Feedback Error Quantification in Adaptive Modulation over Fading Channels

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***Abstract*—In this work, we consider imperfectness of feedback channels in the adaptive transmission scheme which was previously studied with an assumption of error-free feedback channels. New method of mapping the modulation index into the feedback channel symbols and quantifying feedback error over fading channels are proposed. The presented method and results are expected to offer valuable tools for the system designer to efficiently implement adaptive diversity schemes to compensate for the performance degradation due to feedback error.**

***Index Terms*—Adaptive modulation, feedback error, fading channels.**

I. INTRODUCTION

BECAUSE of the growing demand for a higher spectral efficiency as well as a good link reliability in wireless communications, many new technologies have been proposed over the last few decades. Among them, adaptive transmission aims to optimize the transmission rate according to the fading channel variations. It has been shown that a considerable gain in throughput can be achieved by using adaptive transmission while maintaining a certain target error rate performance [1, 2].

In most of the published papers dealing with adaptive modulation, it is generally assumed for analytical tractability that the feedback channel is error-free. However, there are a number of real-life scenarios in which this ideal assumption is not valid, especially in the case that sufficient and powerful error control method cannot be implemented over the feedback channel. As such, the study of the impact of imperfect feedback channels is very important to investigate novel schemes to mitigate the performance degradation and to reduce the outage region due to feedback errors.

In this paper, we suggest a method to map the modulation mode into the feedback channel symbols. Based on the assumption that the adaptive modulation mode used for transmission may be different from the one selected by the receiver due to the imperfect feedback channels, we also show how to quantify feedback error over fading channels.

Manuscript received March 13, 2011; revised March 29, 2011; accepted April 12, 2011.

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II. CHANNEL AND SYSTEM MODEL

We assume that the adaptive modulation is implemented in a discrete-time fashion. More specifically, short guard periods are periodically inserted into the transmitted signal. During these guard periods, the receiver performs a series of operations, including path estimations and combined signal-to-noise ratio (SNR) comparisons with respect to the predetermined SNR threshold. After determining the most appropriate diversity combiner structure and adaptive modulation mode to be used during the subsequent data burst, the receiver sends back the adaptive modulation mode to the transmitter via reverse link before the guard period ends. Note that because of feedback channel imperfection, the modulation mode used for transmission may be different from the one selected by the receiver.

The fading conditions are assumed to follow the Rayleigh model and to be independent and identically distributed (i.i.d.) across the diversity paths and between different guard periods and data bursts. Hence, if we let γ_i denote the instantaneous received SNR of the ith path, $i = 1, 2, \cdots, L$, where L is the number of available diversity paths at the receiver, then the faded SNR, γ_i, follows the same exponential distribution, with common probability density function (PDF) and cumulative distribution function (CDF) given as [3, Eq. (6.5)]

$$f_{\gamma_i}(x) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{x}{\bar{\gamma}}\right), \quad x \geq 0 \tag{1}$$

and

$$F_{\gamma_i}(x) = 1 - \exp\left(-\frac{x}{\bar{\gamma}}\right), \quad x \geq 0 \tag{2}$$

respectively, where $\bar{\gamma}$ is the common average faded SNR.

We adopt the constant-power variable-rate uncoded M-ary quadrature amplitude modulation (M-QAM) scheme, where the modulation mode, M, is restricted to a power of 2, 2^n. Assume that the SNR range is divided into N+1 regions and each region is associated with a particular QAM signal constellation. The region boundaries, denoted by γ_{T_n}, are set to the SNR required to achieve the target BER, denoted by BER0, using 2^n-QAM over an additive white Gaussian noise (AWGN) channel. It has been shown that the instantaneous BER of 2^n-QAM with two-dimensional Gray coding over an AWGN channel with SNR of γ can be well approximated by [1, Eq.

(28)]

$$\mathrm{BER}_n(\gamma) = \frac{1}{5}\exp\left(-\frac{3\gamma}{2(2^n-1)}\right), \quad n \geq 1, 2, \ldots, N. \quad (3)$$

Therefore, the boundary thresholds can be calculated in terms of a target BER, BER0, as

$$\gamma_{T_n} = \begin{cases} -\frac{2}{3}\ln(5\mathrm{BER}_0)\,(2^n - 1), & n = 0, 1, \ldots, N; \\ \infty, & n = N + 1. \end{cases} \quad (4)$$

The receiver chooses the most suitable modulation mode, n, or constellation size, $M = 2^n$, based solely on the fading channel conditions. This is done by first examining the received SNR and then finding a proper region in which its estimated SNR falls. If the estimated SNR is in the nth region, the receiver informs via feedback path the corresponding modulation mode to the transmitter so that the constellation size of 2^n is used during the subsequent data.

