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# Design of compact phase controller for pulse tube refrigerator

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*Abstract*-- **A compact phase controller of pulse tube refrigerator is proposed in this paper. Most pulse tube refrigerators available now consist of a long inertance tube and reservoir as the phase controller. The long inertance tube and reservoir present a challenge for compact packaging in some applications. To overcome this disadvantage, the long inertance tube and reservoir are replaced with the compact phase controller consisted of mass, spring and damper in pulse tube refrigerator. This process is achieved using similarity of mechanical, electrical, and acoustic system and the specific configuration of the compact phase controller is designed. From the simulation code in this paper, the performance of pulse tube refrigerator with the designed compact phase controller is confirmed to be comparable to pulse tube refrigerator with the long inertance tube and reservoir.**

## 1. INTRODUCTION

Pulse Tube Refrigerator (PTR) plays an important role for cryogenic cooling of many applications where the cryogenic refrigerator having high efficiency, low vibration, and high reliability is needed. One of the technologies developed to create an efficient PTR is a proper phase controller which creates a favorable phase shift between the mass flow rate and the pressure in a cold-end heat exchanger [1].

Fig. 1 shows several types of phase controller. Each phase controller has a different maximum phase shift angle. The inertance-type phase controller is widely used for efficient PTRs because it can induce a phase shift of wide range without additional losses. The inertance-type phase controller consists of a long inertance tube (up to length of several meters) and a large reservoir (up to volume of several liters). The awkwardness of involving large volume sometimes presents a challenge for packaging in cryocooler applications. However, there is little effort for reducing the volume or replacing the configuration of inertance-type phase controller.

Recently, an invention what replaces the long inertance tube with the inertance gap of several inches long operated by the linear motor is proposed [2]. The invention shows the PTR with inertance gap offers comparable performance to the PTR with a long inertance tube using the SAGE software, however the reservoir with large volume is still necessary and use of a linear motor induces an additional electrical power.

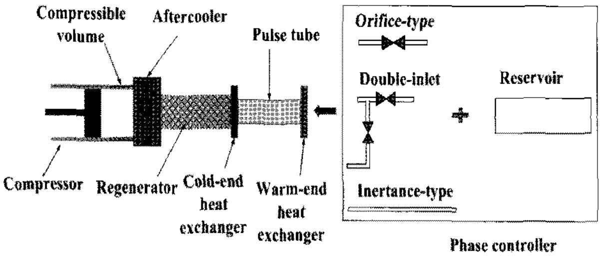

Fig. 1. Phase controllers of pulse tube refrigerator.

In this paper, a compact phase controller having small volume is suggested and designed. A compact mass, spring, and damper system are used in place of the inertance-type phase controller using a concept of similarity of electrical and mechanical systems. The configuration of compact mass, spring, and damper system is specifically proposed and the performance of this new concept is estimated in comparison with an inertance-type phase controller.

## 2. COMPACT PHASE CONTROLLER

### 2.1. Inertance-type phase controller

An inertance-type phase controller can be designed by an analytical model based on the similarity of an electrical system, because the equations governing the acoustic system (inertance-type phase controller) have the same form as those for the electrical system. The design concept is experimentally verified by Radebaugh et al. [3]. Fig. 2 shows the schematic diagram of inertance-type phase

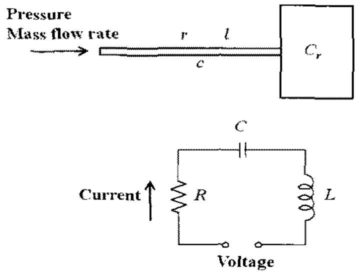

Fig. 2. Schematic diagram of inertance-type phase controller and RLC circuit.

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TABLE I
ANALOGOUS QUANTITIES OF MECHANICAL AND ELECTRICAL SYSTEMS.

| Electrical quantity | Mechanical analogy |
|---|---|
| Voltage | Force |
| Current | Velocity |
| Resistance ( $R$) | Damping coefficient ($c_m$) |
| Inductance ($L$) | Mass ($m$) |
| Capacitance ($C$) | Inverse spring coefficient ($1/k$) |

controller and RLC circuit. For acoustic systems, the impedance is the quotient of the complex pressure divided by the complex mass flow rate:

$$Z = \rho_0 \frac{P}{\dot{m}} = \rho_0 Z_m \tag{1}$$

The complex impedance of inertance-type phase controller having length L and inner diameter D is expressed by [3]:

$$Z_m(D,x) = Z_0(D)\left\{\frac{Z_r + Z_0(D)\tanh[(k(D)(L-x)]}{Z_0(D) + Z_r\tanh[(k(D)(L-x)]}\right\} \tag{2}$$

where the complex characteristic impedance and propagation function are given by:

$$Z_0(D) = \sqrt{\frac{r(D)+i\omega l(D)}{i\omega c(D)}}$$

$$k(D) = \sqrt{[r(D)+i\omega l(D)\cdot i\omega c(D)} \tag{3}$$

and the complex impedance determined by a reservoir is:

