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Performance Analysis of Coded Cooperation Protocol with Reactive and Proactive Relay Selection

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Abstract

Coded cooperation that integrates channel coding in cooperative transmission has gained a great deal of interest in wireless relay networks. The performance analysis of coded cooperation protocol with multiple relays is investigated in this paper. We show that the diversity order achieved by the coded cooperation in a multi-relay wireless network is not only dependent on the number of cooperating relays but is also dependent on the code-rate of the system. We derive the code-rate bound, which is required to achieve the full diversity gain of the order of cooperating nodes. The code-rate required to achieve full diversity is a linearly decreasing function of the number of available relays in the network. We show that the instantaneous channel state information (CSI)-based relay selection can effectively alleviate this code-rate bound. Analysis shows that the coded cooperation with instantaneous CSI-based relay selection can achieve the full diversity, for an arbitrary number of relays, with a fixed code-rate. Finally, we develop tight upper bounds for the bit error rate (BER) and frame error rate (FER) of the relay selection based on coded cooperation under a Rayleigh fading environment. The analytical upper bounds are verified with simulation results.

Key words : Channel Coding, Coded Cooperation, Cooperative Diversity, Relay Network, Fading Channel, Error Performance.

Ⅰ. Introduction

Cooperative diversity and relaying protocols can provide similar diversity multiplexing tradeoff in a multiple input multiple-output (MIMO) system without requiring multiple antennas at each receiver and transmitter [1]. The performance and implementation issues of various cooperative transmission protocols have been studied in the literature [1]~[7]. The concept of coded cooperation, introduced by Hunter in [6], integrates cooperative diversity with channel coding. The main idea of coded cooperation is that the relay nodes, instead of repeating the received bits, transmit incremental redundant bits through a different and independent fading channel. Therefore, the coded cooperation can achieve both coding and diversity gain.

It has been shown in [8] that the coded cooperation achieves coded diversity similar to the Block-fading channel proposed by Knopp et al. in [9]. The diversity order of the block fading channel is limited by the Singleton Bound [9]. In [7], a scheme to achieve coded diversity in cooperative MISO-based wireless sensor networks has been investigated through random interleaving of the codewords. It has been shown in [7] that the Singleton Bound also restricts the diversity order of the cooperative MISO protocol with cooperative channel coding.

To loosen the diversity bound, we propose a coded cooperation protocol with instantaneous channel state information (CSI)-based relay selection. Instantaneous CSI-based relay section was proposed and analyzed in [3]~[5].

It has been shown in [3] that a single relay selection can significantly reduce the bandwidth penalty of orthogonal transmissions and the synchronization difficulties of distributed-space-time-codes (DSTC) [10]. Moreover, the relay selection schemes that select a single relay to cooperate on information transmission, can efficiently use the transmit power, which leads to an outage optimality for the protocols [3], [4]. Pairwise error probability analysis shows that such instantaneous CSI-based relay selection can also relax the code-rate bound on the diversity order of the coded cooperation with multiple relays. For a binary coded system with a single relay selection, the code-rate required to achieve full diversity is less than or equal to 1/2 for any L. Consequently, the code- rate bound is constant for any arbitrary number of relays.

The diversity multiplexing trade-off of the instantaneous relay selection-based protocols has been analyzed in [3]~[5]. The overhead required to exchange the CSI and implement such a protocol has also been inves-

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tigated in [2] and [5]. The outage probability and bit error probability of such a protocol using decode and forward relaying has been presented in [11]. In conventional amplify-and-forward (AF) and decode-and-forward (DF) protocols, the relay node repeats the source information. Whereas, in coded cooperation protocol [6], the relay node transmits the incremental redundancy for the source instead of repeating the same bits.

In this paper, we show that the diversity order of the coded cooperation with multiple relays, similar to the block fading channel, is a function of the code-rate due to the Singleton Bound. We derive the code-rate bound and show that the code-rate required to achieve the full diversity (diversity of the order of the number of cooperating antennas), for a binary coded system with L+1 cooperating nodes (source and L relays), is less than or equal to (L+1). Hence, the achievable diversity order is a function of the code-rate and the number of relays. This paper investigates the pairwise-error-probability (PEP), BER, and FER of the coded cooperation scheme using a relay selection. We investigate both proactive and reactive relay selection schemes of [3] with coded cooperation. For diversity analysis, we consider the high signal-to-noise-ratio (SNR) slope of the so called PEP. We develop the upper bounds for the BER and FER of the proactive and reactive relay selection-based coded cooperation, assuming binary-phase-shift-keying (BPSK) modulation. The analytical results are compared with the results from a Monte Carlo simulation. A comparison shows that the analytical results matched well with the simulation results. A comparison among proactive and reactive relay selection-based coded cooperation shows that the performance of the reactive protocol is slightly better than that of the proactive one and this performance gap increases with the number of relays.

This paper is organized as follows. In section 2, we describe the system and channel models. Section 3 presents the performance analysis of the proposed protocols. The numerical results and a discussion are given in section 4. Finally, we conclude this paper in section 5.

