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A Reliability Sampling Plan Based on Progressive Interval Censoring Under Pareto Distribution of Second Kind

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Abstract. In this paper, a reliability sampling plan under progressively type-1 interval censoring is proposed when the lifetime of products follows the Pareto distribution of second kind. We use the maximum likelihood estimator for the median life and its asymptotic distribution. The cost model is proposed and the design parameters are determined such that the given producer's and the consumer's risks are satisfied. Tables are given and the results are explained with examples.

Keywords: Acceptance Sampling, Cost Minimization, Pareto Distribution of Second kind, Maximum Likelihood Estimation

1. INTRODUCTION

The censoring schemes such as time censored (Type-1) and failure-censored (Type-II) are commonly used to reduce the time and the cost of the life test. In these censoring schemes the surviving items are only removed at the end of the life test. However, there is a situation in which experimenter needs to remove a part of the surviving items at time points before the termination time. This type of life test is called a progressive censoring. The basic advantage of progressive censoring scheme is that the removed items can be used for other experiment. Secondly, this censoring scheme can be used to save the cost and the time of the experiment when the items under inspections are costly. For more details about the advantages of this scheme, reader may refer to Cohen (1963).

Many authors have studied on the inference of the parameters of various lifetime distributions under the progressively censoring. See for example, Ali Mousa and Jaheen (2002), Gouno *et al.* (2004), Guilbaud (2001), Li *et al.* (2007), Lin *et al.* (2006), Soliman (2005), Tse and Yuen (1998). Recently, Wu *et al.* (2008) proposed the progressively group-censoring life test considering

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the cost model for the Weibull distribution. Huang and Wu (2008) developed the reliability plans under the progressively Type-I interval censoring using the cost function. They determined the design parameter using the maximum likelihood estimation (MLE) method which minimizes the total cost of the experiment under the specified producer's and the consumer's risks. Wu and Huang (2009) developed the progressively Type-1 group censoring using the MLE to find the parameter of Weibull distribution. The design parameters using the cost model are determined and the sensitivity analysis is investigated. Recently, Lio *et al*. (2010a, b) proposed acceptance sampling plans for the Birnbaum-Saunders distributions and Burr type XII distributions using the median as the quality parameter. They showed and justified the use of median in area of reliability and argued that as distributions under study are skewed and for skewed distributions median provides better results than the mean as quality parameter. For more detail about these papers reader can refer to Lio *et al*. (2010a, b).

Exploring the literature about the progressively Type-1 censoring we see that most of the authors used the Weibull distribution to develop the plan based on this scheme. No attention has been paid to develop an acceptance sampling plan for the Pareto distribution of the second kind. So, the purpose of this paper is to design an acceptance sampling plan based on the progressive Type-I censoring scheme for the Pareto distribution of the second. The minimum sample size, the number of inspections and the length of the inspection interval will be determined to minimize the total cost while satisfying the given producer's and the consumer's risks. We use the median life of the Pareto distribution as a reliability measure.

2. PROPOSED SAMPLING PLAN

An acceptance sampling plan is used to test the quality or reliability of the submitted lots of items before it is released for consumer's use. This quality is tested on the basis of a few items taken from a lot. We use the estimate of the median life as the test statistic

2.1 Pareto Distribution of Second Kind

Suppose that an experimenter wants to conduct a life test under the assumption that the lifetime of products under inspection is independently and identically distributed as Pareto distribution of the second kind with the probability distribution function (pdf) and cumulative distribution function (cdf) given as

$$f(t) = \frac{\lambda t}{\sigma}\left(1+\frac{t}{\sigma}\right)^{-(\lambda+1)} \quad t>0,\ \sigma,\ \lambda>0 \qquad (1)$$

and

$$F(t) = 1-\left(1+\frac{t}{\sigma}\right)^{-(\lambda)} \quad t>0,\ \sigma,\ \lambda>0 \qquad (2)$$

where λ is a shape parameter and σ is a scale parameter of the distribution. In this study we assume that λ is known. This distribution is frequently used in quality control and the reliability applications. Bain and Engelhardt (1992) discussed the application of this distribution in the field of bio-medical sciences. Baklizi (2003) proposed the ordinary single acceptance sampling plans based on truncated life test under the Pareto distribution of the second kind. More recently, Aslam *et al*. (2010) proposed the group acceptance sampling plans for the Pareto distribution of the second kind.

As stated in the above procedure we are using the median life of the distribution as a reliability measure. The median for the Pareto distribution is derived by

$$median = \sigma(2^{1/\lambda}-1) \qquad (3)$$

Particularly for $\lambda = 1$, the median is just σ. When the shape parameter λ is known, the estimate of the median is just the constant times the estimate of σ. Therefore, we may assume that $\lambda = 1$ without loss of generality.

2.2 Procedure of the Proposed Plan

Let us consider the following sampling plan based on the life test with type-1 progressively interval censoring for the items whose life follows the Pareto distribution of the second kind given in Eq. (1) or Eq. (2). In this plan acceptance number c acts as the lower specification limit.

Step 1: Draw a sample of size n and put them on test at time 0.
Step 2: Items are inspected at pre-determined times $\tau_1, \tau_2, \cdots, \tau_k$, where $0<\tau_1<\tau_2<\ldots<\tau_k$.
Step 3: At the i-th inspection time the number of failed items (n_i) are counted, and r_i surviving items are removed for future use ($i = 1, 2, \cdots, k$).
Step 4: Estimate the median of the distribution using the maximum likelihood estimation.
Step 5: Accept the lot if the estimated median is greater than or equal to the acceptance number c. Reject the lot, otherwise.

