3\zeta}{\partial x^3}+b\frac{\partial^3\zeta}{\partial y^3}\right)+\left(a\frac{\partial^3\zeta}{\partial x\partial y^2}+b\frac{\partial^3\zeta}{\partial x^2\partial y}\right)\right]=O^3\qquad(9c)$$

By comparing this with the modified governing equation, which is Eq. (5), the first three terms of Eq. (9-a) is identical to the corresponding governing equation and the last two terms can represent the last terms of the governing equation by controlling dispersion-correction factors, α and γ. Eq. (9-b) can be neglected because a temporal grid size is much smaller than a spatial grid size in the tsunami propagation model, however, Eq. (9c) may play an important role during computation because bottom slope coefficients cannot be ignored. Therefore, Eq. (9c) should be considered in numerical simulation and additional dispersion-correction factors are employed to control this numerical dispersion.

As a result, following modified dispersion-correction schemes are proposed. In Eqs. (10) and (11), dispersion-correction factors, α and γ are same with those of the practical scheme (Cho et al., 2007) and same functions are used here. On the other hand, added factors, β and δ should be adjusted to match the governing equation. The new scheme is fully explicit scheme. Therefore, the equations can be solved directly at each time step.

$$
\begin{aligned}
&\frac{P_{i+1/2,j}^{n+1}-P_{i+1/2,j}^{n}}{\Delta t}+gh_{i+1/2,j}\frac{\zeta_{i+1,j}^{n+1/2}-\zeta_{i,j}^{n+1/2}}{\Delta x}\\
&+\frac{\alpha}{12\Delta x}gh_{i+1/2,j}\left(\zeta_{i+2,j}^{n+1/2}-3\zeta_{i+1,j}^{n+1/2}+3\zeta_{i,j}^{n+1/2}-\zeta_{i-1,j}^{n+1/2}\right)\\
&+\frac{\gamma}{12\Delta x}gh_{i+1/2,j}\left[\left(\zeta_{i+1,j+1}^{n+1/2}-2\zeta_{i+1,j}^{n+1/2}+\zeta_{i+1,j-1}^{n+1/2}\right)-\left(\zeta_{i,j+1}^{n+1/2}-2\zeta_{i,j}^{n+1/2}+\zeta_{i,j-1}^{n+1/2}\right)\right]\\
&+\frac{\beta}{12}gb\left(\zeta_{i,j+1}^{n+1/2}-2\zeta_{i,j}^{n+1/2}+\zeta_{i,j-1}^{n+1/2}\right)-\frac{\delta}{12}gb\left(\zeta_{i+1,j}^{n+1/2}-2\zeta_{i,j}^{n+1/2}+\zeta_{i-1,j}^{n+1/2}\right)=0
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
&\frac{Q_{i,j+1/2}^{n+1}-Q_{i,j+1/2}^{n}}{\Delta t}+gh_{i,j+1/2}\frac{\zeta_{i,j+1}^{n+1/2}-\zeta_{i,j}^{n+1/2}}{\Delta y}\\
&+\frac{\alpha}{12\Delta y}gh_{i,j+1/2}\left(\zeta_{i,j+2}^{n+1/2}-3\zeta_{i,j+1}^{n+1/2}+3\zeta_{i,j}^{n+1/2}-\zeta_{i,j-1}^{n+1/2}\right)\\
&+\frac{\gamma}{12\Delta y}gh_{i,j+1/2}\left[\left(\zeta_{i+1,j+1}^{n+1/2}-2\zeta_{i,j+1}^{n+1/2}+\zeta_{i,j-1}^{n+1/2}\right)-\left(\zeta_{i+1,j}^{n+1/2}-2\zeta_{i,j}^{n+1/2}+\zeta_{i-1,j}^{n+1/2}\right)\right]\\
&+\frac{\beta}{12}ga\left(\zeta_{i+1,j}^{n+1/2}-2\zeta_{i,j}^{n+1/2}+\zeta_{i-1,j}^{n+1/2}\right)-\frac{\delta}{12}gb\left(\zeta_{i,j+1}^{n+1/2}-2\zeta_{i,j}^{n+1/2}+\zeta_{i,j-1}^{n+1/2}\right)=0
\end{aligned}
\tag{11}
$$

By comparing a modified equation of the scheme with the governing equation, following conditions are required to match numerical dispersion with physical dispersion effects. If the new scheme has a convergent solution, both consistency and stability should be agreeable. With these dispersion correction factors, the modified equations represent well the governing equation. However, the present scheme is a little more sensitive than previous schemes and needs slightly smaller time step size than those of previous schemes has to be used during simulation.

$$\alpha=\frac{4h^2+gh(\triangle t)^2}{(\Delta x)^2}-1,\quad \gamma=\alpha+1 \tag{12}$$

$$\beta=\frac{-20h^2+gh(\triangle t)^2}{(\Delta x)^2}+1 \tag{13}$$

$$\delta=1-\frac{24h^2}{(\Delta x)^2} \tag{14}$$

4. RESULTS AND DISCUSSIONS

The present numerical scheme is employed to simulate a virtual tsunami propagating over submerged circular shoal. Since there is no analytic solution for a varying bathymetry, free surface displacements are computed and compared with those of Imamura and Goto (1988), Cho et al. (2007) and well known Boussinesq wave model, FUNWAVE. FUNWAVE is a fully nonlinear Boussinesq wave model with improved dispersion relationships for short waves (Peregrine, 1967). The accuracy of FUNWAVE has been verified for various coastal problem such as shoaling, refraction, diffraction and breaking of waves. Table 1 shows conditions of the numerical experiments and Fig. 3 shows the wave basin used in numerical simulations.

