International Journal of Fuzzy Logic and Intelligent Systems, vol. 11, no. 2, June 2011, pp. 108-112
DOI : 10.5391/IJFIS.2011.11.2.108

Some Common Fixed Point Theorems using Compatible Maps in Intuitionistic Fuzzy Metric Space

Jong Seo Park*

***Department of Mathematic Education, Chinju National University of Education,**
Jinju 660-756, South Korea

Abstract

Kaneko et al.[4] etc many authors extended with multi-valued maps for the notion of compatible maps in complete metric space. Recently, O'Regan et al.[5] presented fixed point and homotopy results for compatible single-valued maps on complete metric spaces. In this paper, we will establish some common fixed point theorems using compatible maps in intuitionistic fuzzy metric space.

Key Words : Common fixed point theorem, compatible mapping, intuitionistic fuzzy metric space.

1. Introduction

In 1986, Jungck[3] introduced the notion of compatible maps, Beg and Azam[1], Kaneko et al.[4] extended to multi-valued maps for for the notion of compatible maps in complete metric space.

Recently, O'Regan et al.[5] presented fixed point and homotopy results for compatible single-valued maps on complete metric spaces.

In this paper, we will establish some common fixed point theorem for compatible maps in intuitionistic fuzzy metric space.

2. Preliminaries

Throughout this paper, $\mathbf{N}$ denote the set of all positive integers. Now, we begin with some definitions, properties in intuitionistic fuzzy metric space as following:

Let us recall(see [9]) that a continuous $t-$norm is a operation $* : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions: (a)$*$ is commutative and associative, (b)$*$ is continuous, (c)$a * 1 = a$ for all $a \in [0,1]$, (d)$a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$. Also, a continuous $t-$conorm is a operation $\diamond : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions: (a)$\diamond$ is commutative and associative, (b)$\diamond$ is continuous, (c)$a \diamond 0 = a$ for all $a \in [0,1]$, (d)$a \diamond b \geq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Also, let us recall (see [6]) that the following conditions are satisfied: (a)For any $r_1, r_2 \in (0,1)$ with $r_1 > r_2$, there exist $r_3, r_4 \in (0,1)$ such that $r_1 * r_3 \geq r_2$ and $r_4 \diamond r_2 \leq r_1$; (b)For any $r_5 \in (0,1)$, there exist $r_6, r_7 \in (0,1)$ such that $r_6 * r_6 \geq r_5$ and $r_7 \diamond r_7 \leq r_5$.

Definition 2.1. ([8])The $5-$tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous $t-$norm, $\diamond$ is a continuous $t-$conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions; for all $x, y, z \in X$, such that

(a)$M(x, y, t) > 0$,
(b)$M(x, y, t) = 1 \Longleftrightarrow x = y$,
(c)$M(x, y, t) = M(y, x, t)$,
(d)$M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(e)$M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous,
(f)$N(x, y, t) > 0$,
(g)$N(x, y, t) = 0 \Longleftrightarrow x = y$,
(h)$N(x, y, t) = N(y, x, t)$,
(i)$N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,
(j)$N(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.

Note that (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Let X be an intuitionistic fuzzy metric space. For any $t > 0$, the open ball $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r,\ \ N(x, y, t) < r\}$$

Let X be an intuitionistic fuzzy metric space. Let τ be the set of all $A \subset X$ with $x \in A$ if and only if there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Then τ is a topology on X(induced by the intuitionistic fuzzy metric (M, N)).

Definition 2.2. ([8]) Let X be an intuitionistic fuzzy metric space. Then

Manuscript received Oct. 18, 2010; revised Mar. 16, 2011.

(a) A sequence $\{x_n\} \subset X$ is convergent to x in X if $\lim_{n\to\infty} M(x_n, x, t) = 1$, $\lim_{n\to\infty} N(x_n, x, t) = 0$ for each $t > 0$.

(b) A sequence $\{x_n\} \subset X$ is called Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$, $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$ for each $t > 0$ and $p \in \mathbf{N}$.

(c) X is complete if every Cauchy sequence in X is convergent.

Lemma 2.3. [7]Let X be an intuitionistic fuzzy metric space. If a sequence $\{x_n\} \subset X$ is such that for any $n \in \mathbf{N}$,

$$M(x_n, x_{n+1}, t) \geq M(x_0, x_1, k^n t),$$
$$N(x_n, x_{n+1}, t) \leq N(x_0, x_1, k^n t)$$

for all $k > 1$, then the sequence $\{x_n\} \subset X$ is a Cauchy sequence.