Ⅱ. System Model

2-1 Channel Model

We consider a system model similar to [2], where a single source (S) communicates with a destination (D) and a set of M relays, $M \in \{R_1, R_2, \cdots, R_M\}$ are available in the system to achieve cooperative diversity. Consider that the channels between any two nodes are subjected to flat Rayleigh fading plus additive-white-Gaussian-noise (AWGN). Each node has a single half duplex radio and a single antenna. The fading coefficients are assumed to be constant over the channel coherence time of N symbol periods. The baseband equivalent received signal at node j due to the transmission of node i for symbol n can be given as

$$r_{ij}(n) = \sqrt{E_b} h_{ij} s_i(n) + \eta_j(n), \tag{1}$$

where $\eta_j(n)$ are the AWGN samples with a variance $\sigma_\eta^2/2$ per dimension at terminal j, h_{ij} is the fading coefficient between node i and j, $s_i(n)$ is the signal transmitted by node i, and E_b is energy per channel bit. Because of BPSK modulation, the energy per channel bit and the energy per symbol are the same. We consider flat Rayleigh fading, hence h_{ij} is modeled as independent samples of a zero mean complex Gaussian random variable with a variance of σ_{ij}^2. The instantaneous received SNR at node j can be given as

$$\gamma_{ij} = \frac{E_b}{\sigma_\eta^2} |h_{ij}|^2. \tag{2}$$

The average SNR of the corresponding links is $\bar{\gamma}_{ij} = \frac{E_b}{\sigma_\eta^2} \sigma_{ij}^2$. In this paper, we consider that M relays are selected by a higher layer protocol, based on an average signal-to-noise-ratio (SNR) of source-to-relay and relay-to-destination links [12]. We assume that for all relays, the average SNR of the source-to-relay and relay-to-destination links are $\bar{\gamma}_{SR}$ and $\bar{\gamma}_{RD}$, respectively. The average SNR of the source-to destination link is $\bar{\gamma}_{SD}$.

2-2 Coded Cooperation

We consider a similar frame structure of coded cooperation to that proposed in [6], with rate-compatible-puncture-convolutional (RCPC) codes. The source message is segmented into blocks of B bits (length of one packet). Each block is then augmented with a cyclic-redundancy-check (CRC) of k bits and is encoded using a designated code from a family of RCPC codes [13]. For the overall code-rate R, each codeword has $N=(B+k)/R$ bits. The codewords are divided into parts denoted as N_m by means of puncturing [13]. The first part is a punctured rate R_0 codeword with $N_0=(B+k)/R_0$ bits. Here, N_0 must be a valid codeword length with rate R_0 so that the relay nodes can decode the source message correctly. The other parts of the codeword N_m for $m \neq 0$ are the redundant information from N_0.

2-2-1 Coded Cooperation without Relay Selection

In the first phase, a codeword of N_0 bits is trans-

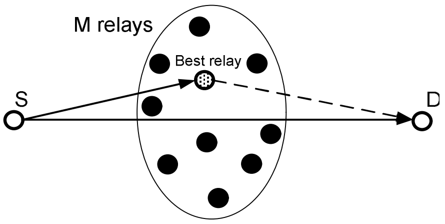

Fig. 1. System model.

mitted by the source, and all relays and destinations receive this codeword. The relays try to decode the received codeword and check it using CRC. The relaysthat decode the source message correctly are members of decoding set, *D*. Assume, L is the cardinality of the decoding set. Relay l is a member of the decoding set and it generates an incremental redundant codeword of N_l bits. In the next L phases, the relays of the decoding set transmit the incremental codeword towards the destination in an orthogonal channel. The destination combines these L+1 portions of the codeword by a de-puncturing [13] operation to generate a low rate code-word of N bits before Viterbi decoding. Clearly, total coderate (R) and the RCPC frame structure of this protocol is a function of the cardinality of the decoding set. The implementation of this type of distributed cooperative coding scheme requires an exchange of information among the relay nodes. Also a centralized controller is required to coordinate the relays in the decoding set and to assign their role in the cooperative coding, similar to the distributed space-time coding scheme of [10]. The centralized controller should inform the mother generator polynomial and the puncturing table of each cooperating relay.

2-2-2 Coded Cooperation with Relay Selection

For relay selection, we consider both the reactive and proactive protocols proposed in [3]. In [2], the proactive relay selection is called opportunistic relaying and in [4], the reactive relay selection is called selection cooperation. This work follows the same trend of [3] and labels the schemes as proactive and reactive relay selection. In proactive relay selection, the relay that maximizes two-hop end-to-end mutual information is selected as the best relay before the source transmission. After selecting the best relay, the source transmits the message to the best relay in the first time slot. If the best relay decodes the message successfully then it forwards it to the destination in the second time slot. Otherwise, it remains silent and the system declares an outage. In a reactive relay selection scheme, the best relay is selected at the end of the first time slot. In the first time slot, the source transmits the message and all the relays receive this message. The relays that successfully decode the source message are members of the decoding set, *D*. The relay that maximizes the mutual information between the relay and destination is selected from the decoding set. If the decoding set is empty, the system declares an outage. Otherwise, the best relay forwards the decoded message.