$$Z_r = \frac{1}{i\omega C_r} \tag{4}$$

In Eqs. (2-4), the resistance ($r$), inertance ($l$), and compliance ($c$, $C_r$) are :

$$r(D) = \frac{2}{\pi}\left(\frac{32 f_r |\dot{m}|}{\pi^2 \rho_0 D^5}\right)$$

$$l(D) = \frac{4}{\pi D^2} \tag{5}$$

$$c(D) = \frac{\pi D^2}{4\gamma RT}$$

$$C_r = \frac{V_r}{\gamma RT}$$

where $f_r$ is Fanning friction factor, $\gamma$ is the ratio of specific heat, and $V_r$ is the volume of reservoir.

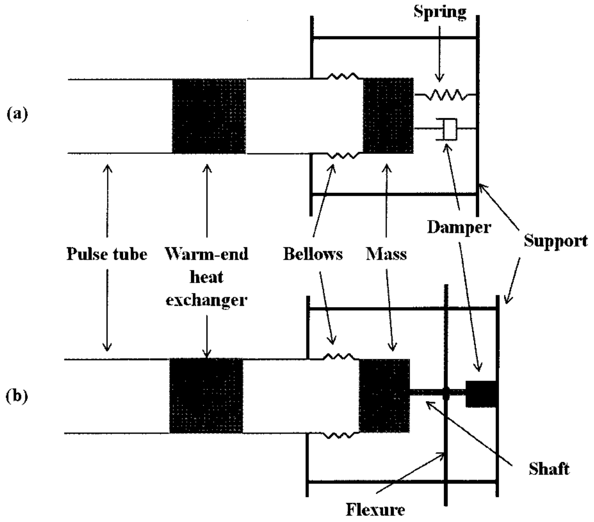

Fig. 3. Configuration of compact phase controller.

The total impedance of inertance-type phase controller can be obtained as input of $x = 0$ in Eq. (2). Therefore, if the optimal phase difference between the pressure and mass flow rate at the inlet of inertance-type phase controller is known, the total resistance, inductance, and capacitance of RLC circuit can be exactly determined and the configuration of inertance-type phase controller can be designed.

### 2.2. Similarity of mechanical and electrical system

As the acoustic and the RLC systems are identically expressed from the similarity, the mechanical system of mass-spring-damper (MSD) and the RLC system have also the similarity because the equations governing the systems have the same forms as follows [4].

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V$$
$$m\frac{d^2x}{dt^2} + c_m\frac{dx}{dt} + kx = F \tag{6}$$

where $q$ is charge, $m$ is mass, $c_m$ is damping coefficient, $k$ is spring coefficient, and $F$ is force.

Table I shows the analogous quantities of mechanical and electrical systems. For satisfying the required resistance, inductance, and capacitance in system analyzed by a viewpoint of similarity, the analogous mechanical system is possible to organize more compact system than the analogous acoustic system [4]. It is because the required volumes of inertance tube and reservoir are larger than the volume of MSD system for attaining the same resistance, inductance, and capacitance. Therefore, the phase controller composed of MSD system can be compact and enables a PTR to operate on the same mechanism.

### 2.3. Configuration of compact phase controller

Fig. 3 shows the configuration of the proposed compact phase controller. The compact phase controller simply consists of a mass, spring, bellows, damper, and support as

TABLE II
CONFIGURATION AND OPERATING CONDITIONS OF PTR.

| Configuration | Inertance-type | Compact |
|---|---|---|
| Compressible volume | 57.15 mm (OD), | 38.4 mm (L) |
| Regenerator | 31.75 mm (OD), | 78 mm (L) |
| Pulse tube | 12.7 mm (OD),<br>154 mm (L) | 12.7 mm (OD),<br>167 mm (L) |
| Inertance tube 1<br>or mass | 4.76 mm (OD),<br>0.42 m (L) | 0.45 kg |
| Inertance tube 2<br>or spring coefficient | 6.35 mm (OD),<br>2.47 m (L) | 37370 N/m |
| Reservoir volume<br>or damping coefficient | 1000 cm$^3$ | 27 Ns/m |
| Operating condition and result | Inertance-type | Compact |
| Frequency | 58.7 Hz | 57 Hz |
| Charging pressure | 34 bar | 31 bar |
| Cooling capacity | 8 W @ 50 K | 8.3 W @ 50 K |

OD and L mean outside diameter and length, respectively.

shown in Fig. 3(a). Fig. 3(b) shows a realistic configuration of mechanical components for the compact phase controller. The warm-end heat exchanger and the mass are connected by the bellows, and the mass and the support are connected by the shaft and damper. The mechanical spring is replaced with the flexure and the flexure is connected together with the shaft and the support. Since the flexure has large spring coefficient to radial direction, movement of mass is easily maintained to axial direction.