Lemma 2.4. ([2])Let the function $\phi(t) : [0, \infty) \to [0, \infty)$ satisfying the following condition:

$\phi(t)$ is strictly increasing, $\phi(0) = 0$ and $\lim_{n\to\infty} \phi^n(t) = \infty$ for all $t > 0$, where $\phi^n(t)$ denotes the $n - th$ iterative function of $\phi(t)$.

Then $\phi(t) > t$, $\phi^n(t) > \phi^{n-1}(t)$ for all $t > 0$ and $n \in \mathbf{N}$.

3. Main Results

Definition 3.1. [7]Let F, G be mappings from an intuitionistic fuzzy metric space X into itself. Then the mappings F and G are said to be compatible if for all $t > 0$,

$$\lim_{n\to\infty} M(FGx_n, GFx_n, t) = 1,$$
$$\lim_{n\to\infty} N(FGx_n, GFx_n, t) = 0$$

whenever $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gx_n = x$ for some $x \in X$.

Let X be a complete intuitionistic fuzzy metric space and $G : X \to X$. If $x_0 \in X$ and $r > 0$, we set

$$B(Gx_0, r, t)$$
$$= \{x \in X | M(Gx_0, x, t) > 1 - r, N(Gx_0, x, t) < r\},$$
$$B(x_0, r, t)$$
$$= \{x \in X | M(x_0, x, t) > 1 - r, N(x_0, x, t) < r\}$$

and

$$G^{-1}(B(Gx_0, r, t) = \{x \in X | Gx \in B(Gx_0, x, t)\}.$$

Let

$$F : (\overline{B(Gx_0, r, t)} \cup G^{-1}(B(Gx_0, r, t))) \to X$$

satisfying $FG^{-1}(B(Gx_0, r, t)) \subseteq G(X)$. Then F and G are said to be compatible on $\overline{B(Gx_0, r, t)}$ if

$$\lim_{n\to\infty} M(FGx_n, GFx_n, t) = 1,$$

$$\lim_{n\to\infty} N(FGx_n, GFx_n, t) = 0$$

whenever $\{x_n\} \subset G^{-1}(B(Gx_0, r, t))$ such that

$$\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gx_n = z$$

for some $z \in \overline{B(Gx_0, r, t)}$

Remark 3.2. If F and G are compatible and $Fx = Gx$ for some $x \in G^{-1}(B(Gx_0, r, t))$, then $FGx = GFx$. That is, F and G commute at coincidence point.

Theorem 3.3. Let X be a complete intuitionistic fuzzy metric space with $t * t \geq t$ and $t \diamond t \leq t$ for all $t \in [0, 1]$. Let $F : (\overline{B(Gx_0, r, t)} \cup G^{-1}(B(Gx_0, r, t))) \to X$ and $G : X \to X$ are compatible maps for $x_0 \in X$, $r > 0$ on $\overline{B(Gx_0, r, t)}$ and $FG^{-1}(B(Gx_0, r, t)) \subseteq G(X)$. Suppose that G is continuous and there exists a continuous strictly increasing function $\phi : [0, \infty) \to [0, \infty)$ satisfying $\phi(t) > t$ and $\lim_{t\to\infty}[\phi(t) - t] = \infty$ for $t > 0$ such that for $x, y \in \overline{B(Gx_0, r, t)} \cup G^{-1}(B(Gx_0, r, t)))$, we have

$$M(Fx, Fy, t) \geq M(x, y, \phi(t)), \tag{1}$$
$$N(Fx, Fy, t) \leq N(x, y, \phi(t))$$

where

$$M(x, y, t)$$
$$\geq \min\{M(Gx, Gy, \phi(t)), M(Gx, Fx, \phi(t))$$
$$, M(Gy, Fy, \phi(t)),$$
$$M(Gx, Fy, \phi(t)) * M(Gy, Fx, \phi(t)\},$$
$$N(x, y, t)$$
$$\leq \max\{N(Gx, Gy, \phi(t)), N(Gx, Fx, \phi(t))$$
$$, N(Gy, Fy, \phi(t)),$$
$$N(Gx, Fy, \phi(t)) \diamond N(Gy, Fx, \phi(t)\}.$$

Also, suppose

$$M(Gx_0, Fx_0, \phi(t)) > 1 - r,\ N(Gx_0, Fx_0, \phi(t)) < r. \tag{2}$$

Then there exists a unique $x \in \overline{B(Gx_0, r, t)}$ with $x = Fx = Gx$.