III. FEEDBACK ERROR AND ITS QUANTIFICATION

In this section, we present as an example a modulation symbol labeling scheme for the feedback channel and show how to quantify feedback error over fading channels.

Assume that after performing a series of operations described in the previous section, the receiver decides to use the modulation mode j ($0 \leq j \leq N$).The receiver sends the index of this modulation mode, j, to the transmitter via the feedback channel but due to a possible feedback error the transmitter may end up using mode n ($0 \leq n \leq N$) instead of j for transmission. To evaluate the effect of the feedback error, we need to determine the probability that the transmission mode n is used while mode j was selected by the receiver due to feedback error. In what follows, we denote these mode transition probabilities by $q_{n,j}$. Note that these probabilities depend on the quality of the feedback channel and the signaling scheme used. In this work, we assume a phase shift keying (PSK) based signaling scheme over the feedback channel. More specifically, we use N+1 different PSK symbols to represent the total N+1 transmission modes. The receiver needs to transmit only one symbol during the guard period to indicate the selected mode.

If we assume that each transmission mode is sequentially mapped to each PSK symbol, then the decision region of the ith symbol representing the ith transmission mode will be the ith wedge-shaped area in the PSK signal space. Therefore, the mode transition probability, $q_{n,j}$, is equal to the average probability that the decision variable for the feedback channel detection falls erroneously in the nth wedge-shaped area instead of the jth wedge-shaped area. Since the transition probability, $q_{n,j}$, can be expressed as a function of the difference of mode indices, j and n, which can be calculated in a circular fashion as

$$a = \min\{|n - j|, N + 1 - |n - j|\} \quad (5)$$

the instantaneous transition probability is equal to the probability that the decision variable falls in the wedge-shaped region between two different phase angles,

$$\theta_1 = \frac{(2a-1)\pi}{N+1} \text{and} \theta_2 = \frac{(2a+1)\pi}{N+1},$$

when the symbol with phase zero was transmitted. It can be shown that this probability can be calculated as [4, Eq. (17)]

$$\begin{aligned} &\Psi(\gamma_s;\ \theta_1, \theta_2) \\ &= Q\left(\sqrt{2\gamma_s}\sin\theta_1\right) - Q\left(\sqrt{2\gamma_s}\sin\theta_2\right) \\ &+ Q\left(-\sqrt{2\gamma_s}\sin\theta_{1,0}; \frac{\left(\sqrt{2\gamma_s}\sin\theta_2 - \cos(\theta_2-\theta_1)\sqrt{2\gamma_s}\sin\theta_1\right)\mathrm{sgn}\left(\sqrt{2\gamma_s}\sin\theta_1\right)}{\sqrt{2\gamma_s\sin^2\theta_1 - 4\cos(\theta_2-\theta_1)\gamma_s\sin\theta_1\sin\theta_2 + 2\gamma_s\sin^2\theta_1}}\right) \\ &+ Q\left(-\sqrt{2\gamma_s}\sin\theta_{2,0}; \frac{\left(\sqrt{2\gamma_s}\sin\theta_1 - \cos(\theta_2-\theta_1)\sqrt{2\gamma_s}\sin\theta_2\right)\mathrm{sgn}\left(\sqrt{2\gamma_s}\sin\theta_2\right)}{\sqrt{2\gamma_s\sin^2\theta_1 - 4\cos(\theta_2-\theta_1)\gamma_s\sin\theta_1\sin\theta_2 + 2\gamma_s\sin^2\theta_1}}\right) \\ &- \Lambda(-2\gamma_s\sin\theta_1\sin\theta_2) \end{aligned} \quad (6)$$

Where γ_s is the instantaneous symbol SNR of the feedback channel, $Q(\cdot)$ and $Q(\cdot, \cdot; \cdot)$ are the 1-D and 2-D Gaussian Q-functions [5, Eq. (26.3.3)], respectively,

$$\mathrm{sgn}(x) = \begin{cases} 1, & x \geq 0; \\ -1, & x < 0, \end{cases} \quad (7)$$

and

$$\Lambda(x) = \begin{cases} 0, & x \geq 0; \\ 1/2, & x < 0, \end{cases} \quad (8)$$

Finally, noting that due to Rayleigh fading, the instantaneous SNR, γ_s, of the feedback channel is a random variable, we can obtain the mode transition probability, $q_{n,j}$, by averaging over its distribution as

$$q_{n,j} = \int_0^\infty \Psi\left(\gamma_s;\ \frac{(2a-1)\pi}{N+1}, \frac{(2a+1)\pi}{N+1}\right) f_{\gamma_s}(\gamma_s)\,d\gamma_s \quad (9)$$