In such instantaneous CSI-based relay selection processes, each relay node has to know its incoming channel from the source as well as its outgoing channel to the destination. In both relay selection schemes, ready-to-send (RTS) and clear-to-send (CTS)-based link layer signaling can be used to measure the CSI to calculate the mutual information [3]. Then, a distributed trimmer-based scheme is used to select the best relay. Our main focus is to show that such instantaneous relay selection alleviates the code-rate bond of a cooperative coding scheme.

Ⅲ. Performance Analysis

3-1 Diversity Analysis

For the diversity analysis, we employ the so-called pairwise-error-probability (PEP), the probability of choosing one symbol sequence over another for a given pair of possible transmitted symbol sequences. Assume that the Viterbi decoder chooses the coded sequence $\hat{c}=(\hat{c}_1, \hat{c}_2, \hat{c}_3,\ldots,\hat{c}_N)$ when the transmitted codeword was $c=(c_1, c_2, c_3,\ldots,c_N)$; given that these are the only two possible choices. The pairwise error probability, between any two nodes i and j, for convolutionally encoded and BPSK modulated signals conditioned on the instantaneous signal to noise ratio can be written using the approach of [14] and [15], as

$$P(c \rightarrow \hat{c}|\gamma_{ij}) = Q\left(\sqrt{2\textstyle\sum_{n\in\Omega}\gamma_{ij}(n)}\right), \tag{3}$$

where the set Ω is the set of all n for which $\hat{c}_n \neq c_n$, Q(x) is the Gaussian Q function and $\gamma_{ij}(n)$ is the instantaneous signal-to-noise-ratio (SNR) of the received signal at instance n. The diversity order is evaluated by using the high SNR slope of the so-called PEP curve. Hence, to derive the diversity order, we need to approximate the PEP probability of the protocols at a high SNR. For this analysis, with the assumption that SNR→∞, we can assume that the symmetrical uplink channels, i.e., $\bar{\gamma}_{SD}=\bar{\gamma}_{RD}=\bar{\gamma}$.

3-1-1 Coded Cooperation without Relay Selection

Assume that a set of L relays among total M relays are able to decode the source message correctly. For linear codes, the PEP depends on the Hamming distance d between $\hat{c}$ and c, not on the codewords [16]. In the case of coded cooperation without relay selection, the total codeword is the recombination of all N_m bits for m=0, 1, 2,⋯, L [7]. So we can assume that the Hamming distance between transmitted and received codewords ($\hat{c}$ and c) is divided over L+1 blocks as

$$d = d_0 + d_1 + d_2 + \ldots + d_L, \tag{4}$$

where d_m is the Hamming distance between the portion of the transmitted and received code-words corresponding to N_m bits. In this scheme, the destination receives a codeword of N bits and L+1L portions of the received codeword experience independent fading path. The conditional PEP of the recombined codeword can be simply written as [6]

$$P\left(d\middle|\{\gamma_{R_mD}\}\right) = Q\left(\sqrt{2\sum_{m=0}^{L} d_m\gamma_{R_mD}}\right), \tag{5}$$

where γ_{RmD} is the instantaneous SNR at the destination due to the transmission of the cooperating node R_m. For simplicity of presentation, in this subsection, we use the index 0 for the source in some cases (for example, γ_{R_0D} indicates the γ_{SD} and 1, 2,⋯, L for L relays that are members of the decoding set.

We can get the unconditional PEP over the probability density function (PDF), p(·), of the instantaneous received SNR by averaging eq. (5). Considering the alternative form of the Q function [17] and following the same procedures as in [6] and [15], we can find the exact unconditional PEP as

$$P(d) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left[\int_0^{\infty}\exp\left(-\frac{d_0\gamma_{R_0D}}{\sin^2\theta}\right)p(\gamma_{R_0D})d\gamma_{R_0D}\right]\times \left[\int_0^{\infty}\exp\left(-\frac{d_1\gamma_{R_1D}}{\sin^2\theta}\right)p(\gamma_{R_1D})d\gamma_{R_1D}\right]\times \ldots\times\left[\int_0^{\infty}\exp\left(-\frac{d_0\gamma_{R_0D}}{\sin^2\theta}\right)p(\gamma_{R_0D})d\gamma_{R_0D}\right]. \tag{6}$$

For Rayleigh fading, γ_{RmD} is a random variable with an exponential distribution. Using the integral I_1 in the Appendix A, with a high SNR approximation, we can write

$$P(d) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{l=0}^{L}\left\{\left(1+\frac{d_m\bar{\gamma}}{\sin^2\theta}\right)^{-1}\right\}. \tag{7}$$

It is clear from eq. (7) that coded cooperation without relay selection achieves a diversity order of L+1 when $d_m \neq 0$ for all m. As defined in (4), d_m is the Hamming distance between the transmitted and received blocks corresponding to N_m bits. Therefore, the diversity order of the multi-relay coded diversity scheme depends on the number of nonzero d_m and the number of cooperative antennas.