Proof. Since $FG^{-1}(B(Gx_0, r, t)) \subseteq G(X)$ and $Gx_0 \in B(Gx_0, r, t)$ let $Gx_1 = Fx_0$ for some $x_1 \in X$. Then by (2),

$$M(Gx_1, Gx_0, t) > 1 - r,\ N(Gx_1, Gx_0, t) < r.$$

and so $Gx_1 \in B(Gx_0, r, t)$. Also, since $Gx_1 \in B(Gx_0, r, t)$ and $FG^{-1}(B(Gx_0, r, t)) \subseteq G(X)$, we can set $Gx_2 = Fx_1$. For $n \in \mathbf{N}(n \geq 3)$, we let $Gx_n = Fx_{n-1}$. Now we show for $n \in \mathbf{N}$,

$$M(Gx_n, Gx_{n+1}, t) \geq M(Gx_{n-1}, Gx_n, \phi(t)),$$
$$N(Gx_n, Gx_{n+1}, t) \leq N(Gx_{n-1}, Gx_n, \phi(t))$$

and $Gx_i \in B(Gx_0, r, t)$ for $i \in \{0, 1, 2, \cdots, \}$ and by Lemma 2.4,

$$\begin{aligned}
&M(Gx_1, Gx_2, t)\\
&= M(Fx_0, Fx_1, t)\\
&\geq M(x_0, x_1, \phi(t))\\
&\geq \min\{M(Gx_0, G_1, \phi(t)), M(Gx_1, Gx_2, \phi(t))\\
&\qquad M(Gx_0, Gx_1, \phi(t) - t) * M(Gx_1, Gx_2, t\}\\
&\geq M(Gx_0, G_1, \phi(t)) > 1 - r,\\
&N(Gx_1, Gx_2, t)\\
&= N(Fx_0, Fx_1, t)\\
&\leq N(x_0, x_1, \phi(t))\\
&\leq \max\{N(Gx_0, G_1, \phi(t)), N(Gx_1, Gx_2, \phi(t))\\
&\qquad N(Gx_0, Gx_1, \phi(t) - t) \diamond N(Gx_1, Gx_2, t\}\\
&\leq N(Gx_0, G_1, \phi(t)) < r.
\end{aligned}$$

Hence

$$\begin{aligned}
&M(Gx_0, Gx_2, \phi(t))\\
&\geq M(Gx_0, Gx_1, \phi(t) - t) * M(Gx_1, Gx_2, t)\\
&\geq M(Gx_0, Gx_1, \phi(t)) > 1 - r,\\
&N(Gx_0, Gx_2, \phi(t))\\
&\leq N(Gx_0, Gx_1, \phi(t) - t) \diamond N(Gx_1, Gx_2, t)\\
&\leq N(Gx_0, Gx_1, \phi(t)) < r.
\end{aligned}$$

Therefore $Gx_2 \in B(Gx_0, r, t)$.

Essentially the same argument as above yields

$$M(Gx_2, Gx_3, t) > 1 - r,\ N(Gx_2, Gx_3, t) < r.$$

Suppose that there exists $k \in \{2, 3, \cdots\}$ with

$$M(Gx_m, Gx_{m+1}, t) \geq M(Gx_{m-1}, Gx_m, \phi(t)),$$

$$N(Gx_m, Gx_{m+1}, t) \leq N(Gx_{m-1}, Gx_m, \phi(t))$$

and $Gx_m \in B(Gx_0, r, t)$ for $m \in \{1, 2, \cdots, k\}$.

Now, we will prove $Gx_{k+1} \in B(Gx_0, r, t)$ using

$$M(Gx_k, Gx_1, \phi(t)) > 1 - r,\ N(Gx_k, Gx_1, \phi(t)) < r.$$

If $k = 2$, this is obvious.