Where $f_{\gamma_s}(\cdot)$ is the PDF of the faded SNR of the feedback channel, givenin (1) but with average $\bar{\gamma}_s$. After substituting (1) and (6) with (5) into(9), the transition probability, $q_{n,j}$, can be shown to be a function of the average SNR, $\bar{\gamma}_s$, of the feedback channel. One can easily relate this average SNR with the average BER of (N+1)-PSK over Rayleigh fading channels, denoted by Pb. For example, when N = 3, we have $P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$ where $\bar{\gamma} = \bar{\gamma}_s/2$ [6, Eq. (8.104)]. In Table I, the transition probability, $q_{n,j}$, for this PSK-based sequential labeling scheme is calculated as a function of the average BER, P_b, and the required average symbol SNR, $\bar{\gamma}_s$, of the feedback channel when $N = 3$.

Note that with this sequential labeling scheme, the symbol for the highest transmission mode is adjacent to the symbol for the lowest mode. One may think of other labeling schemes in which no adjacent symbols for the highest and lowest modes exist. In this case, because of the lower chance of transition between the highest and the lowest modulation indices, we can in general expect a higher spectral efficiency with more combined paths than the sequential labeling scheme in the high SNR region and a lower spectral efficiency with less combined paths in the low SNR region. This difference will be increasing as feedback error increases.

TABLE I
TRANSITION PROBBILITY, $q_{n,j}$, OVER RAYLEIGH FADING CHANNELS FOR DIFFERENT AVERAGE BERS, P_b, WITH REQUIRED AVERAGE SNRS, $\bar{\gamma}_s$, OF THE FEEDBACK CHANNEL WHEN $N = 3$.

	$P_b = 10^{-1}$ $\bar{\gamma}_s$=5.51 dB	P_b= 10-2 $\bar{\gamma}_s$=16.86 dB	$P_b = 10^{-3}$ $\bar{\gamma}_s$=26.98 dB
$\mid n - j \mid = 0$	0.8218206	0.9818155	0.9981819
$\mid n - j \mid = 1, 3$	0.0781789	0.0081057	0.0008180
$\mid n - j \mid = 2$	0.0218214	0.0018491	0.0001820

IV. APPLICATION TO AMDC SCHEME

In this section, we illustrate how to use the transition probability we obtained in the previous section. First, we consider adaptive modulation and diversity combining (AMDC) schemes in [7], [8]. With this scheme, the receiver jointly decides the proper modulation mode and diversity combiner structure based on the channel quality and the target error rate requirement. Different modes of operations have been considered based on the primary optimization criteria of the joint design. For example, the power-efficient AMDC scheme leads to a high processing power efficiency, the bandwidth-efficient AMDC scheme leads to a high bandwidth efficiency, and the bandwidth-efficient and power-greedy AMDC scheme leads to a high bandwidth efficiency as well as an improved power efficiency at the cost of a higher error rate in comparison with the bandwidth-efficient AMDC scheme. We also consider the effect of feedback imperfection and its mitigation in the context of AMDC systems. Specifically, we assume that feedback error causes the adaptive modulation mode used for transmission to be different from the one selected by the receiver after adaptive combining. Now, we can propose to use diversity path adjustment at the receiver to mitigate the error performance degradation or explore additional power savings. In particular, the receiver may combine more diversity paths, if possible, to compensate for the bit error rate (BER) degradation when the transmitter sends data with a higher modulation mode than the one selected by the receiver. On the other hand, the receiver may use less diversity paths to save the receiver processing power when a lower modulation mode is used instead. Hence, the number of combined paths can be adaptively changed depending on the nature of feedback error.

Hence, to accurately evaluate the effect of the feedback error and the compensation strategy, we need to determine the probability that the transmission mode n is used while mode j was selected by the receiver due to feedback error. We have already denoted and obtained these mode transition probabilities by $q_{n,j}$ in the previous section.

V. CONCLUSIONS

In this paper, we considered the adaptive transmission scheme over fading channels taking into account imperfect feedback channel conditions. More specifically, it is assumed that the modulation mode the transmitter receives from the receiver can be perturbed by some feedback errors that can result in the usage of the wrong transmission mode between the transmitter and the receiver. We proposed a method of quantifying this feedback error which can be valuably utilized to implement new diversity combining structures at the receiver to mitigate the performance impediment according to the nature of feedback error. Based on this result, as a future work, we will consider adaptive diversity schemes to compensate for the performance degradation due to feedback error.

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