We assume that the minimum number of nonzero d_m for two arbitrary codewords of a truncated convolutional code is λ. The minimum pairwise λ for any binary code of rate R with L+1 blocks can be found by using the Singleton Bound given in [9] as

$$\lambda \le 1 + \left\lfloor (L+1)\left(1-\frac{R}{\log_2|S|}\right)\right\rfloor, \tag{8}$$

where |S| is the alphabet size. Eq. (7) depicts how coded cooperation without relay selection can obtain full coded diversity when λ=L+1. Applying this condition to eq. (8), we can get the code-rate bound to obtain full diversity as

$$R \le \frac{\log_2|s|}{L+1}. \tag{9}$$

For binary modulation (|s|=2) the condition becomes, R≤1/(L+1). For a binary code with R≤1/(L+1), each block of N_m bits can carry sufficient information to decode the source message correctly. Whereas, for rate R>1/(L+1), more than one block is required to carry the decodable information. This fact restricts the achievable diversity order of the proposed coded diversity schemes. For a large number of relays, the required code-rate of the system is very low to achieve full diversity. Since the coded cooperation protocol results a block-fading channel with L+1 blocks, the block-fading code-design framework of [9] can be used in such a scenario. Obviously, such codes can obtain diversity of the order L+1. However, the code design for coded cooperation involves some additional features that are not present in the traditional blockfading model. In coded cooperation, the independent fading channels are provided by the distributed cooperating nodes and the design should also be distributed. Moreover, as we mentioned earlier, the code-rate of the system is dependent on the cardinality of the decoding set, and the decoding set is highly dependent on the portion of codeword (N_0 bits) transmitted by the source [8]. To implement a block-fading code-design framework in a coded cooperation scenario, the cooperating nodes need to know the status of the other nodes (whether they decode the source message correctly or not) to set their own role in code design. Alternatively, a centralized controller, possibly the source node, can collect the information about the status of the cooperating nodes and inform their rule in code design. Both of these procedures require a huge amount of control information exchange among the nodes.

To simplify the code design, we investigate a coded cooperation protocol with the instantaneous CSI-based relay selection in the following subsections. Such relay selection can be implemented through a link layer RTS and CTS signaling without any other signaling overhead [2]. In relay selection schemes, only the selected best relay takes part into the cooperation and the code design criterion are simple. We can use the same codes as in [8] for a single relay system or the RCPC codes used in [6]. Moreover, analysis presented in the next subsection shows that coded cooperation with single relay selection can achieve full diversity by effectively relaxing the code-rate bound explained above.

3-1-2 Coded Cooperation with Proactive Relay Selection

In proactive relay selection, the best relay is selected before starting the data transmission. This protocol selects the best relay that satisfies the following condition [3]

$$R_b = \arg\max_{m \in M}(\min\{\gamma_{SR_m}, \gamma_{R_mD}\}). \tag{10}$$

In the first phase, a packet of N_0 bits is transmitted by the source towards the destination and the best relay. The best relay decodes the source information from the received codeword of N_0 bits and checks whether it is correct or not. If decoding at the best relay is correct, it re-encodes the source information using an RCPC encoder to generate N_1 incremental bits [6]. In the second phase, the best relay transmits the incremental codeword (N_1 bits) towards the destination. If decoding at the best relay is wrong, the source transmits the incremental codeword. Similar to [6], the source can be aware of this situation through a single bit feedback from the best relay.

When the best relay decodes the source information correctly, the destination receives two portions of the codeword from the source and the best relay. Therefore, the PEP of a coded cooperation with proactive relay selection can be given similarly to eq. (6) as

$$P(d) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left[\int_0^{\infty}\exp\left(-\frac{d_0\gamma_{SD}}{\sin^2\theta}\right)p(\gamma_{SD})d\gamma_{SD}\right] \times\left[\int_0^{\infty}\exp\left(\frac{d_1\gamma_{R_bD}}{\sin^2\theta}\right)p\left(\gamma_{R_bD}\right)d\gamma_{R_bD}\right]d\theta. \tag{11}$$

For Rayleigh fading γ_{SD}, and γ_{R_mD} are exponential random variables with hazard rate $1/\bar{\gamma}$. Here, the notation R_m represents the relay-m and R_b represents the best relay. The distribution of γ_{R_bD} is dependent on the relay selection policy. The best relay is successful in decoding the source information if it is a member of the decoding set. The exact PDF of the random variable (rv) γ_{bD} is difficult to obtain but we can easily approximate this rv, conditioned on *D*, as

$$\gamma_{R_bD} \le \max_{m \in \mathcal{D}}\{\min(\gamma_{SR_m}, \gamma_{R_mD})\}. \tag{12}$$

The minimum of two independent exponential rvs is again an exponential rv with a hazard rate equal to the sum of two hazard rates [18]. Therefore, the rv min $(\gamma_{SR_m}, \gamma_{R_mD})$ is exponentially distributed with a hazard rate $2/\bar{\gamma}$. Consider that L is the cardinality of *D*. Now, γ_{R_bD} is a rv, which is the maximum of L exponential random variables with a hazard rate $2/\bar{\gamma}$. The PDF of the random variable γ_{R_bD} can be given as eq. (A2) in the Appendix with $y=\gamma$ and $\bar{y}=\bar{\gamma}/2$. Now, using the high SNR approximation of the integrals I_1 and I_2 the PEP of eq. (11) can be written as

$$P(d) \le \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{1}{d_0\bar{\gamma}}\times\frac{L2^L}{(d_1\bar{\gamma})^L}\Gamma[L]d\theta = \frac{L2^L}{2(d_0\bar{\gamma})(d_1\bar{\gamma})^L}\Gamma[L] = \frac{C_1}{\bar{\gamma}^{L+1}}, \tag{13}$$

where $\Gamma[L]$ is the Gamma function and $C_1=2^L L\Gamma[L]/2d_0d_1$, which is not a function of the average SNR.