For $k = 3$, since $Gx_2, Gx_3 \in B(Gx_0, r, t)$ and

$$M(Gx_3, Gx_2, t) \geq M(Gx_1, Gx_0, \phi^3(t)),$$
$$N(Gx_3, Gx_2, t) \leq N(Gx_1, Gx_0, \phi^3(t)),$$

thus

$$\begin{aligned}
&M(Gx_3, Gx_1, t)\\
&= M(Fx_2, Fx_0, t)\\
&\geq M(x_2, x_0, \phi(t))\\
&\geq \min\{M(Gx_2, Gx_0, \phi(t)), M(Gx_2, Fx_2, \phi(t))\\
&\qquad , M(Gx_0, Fx_0, \phi(t))\\
&\qquad , M(Gx_2, Fx_0, \phi(t)) * M(Gx_0, Fx_2, \phi(t))\},\\
&N(Gx_3, Gx_1, t)\\
&= N(Fx_2, Fx_0, t)\\
&\leq N(x_2, x_0, \phi(t))\\
&\leq \max\{N(Gx_2, Gx_0, \phi(t)), N(Gx_2, Fx_2, \phi(t))\\
&\qquad , N(Gx_0, Fx_0, \phi(t))\\
&\qquad , N(Gx_2, Fx_0, \phi(t)) \diamond N(Gx_0, Fx_2, \phi(t))\}.
\end{aligned}$$

Since $\phi^3(t) \geq t$, we have

$$M(Gx_3, Gx_1, t) > 1 - r,\ N(Gx_3, Gx_1, t) < r.$$

If $k = 4$, then since $Gx_3, Gx_4 \in B(Gx_0, r, t)$,

$$\begin{aligned}
&M(Gx_4, Gx_1, t)\\
&= M(Fx_3, Fx_0, t)\\
&\geq M(x_3, x_0, \phi(t))\\
&\geq \min\{M(Gx_3, Gx_0, \phi(t)), M(Gx_3, Fx_3, \phi(t))\\
&\qquad , M(Gx_0, Fx_0, \phi(t))\\
&\qquad , M(Gx_3, Fx_0, \phi(t)) * M(Gx_0, Fx_3, \phi(t))\}\\
&> 1 - r,\\
&N(Gx_4, Gx_1, t)\\
&= N(Fx_3, Fx_0, t)\\
&\leq N(x_3, x_0, \phi(t))\\
&\leq \max\{N(Gx_3, Gx_0, \phi(t)), N(Gx_3, Fx_3, \phi(t))\\
&\qquad , N(Gx_0, Fx_0, \phi(t))\\
&\qquad , N(Gx_3, Fx_0, \phi(t)) \diamond N(Gx_0, Fx_3, \phi(t))\}\\
&< r.
\end{aligned}$$

Continuing this process, we obtain for $k \in \{5, 6, \cdots\}$,

$$M(Gx_k, Gx_1, t) > 1 - r,\ N(Gx_k, Gx_1, t) < r.$$

Also, since $Gx_m \in B(Gx_0, r, t)$ for $m \in \{1, 2, \cdots, k\}$,

$$\begin{aligned}
&M(Gx_0, Gx_{k+1}, \phi(t))\\
&\geq M(Gx_0, Gx_1, \phi(t) - t) * M(Gx_1, Gx_{k+1}, t)\\
&\geq M(Gx_0, Gx_1, \phi(t) - t) * M(Fx_0, Fx_k, t)\\
&\geq M(Gx_0, Gx_1, \phi(t) - t) * M(x_0, x_k, \phi(t))
\end{aligned}$$