To design and confirm the performance of the proposed compact phase controller, we use the simulation code that is capable of predicting the behavior of PTR with the inertance-type phase controller [5].

## 3. SIMULATION

The simulation code of PTR is modified by replacing the governing equations of inertance-type phase controller with governing equations of MSD system. We want to confirm that the PTR with the compact phase controller has the same performance with PTR having the inertance-type phase controller. From this simulation, the exact values of mass, spring, and damper can be obtained.

### 3.1. Initial values

The approximate configuration of MSD system should be designed before running the simulation code. The impedance of inertance-type phase controller in our previous simulation is calculated by Eqs. (2)-(5). RLC circuit is first expressed from the calculation result and then changed into MSD system. From this preprocessing procedure, the values of mass, spring coefficient, and damping coefficient are obtained and these values are used as the initial values of simulation code.

### 3.2. Simulation

The configuration and the operating conditions of PTR with the compact phase controller are optimized to maximize the cooling capacity of cold-end heat exchanger at 50 K.

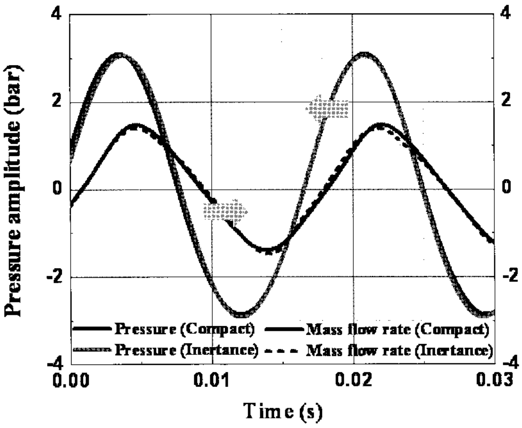

Fig. 4. Curves of pressure and mass flow rate in cold-end of pulse tube.

Table II lists the detailed parameters of the PTRs designed finally from the simulation code. When the PTR with the inertance-type phase controller [6] is compared, the configuration and the operating conditions of the PTR with the compact phase controller are slightly changed. The charging pressure is decreased by the exclusion of volumes of inertance and reservoir and the length of pulse tube is extended by the decrease of connecting space between the warm-end heat exchanger and the inertance tube. The obtained mass, spring coefficient and damping coefficient are 0.45 kg, 37370 N/m, 27 Ns/m, respectively. The compact phase controller vibrates with the amplitude of 5.2 mm. The values are small enough to be able to make a compact structure.

In the simulation result, PTR with the compact phase controller results in the same cooling capacity of PTR with the inertance-type phase controller. It is because the same cooling effect is generated in the cold-end of PTRs according to having the same phase shift at the cold-end of PTRs. The time-averaged PV power generating the cooling effect in the cold-end of PTR is given by:

$$<\dot{W}>_{PV} = \frac{1}{2} R T_0 \frac{|P|}{P_0} |\dot{m}| \cos\theta \qquad (7)$$

where $T_0$ is the average temperature, $P_0$ is the average pressure, and $\theta$ is the phase difference between the pressure and the mass flow rate at the cold-end of pulse tube. Fig. 4 and Table III show the curves of pressure and mass flow rate and the calculated PV power at the cold-end of pulse tube, respectively. Since the PTRs with the compact and the inertance-type phase controller have the

TABLE III
PV POWER IN COLD-END OF PULSE TUBE.

| Phase controller | PV power |
|---|---|
| Inertance-type | 43.6 W |
| Compact | 44.2 W |

similar curves of pressure and mass flow rate at cold-end, the PV power is almost same. The optimal phase difference of PTR is about 29° (pressure leading mass flow rate), which is typically achieved with a Stirling refrigerator because this phase angle can minimize the regenerator loss. Therefore, we can conclude the optimal points of PTRs are achieved by running the simulation code that generates appropriate mechanism to minimize the regenerator loss.

## 4. CONCLUSION

The compact phase controller concept using MSD system in PTR is proposed by analogy of mechanical, acoustic, and electrical systems. It is consisted of mass, bellows, flexure, damper, shaft, and support. The specific configuration is designed and the performance of compact phase controller in PTR is estimated by the simulation code. As a result, the mass, spring coefficient, and damping coefficient have reasonable values possible to be composed as compact configuration. The PTR with MSD system has the similar performance of the PTR with an inertance-type phase controller. The compact phase controller can replace an inertance-type phase controller and the whole PTR can be compactly assembled.

## 5. FUTURE WORK

A PTR with a compact phase controller will be fabricated and potential problems generated in the progress will be solved. The PTR with the compact phase controller can induce the vibration of system because the inertance tube and the reservoir are replaced with the moving MSD system. However, the MSD system does not have the tolerance gap and mechanical friction. The moving part cannot generate the problem of long term reliability. Therefore, the induced vibration will be the main problem and must be reduced.

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