The final result in (13) clearly indicates that coded cooperation with proactive relay selection achieves diversity of the order L+1 when d_0 and d_1 are not equal to zero. Therefore, we need a codeword with only two nonzero d_m for $m\in\{0, 1\}$. Using the same approach as eq. (9), the code-rate bound for this case can be given as

$$R \le \frac{\log_2|S|}{2}. \tag{14}$$

For binary modulation, any 1/2 rate code is enough to achieve an arbitrary diversity of the order of L+1. Therefore, the proposed relay selection-based coded cooperation effectively loosens up the code-rate bound. More importantly, the required code-rate is no longer a function of the number of available relays in the network.

3-1-3 Coded Cooperation with Reactive Relay Selection

In this protocol, the best relay is selected from the decoding set that satisfies the following condition:

$$R_b = \arg\max_{k \in \mathcal{D}} \gamma_{R_kD}. \tag{15}$$

The exact error probability of the reactive protocol is dependent on the cardinality of decoding set D i.e., the number of relays that successfully decode the source information in the first phase. In reactive relay selection, γ_{R_bD} is rv, which is the maximum L rvs of the exponential distribution. Now, the PDF of γ_{R_bD} can be given as eq. (A2) in the Appendix with $y=\gamma$ and $\bar{y}=\bar{\gamma}$. The destination receives two portions of the codeword from the source and the best relay, hence the PEP of this protocol can given similarly as (11), and the closed form expression can be obtained by the integrals and PDF in the Appendix as

$$P(d) \le \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{1}{d_0\bar{\gamma}} \times \frac{L}{(d_1\bar{\gamma})^L}\Gamma[L]d\theta = \frac{L}{2(d_0\bar{\gamma})(d_1\bar{\gamma})^L}\Gamma[L] = \frac{c_2}{\bar{\gamma}^{L+1}}, \tag{16}$$

where C_2 is not a function of SNR. Consequently, the code-rate bound of both proactive and reactive relay selection-based coded cooperation is the same as that given in eq. (14).

3-2 BER and FER Probability

3-2-1 Coded Cooperation with Proactive Relay Selection

In this protocol, two things can occur at the destination. If decoding at the best relay is successful then a cooperative-transmission (CT) occurs where the destination receives two portions of the codeword from two independent channels. Otherwise, a detect-transmission (DT) between the source and destination takes place. To develop the upper bounds on BER and FER, we employ the so-called pairwise-error-probability (PEP) defined in the previous subsection. The conditional pairwise error probability of CT and DT for convolutionally encoded and BPSK modulated signals conditioned on the instantaneous received SNR can be written as [6]

$$P^{CT}(d) = Q(\sqrt{2d_0\gamma_{SD} + 2d_1\gamma_{R_bD}}) \tag{17}$$

$$P^{DT}(d) = Q(\sqrt{2d\gamma_{SD}}, \tag{18}$$

where d, d_0 and d_1 are the Hamming distance of the transmitted and received codewords corresponding to N, N_0, and N_1 bits with $d=d_1+d_2$.

The conditional BER and FER probability in terms of the PEP of a decoded block of length B can be calculated using the approach in [16] as

$$P_{FER}^{CT}(\gamma_{SD},\gamma_{R_bD}) \le B \sum_{d=d_{free}}^{\infty} a(d)P^{CT}(d) \tag{19}$$

$$P_{BER}^{CT}(\gamma_{SD},\gamma_{R_bD}) \le \frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)P^{CT}(d) \tag{20}$$

$$P_{FER}^{DT}(\gamma_{SD}) \le B \cdot \sum_{d=d_{free}}^{\infty} a(d)P^{DT}(d) \tag{21}$$

$$P_{BER}^{DT}(\gamma_{SD}) \le \frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)P^{DT}(d), \tag{22}$$

where k_c is the number of input bits per branch of the code trellis, d_{free} is the free distance, and a(d) and c(d) are the number of error events and the information error weight with a Hamming distance d of the codeword of N bits [16]. Using the limit before the averaging approach proposed in [14], the unconditional FER and BER, can be given as

$$P_{FER}^{CT} \le \int_{\gamma_{R_bD}}\int_{\gamma_{SD}} \min[1, P_{FER}^{CT}(\gamma_{SD},\gamma_{R_bD})] \times P(\gamma_{SD})P(\gamma_{R_bD})d\gamma_{SD}d\gamma_{R_bD} \tag{23}$$