In this case, the entire computational domain is discretized using the uniform grid system with 500 m mesh sizes for the FUNWAVE model and 2,000 m meshes for tsunami propagation models. The FUNWAVE model using 500 m meshes shows the best result. Since the present model is developed to simulate propagation of distant tsunamis, a grid size of the present model is selected to be 2,000 m, which is used in typical simulations of tsunami propagation on real topographies, and also a same grid size is applied to previous tsunami models.

Table 1. Comparison of the Present Model with Available Models

Model	Imamura and Goto (1988)	Cho and Yoon (1998)	Cho et al. (2007)	Present model
Numerical simulate dispersion term	Using numerical dispersion term	The same dispersion as that from the linear Boussinesq equations in all direction	Improvement applicable to varying bathymetry using dispersion-correction scheme	Dispersion-corrections term have 1st order varying depth
Problem	Cannot simulate dispersion term of Boussinesq equation due to cross-derivative term	The limitation that spatial grid size and time step size should be changed continuously as the water depth changes	Dispersion-corrections term derived from constant depth	-
Assumption of governing equation	Constant depth	Constant depth	Constant depth	1st order varying depth

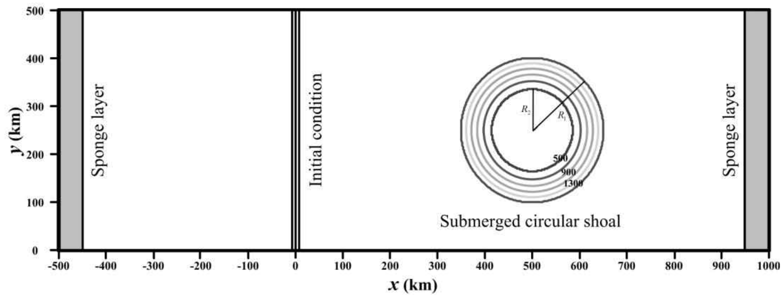


Fig. 3. Numerical Wave Basin (National Emergency Management Agency, 2008)

Table 2. Conditions of the Numerical Experiments

Model	FUNWAVE	Imamura and Goto	Cho et al.	Present Model
Spatial grid size	500 *m*	2,000 *m*	2,000 *m*	2,000 *m*
Number of grids	3001 × 1001	751 × 251	751 × 251	751 × 251
Temporal grid size	2 *sec*	5 *sec*	5 *sec*	5 *sec*
Computation time	156,888 *sec*	49 *sec*	53 *sec*	83 *sec*

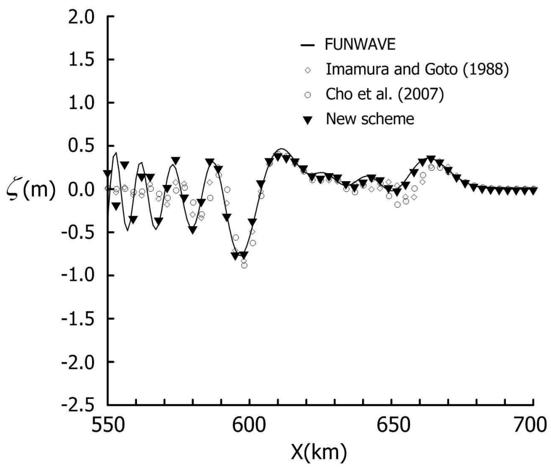


Fig. 4. Comparison of Free Surface Profiles Along the Center Line at 7,200 sec

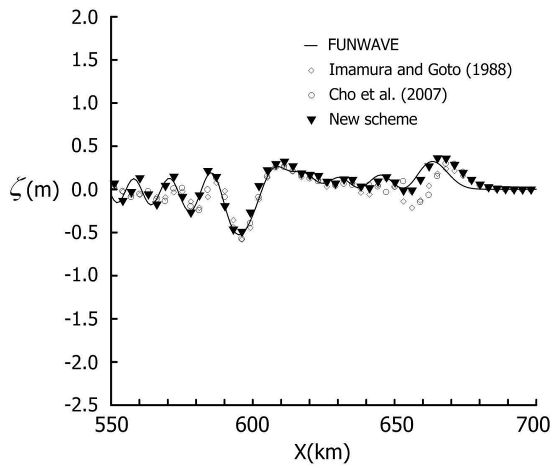


Fig. 5. Comparison of Free Surface Profiles along the Center Line at 8,000 sec

Figs. 4 and 5 show comparison of free surface profiles along the center line, which is $y = 250\,km$. The black line represents FUNWAVE and the small diamond represents Imamura and Goto's model. The small circle represents Cho et al.'s and the black gradient represents the present model. As seen in figures, the present model shows much better results than those of previous tsunami propagation models. Although the present model has to use a bigger time step size and spend a little more computation time than previous models, time difference can be ignored, actually the present model only takes approximately 30 seconds more. Therefore, we can put this disadvantage behind.

5. CONCLUSION

In this study, a modified numerical scheme has been developed and applied to predict tsunami behaviors. Governing equations are slightly modified and additional terms are added to the previous scheme to consider proper bottom slope effects. Although there is a little discrepancy, new numerical model shows much better results compared with the previous tsunami propagation models and can be applied to simulating historical tsunami events to predict maximum runup heights in real topography. However, the new model seems more sensitive than the previous tsunami propagation models and a little smaller time step size should be used in numerical simulations. Even though the present model is performed with a uniform grid size of 2,000 m, the results agree well with FUNWAVE using 500 m grid size. Although the disadvantage has not been focused on because actual time difference is small enough to be neglected in applying the new model to simple cases, it may generate a big problem, such as stability problem or huge computation time, during numerical simulation in real topography. Therefore, additional researches are required to control this disadvantage before applying the new model.

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