$$\begin{aligned}
&\geq M(Gx_0, Gx_1, \phi(t)-t) \\
&\quad * \min\{M(Gx_0, Gx_k, \phi(t)), M(Gx_0, Gx_1, \phi(t)) \\
&\quad , M(Gx_k, Gx_{k+1}, \phi(t)), M(Gx_0, Gx_k, \phi(t)-t) \\
&\quad * M(Gx_k, Gx_{k+1}, t) * M(Gx_k, Gx_1, \phi(t))\} \\
&> 1-r, \\
&N(Gx_0, Gx_{k+1}, \phi(t)) \\
&\leq N(Gx_0, Gx_1, \phi(t)-t) \diamond N(Gx_1, Gx_{k+1}, t) \\
&\leq N(Gx_0, Gx_1, \phi(t)-t) \diamond N(Fx_0, Fx_k, t) \\
&\leq N(Gx_0, Gx_1, \phi(t)-t) \diamond N(x_0, x_k, \phi(t)) \\
&\leq N(Gx_0, Gx_1, \phi(t)-t) \\
&\quad \diamond \max\{N(Gx_0, Gx_k, \phi(t)), N(Gx_0, Gx_1, \phi(t)) \\
&\quad , N(Gx_k, Gx_{k+1}, \phi(t)), N(Gx_0, Gx_k, \phi(t)-t) \\
&\quad \diamond N(Gx_k, Gx_{k+1}, t) \diamond N(Gx_k, Gx_1, \phi(t))\} \\
&< r.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
&M(Gx_k, Gx_{k+1}, \phi(t)) \\
&\geq M(x_k, x_{k+1}, \phi(t)) \\
&\geq \min\{M(Gx_k, Gx_{k+1}, \phi(t)), M(Gx_k, Fx_k, \phi(t)) \\
&\qquad , M(Gx_{k+1}, Fx_{k+1}, \phi(t)) \\
&\quad M(Gx_k, Fx_{k+1}, \phi(t)) * M(Gx_{k+1}, Fx_k, \phi(t))\} \\
&\geq M(Gx_k, Gx_{k+1}, \phi(t)), \\
&N(Gx_k, Gx_{k+1}, \phi(t)) \\
&\leq N(x_k, x_{k+1}, \phi(t)) \\
&\leq \max\{N(Gx_k, Gx_{k+1}, \phi(t)), N(Gx_k, Fx_k, \phi(t)) \\
&\qquad , N(Gx_{k+1}, Fx_{k+1}, \phi(t)) \\
&\quad N(Gx_k, Fx_{k+1}, \phi(t)) \diamond N(Gx_{k+1}, Fx_k, \phi(t))\} \\
&\leq N(Gx_k, Gx_{k+1}, \phi(t)).
\end{aligned}$$

Thus by mathematical induction, $Gx_n \in B(Gx_0, r, t)$ for $n \in \{0, 1, 2, \cdots\}$ and

$$\begin{aligned}
M(Gx_n, Gx_{n+1}, t) &\geq M(Gx_{n-1}, Gx_n, \phi(t)), \\
N(Gx_n, Gx_{n+1}, t) &\leq N(Gx_{n-1}, Gx_n, \phi(t))
\end{aligned}$$

for $n \in \{1, 2, \cdots\}$. This implies that

$$\begin{aligned}
M(Gx_n, Gx_{n+1}, t) &\geq M(Gx_0, Gx_1, \phi^n(t)), \\
N(Gx_n, Gx_{n+1}, t) &\leq N(Gx_0, Gx_1, \phi^n(t)).
\end{aligned}$$

Also, for $n, m \in \mathbf{N}$ and $n < m$,

$$\begin{aligned}
&M(Gx_n, Gx_m, \phi(t)) \\
&= M(Fx_{n-1}, Fx_{m-1}, t) \\
&\geq M(x_{n-1}, x_{m-1}\phi(t)) \\
&\geq M(x_k, x_{k+1}, \phi(t)) \\
&\geq \min\{M(Gx_{n-1}, Gx_{m-1}, \phi(t)) \\
&\quad , M(Gx_{n-1}, Gx_n, \phi(t)), M(Gx_{m-1}, Gx_m, \phi(t)) \\
&\quad , M(Gx_{n-1}, Gx_m, \phi(t)) * M(Gx_m, Gx_n, \phi(t))\}, \\
&N(Gx_n, Gx_m, \phi(t)) \\
&= N(Fx_{n-1}, Fx_{m-1}, t) \\
&\leq N(x_{n-1}, x_{m-1}\phi(t)) \\
&\leq N(x_k, x_{k+1}, \phi(t)) \\
&\leq \max\{N(Gx_{n-1}, Gx_{m-1}, \phi(t)) \\
&\quad , N(Gx_{n-1}, Gx_n, \phi(t)), N(Gx_{m-1}, Gx_m, \phi(t)) \\
&\quad , N(Gx_{n-1}, Gx_m, \phi(t)) \diamond N(Gx_m, Gx_n, \phi(t))\}.
\end{aligned}$$