$$P_{BER}^{CT} \le \int_{\gamma_{R_bD}}\int_{\gamma_{SD}} \min[0.5, P_{BER}^{CT}(\gamma_{SD},\gamma_{R_bD})] \times P(\gamma_{SD})P(\gamma_{R_bD})d\gamma_{SD}d\gamma_{R_bD} \tag{24}$$

$$P_{FER}^{DT} \le \int_{\gamma_{SD}} \min[1, P_{FER}^{DT}(\gamma_{SD})]P(\gamma_{SD})d\gamma_{SD} \tag{25}$$

$$P_{BER}^{DT} \le \int_{\gamma_{SD}} \min[0.5, P_{BER}^{DT}(\gamma_{SD})]P(\gamma_{SD})d\gamma_{SD}, \tag{26}$$

where the rv γ_{R_bD} is approximated in eq. (10) and the PDF of γ_{R_bD} can be given as eq. (A· 2) in the Appendix with $y=\gamma$ and $\bar{y}=2/\bar{\gamma}_{SR}$. Due to minimization, we cannot change the order of integration and summation of eqs. (23)~(26) for slow fading as explained in [14]. To evaluate the end-to-end FER and BER, we need to calculate the probability that the best relay decodes the source information successfully. The probability of incorrect decoding at the best relay (p_1) is the same as the FER at the best relay. The conditional and unconditional FER at the best relay can be calculated by considering the error event $a(d_0)$ corresponding to the codeword of N_0 bits as

$$P_{FER}^{R_b}(\gamma_{SR_b}) \le B \cdot \sum_{d_0=d_{free}}^{\infty} a(d_0)P^{R_b}(d_0|\gamma_{SR_b}) \tag{27}$$

$$p_1 \le \int_{\gamma_{SR_b}} \min[1, P_{FER}^{R_b}(\gamma_{SR_b})]\, p(\gamma_{SR_b})d\gamma_{SR_b}, \tag{28}$$

where $P^{R_b}(d_0|\gamma_{SR_b})Q(\sqrt{2d_0\gamma_{SR_b}})$. The rv γ_{SR_b} also can be approximated as eq. (12) and the PDF of γ_{SR_b} can be given as eq. (A2) in the Appendix with $y=\gamma$ and $\bar{y}=2/\bar{\gamma}_{SR}$. Finally, the overall unconditional BER and FER of the coded cooperation with proactive relay selection are given by

$$P_{FER} = p_1P_{FER}^{DT} + (1-p_1)P_{FER}^{CT} \tag{29}$$

$$P_{BER} = p_1P_{BER}^{DT} + (1-p_1)P_{BER}^{CT}. \tag{30}$$

3-2-2 Coded Cooperation with Reactive Relay Selection

In this protocol, the best relay is selected from the decoding set D as given in eq. (15). The exact error probability of the reactive protocol is dependent on the cardinality of decoding set D i.e., the number of relays that successfully decode the source information in the first phase. The source-to-relay links are assumed to be symmetrical, so that the probability of decoding the source information at each relay is the same. Assume, P_2 is the probability of the incorrect decoding of the source information at each relay, which represents the FER at each relay. Now the probability that L (cardinality of D) relays have decoded the source information correctly can be written as

$$\Pr(L) = \binom{M}{L} P_2^{M-L}(1-p_2)^L. \tag{31}$$

We can calculate p_2 by considering the properties of the codeword transmitted in the first frame (N_0). For this case, the conditional and unconditional FER at relay-m (R_m) can be written as

$$P_{FER}^{R_m}(\gamma_{SR_m}) \le B\sum_{d_0=d_{free}}^{\infty} a(d_0)P^{R_m}(d_0 | \gamma_{SR_m}) \tag{32}$$

$$p_2 \le \int_{\gamma_{SR_m}} \min[1, P_{FER}^{R_m}(\gamma_{SR_m})]\, p(\gamma_{SR_m}) d\gamma_{SR_m}, \tag{33}$$

Now for L>0, the average PEP conditioned on L can be given by

$$P^{CT}(d) = Q(\sqrt{2d_0\gamma_{SD} + 2d_1\gamma_{R_bD}(L)}). \tag{34}$$

For L=0, the decoding set is empty and a direct transmission between the source and destination takes place. The conditional and unconditional BER and FER of direct transmission are derived in Eqs. (21), (22), (25), and (26). The conditional and unconditional BER and FER probability of the reactive protocol, for L>0, can be given as

$$P_{FER}(\gamma_{SD}, \gamma_{R_bD}|L) \le B \sum_{d=d_{free}}^{\infty} a(d)P^{CT}(d) \tag{35}$$

$$P_{BER}(\gamma_{SD}, \gamma_{R_bD}|L) \le \frac{1}{k_c} \sum_{d=d_{free}}^{\infty} c(d)P^{CT}(d) \tag{36}$$

$$P_{FER}(L) \le \int_{\gamma_{R_bD}}\int_{\gamma_{SD}} \min[1, P_{FER}(\gamma_{SD}, \gamma_{R_bD}|L)] \times P(\gamma_{SD})P(\gamma_{R_bD})d\gamma_{SD}d\gamma_{R_bD{}_bD} \tag{37}$$

$$P_{BER}(L) \le \int_{\gamma_{R_bD}}\int_{\gamma_{SD}} \min[0.5, P_{BER}(\gamma_{SD}, \gamma_{R_bD}|L)] \times P(\gamma_{SD})P(\gamma_{R_bD})d\gamma_{SD}d\gamma_{R_bD}. \tag{38}$$

The rv γ_{R_bD} is a rv which is the maximum from L rvs of the exponential distribution with a hazard rate $\bar{\gamma}_{RD}$.

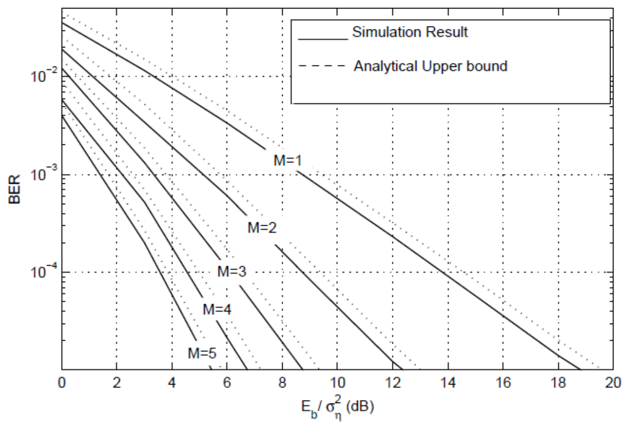

Fig. 2. BER performance with proactive relay selection.

Hence, the PDF of γ_{R_bD} can be given as eq. (A· 2) in the Appendix, with M=L, y=γ and $\bar{y}=\bar{\gamma}_{RD}$.

Finally, the overall unconditional BER and FER of reactive-protocol (RP) can be given by

$$P_{FER,RP} = \sum_{L+0}^{M} P_{FER}(L) \cdot \Pr(L) \tag{39}$$

$$P_{BER,RP} = \sum_{L+0}^{M} P_{BER}(L) \cdot \Pr(L). \tag{40}$$

Ⅳ. Numerical Results

This section provides the BER and FER performance of the coded cooperation with relay selection. We consider the source node with data of a block size B=128 bits. For error checking, a CRC code with a generator polynomial 15935 (hexadecimal number) is used. Puncturing is done for 50 % cooperation ($N_0=N_1$). Corresponding to the codeword of N bits, convolutional code with polynomial generator G=23, 35, 27, 33) (in octal notation) is considered. We consider the fading coefficients to be constant over the transmission of one frame and identically independent from the next. Although the developed upper bounds are valid for any channel variance, for simplicity we consider the variance of the channel coefficients to be σ_{SD}^2=1, σ_{SR}^2=2, and σ_{RD}^2 =1.5 throughout the simulation.

In Fig. 1, we analyze the BER performance of the proposed coded cooperation with proactive relay selection for a different number of relays (M=1, 2, 3, 4, and 5). Similar results using FER probability for reactive relay selection-based protocol are shown in Fig. 2. Figs. 1 and 2 confirm that the performance of both protocols improves as the number of available relays increases due to the diversity gain achieved by the relay selection scheme. In Figs. 1 and 2, we also verify the analytical upper bounds developed in section 3, relative to the simulation results. The summations of BER and

FER equations are truncated to the first 5 terms of d. We use MATLAB function '*distspec*' to evaluate the values of a(d), c(d) and d_{free}. In all cases, the analytical upper bounds agree very well with the Monte Carlo simulation results. For proactive protocol, the upper bounds are quite loose compared with the reactive protocol because of the approximation of eq. (12). For both protocols, the bounds are tight enough to evaluate the performance of the system. Similar results for the FER of the proactive protocol and BER of the reactive protocol are also possible but they do not give any further insightful meaning.

In Fig. 3, we compare the BER probability of coded cooperation with proactive and reactive relay selection. For clarity of presentation, we omit the upper bounds from this figure. The results show that the reactive protocol performs better than the proactive protocol and this improvement increases as the number of relays increases. This result suggests that the reactive relay selection policy is better than the proactive one. Bletsas et al. in [3] show that the opportunistic relaying is outage optimal when only cooperative links are considered i.e. destination decodes the signal received from the best relay. In this paper, we consider that the destination combines the signals received from the source and the best relay through depuncturing [13]. Therefore, the best relay-to-destination channel is not necessarily required to carry all the information because the source-to-destination channel carries a portion of this information. Hence, when the destination optimally combines two links, the reactive relay selection performs better than the proactive protocol. In the proactive protocol, only the best relay needs to overhear the source message. However, in reactive protocol, all possible relays need to overhear the source transmission in order to participate in the relay selection procedure. The reactive protocol provides better error performance at the expense of the extra overhearing.