Therefore

$$M(Gx_n, Gx_m, t) > 1-r, \ N(Gx_n, Gx_m, t) < r.$$

Hence $\{x_n\}$ is a Cauchy sequence. Since X is complete, there exists $x \in \overline{B(Gx_0, r, t)}$ with $Gx_n \to x$ as $n \to \infty$. Also, $Gx_{n+1} = Fx_n \to x$ as $n \to \infty$. Since $\lim_{n\to\infty} Fx_n = x = \lim_{n\to\infty} Gx_n$ and $Fx_n = Gx_{n+1} \in B(Gx_0, r, t)$ for $n \in \{1, 2, \cdots\}$, by the compatibility of F, G and continuity of G,

$$\begin{aligned}
&\lim_{n\to\infty} M(FGx_n, Gx, \phi(t)) \\
&= \lim_{n\to\infty} M(FGx_n, GFx_n, \phi(t)) = 1, \\
&\lim_{n\to\infty} N(FGx_n, Gx, \phi(t)) \\
&= \lim_{n\to\infty} N(FGx_n, GFx_n, \phi(t)) = 0.
\end{aligned}$$

Thus $\lim_{n\to\infty} FGx_n = Gx$. Also, since

$$\begin{aligned}
&M(Fx, Gx, \phi(t)) \\
&\geq M(Fx, FGx_n, t) * M(FGx_n, Gx, \phi(t)-t) \\
&\geq M(x, Gx_n\phi(t)) * M(FGx_n, Gx, \phi(t)-t) \\
&\geq \min\{M(Gx, GGx_n, \phi(t)), M(Gx, Fx, \phi(t)) \\
&\qquad , M(GGx_n, FGx_n, \phi(t)), \\
&\quad M(Gx, Gx_n, \phi(t)) * M(GGx_n, Fx, \phi(t))\} \\
&\quad * M(FGx_n, Gx, \phi(t)-t), \\
&N(Fx, Gx, \phi(t)) \\
&\leq N(Fx, FGx_n, t) \diamond N(FGx_n, Gx, \phi(t)-t) \\
&\leq N(x, Gx_n\phi(t)) \diamond N(FGx_n, Gx, \phi(t)-t) \\
&\leq \max\{N(Gx, GGx_n, \phi(t)), N(Gx, Fx, \phi(t)) \\
&\qquad , N(GGx_n, FGx_n, \phi(t)), \\
&\quad N(Gx, Gx_n, \phi(t)) \diamond N(GGx_n, Fx, \phi(t))\} \\
&\quad \diamond N(FGx_n, Gx, \phi(t)-t).
\end{aligned}$$

From above inequality, taking the limit as $n \to \infty$, since $\lim_{n\to\infty} FGx_n = Gx$ and $\lim_{n\to\infty} Gx_n = x$,

$$\begin{aligned}
M(Fx, Gx, t) &\geq M(Gx, Fx, \phi(t)), \\
N(Fx, Gx, t) &\leq N(Gx, Fx, \phi(t)).
\end{aligned}$$

Furthermore since $\lim_{n\to\infty} FGx_n = Gx$ and $\lim_{n\to\infty} Fx_n = x$,

$$\begin{aligned}
&M(Fx_n, FGx_n, \phi(t)) \\
&\geq \min\{M(Gx_n, GGx_n, \phi(t)), M(Gx_n, Fx_n, \phi(t)) \\
&\qquad , M(GGx_n, FGx_n, \phi(t)) \\
&\qquad M(Gx_n, FGx_n, \phi(t)) * M(GGx_n, Fx_n, \phi(t))\}, \\
&N(Fx_n, FGx_n, \phi(t)) \\
&\leq \max\{N(Gx_n, GGx_n, \phi(t)), N(Gx_n, Fx_n, \phi(t)) \\
&\qquad , N(GGx_n, FGx_n, \phi(t)) \\
&\qquad N(Gx_n, FGx_n, \phi(t)) \diamond N(GGx_n, Fx_n, \phi(t))\}
\end{aligned}$$

Taking limit as $n \to \infty$,

$$\begin{aligned}
M(x, Gx, t) &\geq M(x, Gx, \phi(t)), \\
N(x, Gx, t) &\leq N(x, Gx, \phi(t)).
\end{aligned}$$

Hence $x = Gx = Fx$.