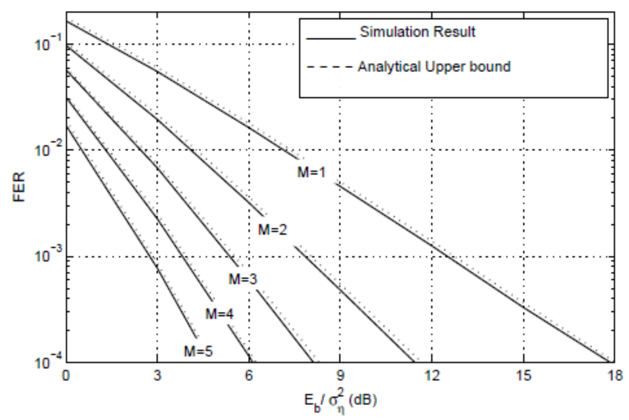

Fig. 3. FER performance with reactive relay selection.

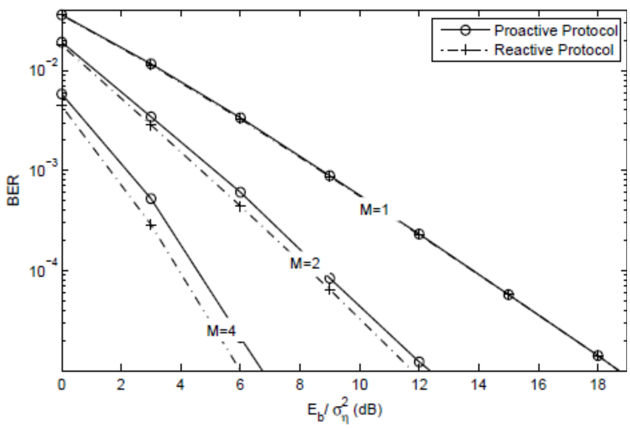

Fig. 4. BER performance comparison of proactive and reactive relay selection.

V. Conclusion

In this paper, we analyzed the performance of coded cooperation protocol in a multi-relay environment. We showed that coded cooperation without relay selection can achieve full diversity when the code-rate is below a certain threshold. Analytical results showed that this code-rate bound is in the order of the number of relays. We have investigated the instantaneous CSI-based relay selection methods to minimize this code-rate bound. We considered both a proactive and reactive relay selection scheme. For both cases, the code-rate bound is significantly relaxed to achieve the full diversity. A method to evaluate the error performance of coded cooperation with single relay selection is also presented. Analysis and simulation results are given to show the effectiveness of the protocols in a Rayleigh fading environment. In this paper, we only consider BPSK modulation and convolutional codes for the purpose of exposition. Extension to other codes and modulation schemes is also possible.

Appendix A

Let, x be an exponential random variable with hazard rate $1/\bar{x}$ and $y=\max(y_1, y_2, y_3,\cdots, y_M)$, where $y_1, y_2, y_3,\cdots, y_M$ are M independent exponential rvs with a common hazard rate $1/\bar{y}$. The PDF of the random variables x and y are

$$p(x) = \frac{1}{\bar{x}}\exp\left(-\frac{x}{\bar{x}}\right) \tag{A1}$$

$$p(y) = M\frac{1}{\bar{y}}\exp\left(-\frac{y}{\bar{y}}\right)\left[1-\exp\left(-\frac{y}{\bar{y}}\right)\right]^{M-1}. \tag{A2}$$

We define two integrals I_1 and I_2 and solve by replacing the PDF of (A1) and (A2) as

$$I_1(x,\bar{x},a) = \int_0^{\infty} \exp\left(-\frac{ax}{\sin^2\theta}\right) p(x)dx = \left(1+\frac{a\bar{x}}{\sin^2\theta}\right)^{-1} \quad \text{(A3)}$$

$$I_1(y,\bar{y},M,b) = \int_0 \exp\left(-\frac{by}{\sin^2\theta}\right) p(y)dy = \int_0^{\infty} \exp(-by)\, p(y)dy$$
$$= \int_0^{\infty} \exp(-by)\, M\frac{1}{\bar{y}}\exp\left(-\frac{y}{\bar{y}}\right)\left[1-\exp\left(-\frac{y}{\bar{y}}\right)\right]^{M-1} dy. \quad \text{(A4)}$$

The upper bounds of the integral I_1 can be obtained by setting son θ=1 and considering that $\bar{x} \gg 1$ as

$$I_1(\bar{x}\to\infty) = \lim_{\bar{x}\to\infty} I(x,\bar{x},a) = \frac{1}{a\bar{x}}. \quad \text{(A5)}$$

Similarly, the upper bounds of the integral I_2 can be obtain by setting $\sin\theta$=1. Considering $\tau \to 0$, we can approximate $e^{\tau} \approx (1+\tau)$ and eq. (A4) can be written as

$$I_2(\bar{y}\to\infty) = \int_0^{\infty} \exp(-by)\, M\frac{1}{\bar{y}}\left(1-\frac{y}{\bar{y}}\right)\left[\frac{y}{\bar{y}}\right]^{M-1} dy$$
$$= \frac{1}{\bar{y}^M} M \int_0^{\infty} \exp(-by)\, y^{M-1}dy = \frac{M}{(by)^M}\Gamma[M], \quad \text{(A6)}$$

where $\Gamma[\cdot]$ is the well known Gamma function.

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