If $y = Gy = Fy$ with $x \neq y$, then

$$\begin{aligned}
&M(x, y, t) \\
&= M(Fx, Fy, t) \geq M(x, y, \phi(t)) \\
&\geq \min\{M(Gx, Gy, \phi(t)), M(Gx, Fx, \phi(t)) \\
&\qquad , M(Gy, Fy, \phi(t)) \\
&\qquad M(Gx, Fy, \phi(t)) * M(Gy, Fx, \phi(t))\} \\
&= M(Fx, Fy, \phi(t)) \\
&\geq M(x, y, \phi^2(t)), \\
&N(x, y, t) \\
&= N(Fx, Fy, t) \leq N(x, y, \phi(t)) \\
&\leq \max\{N(Gx, Gy, \phi(t)), N(Gx, Fx, \phi(t)) \\
&\qquad , N(Gy, Fy, \phi(t)) \\
&\qquad N(Gx, Fy, \phi(t)) \diamond N(Gy, Fx, \phi(t))\} \\
&= N(Fx, Fy, \phi(t)) \\
&\leq N(x, y, \phi^2(t)).
\end{aligned}$$

Therefore $x = y$. Hence there exists a unique $x \in \overline{B(Gx_0, r, t)}$ with $x = Fx = Gx$. □

Corollary 3.4. Let X be a complete intuitionistic fuzzy metric space with $t * t \geq t$ and $t \diamond t \leq t$ for all $t \in [0, 1]$. Let $F : \overline{B(x_0, r, t)} \to X$. Suppose that there exists a continuous strictly increasing function $\phi : [0, \infty) \to [0, \infty)$ satisfying $\phi(t) > t$ and $\lim_{t\to\infty}[\phi(t) - t] = \infty$ for $t > 0$ such that for $x, y \in \overline{B(x_0, r, t)}$, we have

$$\begin{aligned}
M(Fx, Fy, t) &\geq M(x, y, \phi(t)), \\
N(Fx, Fy, t) &\leq N(x, y, \phi(t))
\end{aligned}$$

where

$$\begin{aligned}
&M(x, y, t) \\
&\geq \min\{M(x, y, \phi(t)), M(x, Fx, \phi(t)) \\
&\quad , M(y, Fy, \phi(t)), M(x, Fy, \phi(t)) * M(y, Fx, \phi(t)\}, \\
&N(x, y, t) \\
&\leq \max\{N(x, y, \phi(t)), N(x, Fx, \phi(t)) \\
&\quad , N(y, Fy, \phi(t)), N(x, Fy, \phi(t)) \diamond N(y, Fx, \phi(t)\}.
\end{aligned}$$

Also, suppose

$$M(x_0, Fx_0, \phi(t)) > 1 - r,\ N(x_0, Fx_0, \phi(t)) < r.$$

Then there exists a unique $x \in \overline{B(Gx_0, r, t)}$ with $x = Fx = Gx$.

References

[1] I. Beg, A. Azam, "Fixed points of asymptotically regular multi-valued mappings," *J. Austral. Math. Soc., Series A,* vol. 53, pp. 313–326, 1992.

[2] S.S. Chang, "Fixed point theorems of mappings on probabilistic metric spaces with applications," *Sci. Sinica Series A,* vol. 26, pp. 1144–1155, 1983.

[3] G. Jungck, "Compatible mappings and common fixed points," *Internat. J. Math. Math. Sci.,* vol. 9, pp. 771–779, 1986.

[4] H. Kaneko, S. Sessa, "Fixed theorems for multi-valued and single-valued mappings," *Internat. J. Math. Math. Sci.,* vol. 12, pp. 257–262, 1989.

[5] D. O'Regan, N. Shahzad, R.P. Agarwal, "Common fixed point theory for compatible maps," *Nonlinear Anal. Forum,* vol. 8, no. 2, pp. 179–222, 2003.

[6] J,H. Park, "Intuitionistic fuzzy metric spaces," *Chaos Solitons & Fractals,* vol. 22, no. 5, pp. 1039–1046, 2004.

[7] J.S. Park, S.Y. Kim, "Common fixed point theorem and example in intuitionistic fuzzy metric space," *K.I.I.S.,* vol. 18, no. 4, pp. 524–552, 2008.

[8] J.S. Park, Y.C. Kwun, J.H. Park, "A fixed point theorem in the intuitionistic fuzzy metric spaces," *F.J.M.S.,* vol. 16, no. 2, pp. 137–149, 2005.

[9] B. Schweizer, A. Sklar, "Statistical metric spaces," *Pacific J. Math.,* vol. 10, pp. 314–334, 1960.

Jong Seo Park
Professor of Chinju National University of Education
Research Area: Fuzzy mathematics, Fuzzy fixed point theory, Fuzzy differential equation

E-mail : parkjs@cue.ac.kr