Let m_i be the surviving items just before the i-th inspection time. Then, we have

$$m_i = m_{i-1} - n_{i-1} - r_{i-1},\ \text{i} = 2,\ \cdots,\ \text{k};\ \ m_1 = n$$

The value of r_i, $i = 1,\ 2,\ \cdots k$ are assumed to be determined by pre-fixed proportion p_i ($p_k = 1$) of the remaining items as in Huang and Wu (2008) such that

$$r_i = (m_i - n_i) p_i \text{ , i=1, } \cdots \text{, k}$$

Under a progressively type-1 censoring scheme, we have the fact that

$$n_i \left| n_{i-1}, \; \cdots \; , n_1, r_{i-1}, \; \cdots, \; r_1 \sim binomial\left(m_i, \; q_i\right) \right. \tag{4}$$

where q_i is the probability that an item fails between τ_{i-1} and τ_i and given by

$$q_i = \frac{\left(F(\tau_i) - F(\tau_{i-1})\right)}{1 - F(\tau_{i-1})} \tag{5}$$

For the distribution under study given (2), the Eq. (5) can be written as

$$q_i = \frac{\left(\sigma + \tau_i\right)^\lambda - \left(\sigma + \tau_{i-1}\right)^\lambda}{\left(\sigma + \tau_i\right)^\lambda} \tag{6}$$

Particularly for λ = 1, the above equation can be written as

$$q_i = \frac{\tau_i - \tau_{i-1}}{\sigma + \tau_i} \tag{7}$$

In fact, Step 5 in the above procedure represents the decision rule for the following hypothesis testing:

$$H_0 : \sigma = \sigma_0 \text{ vs } H_1 : \; \sigma = \sigma_1, \; \sigma_0 > \sigma_1 \tag{8}$$

where σ_0 is the acceptable reliability level (ARL) and σ_1 is lot tolerance reliability level (LTRL). As mentioned earlier, two types of risks are always associated with the acceptance sampling plans. The rejection of a good lot is called the producer's risk, say α and the acceptance of a bad lot is called the consumer's risk, say β. We want to find the design parameter for the proposed plan such that the following two inequalities must satisfied

$$P\{\hat{\sigma} > c \,|\, \sigma = \sigma_0\} = 1 - \alpha \tag{9}$$

and

$$P\{\hat{\sigma} > c \,|\, \sigma = \sigma_1\} = \beta \tag{10}$$

where $\hat{\sigma}$ is the estimator of σ.

2.3 Parameter Estimation

Given observations $\left(n_1, \; n_2, \; \cdots, \; n_k\right)$, and $\left(r_1, \; r_2, \; \cdots, \; r_k\right)$, the likelihood function is given as

$$ln L\left(\sigma\right) \propto \sum_{i=1}^{k} n_i \ln q_i + \left(m_i - n_i\right) \ln\left(1 - q_i\right) \tag{11}$$

The maximum likelihood estimator (MLE) of σ can be obtained from the following equation:

$$\frac{\partial \ln L\left(\sigma\right)}{\partial \sigma} = \sum_{i=1}^{k} \frac{-n_i + m_i q_i}{\left(\sigma + \tau_i\right)\left(1 - q_i\right)} = 0 \tag{12}$$

Let $\hat{\sigma}$ be the MLE of σ. Then, the asymptotic distribution of $\hat{\sigma}$ is given as

$$\hat{\sigma} \sim Nor\left(\sigma, \frac{1}{I(\sigma)}\right) \tag{13}$$

where $I(\sigma)$ is the Fisher information which is given as

$$I(\sigma) = -E\left[\frac{\partial^2 \ln L(\sigma)}{\partial \sigma^2}\right] = -E\left[\sum_{i=1}^{k} \frac{n_i - 2m_i q_i + m_i q_i^2}{(1 - q_i)^2 (\sigma + \tau_i)^2}\right] \tag{14}$$

Since $E[n_i] = q_i E[m_i]$ and

$$E\left(m_i\right) = \frac{n\sigma}{\sigma + \tau_{i-1}} \prod_{j=1}^{i-1} \left(1 - p_j\right)$$

$I\left(\sigma\right)$ in (14) reduces to

$$I\left(\sigma\right) = -E\left(\frac{\partial^2 \ln(\sigma)}{\partial \sigma^2}\right) = n\sigma \sum_{i=1}^{k} \frac{\prod_{j=1}^{i-1}\left(1 - p_j\right)}{\left(\sigma + \tau_i\right)^2} \frac{\tau_i - \tau_{i-1}}{\left(\sigma + \tau_{i-1}\right)^2} \tag{15}$$

If we suppose that the inspection interval have the same length and that the percentage of removal from each interval are the same such that $\tau_i = i\tau$ and $p_i = p$, then the Eq. (15) can be written as

$$I\left(\sigma\right) = n\tau\sigma \sum_{i=1}^{k} \frac{\left(1 - p\right)^{i-1}}{\left(\sigma + i\tau\right)^2} \frac{1}{\left(\sigma + \left(i - 1\right)\tau\right)^2} \tag{16}$$

3. DESIGN OF SAMPLING PLAN USING COST FUNCTION

Let us define

$$V = 1 / I(\sigma) \tag{17}$$

According to Huang and Wu (2008), the sample size n and the acceptance number c of the proposed sampling plan satisfying the producer's risk α and the consumer's risk β are given by

$$n = \left(\frac{z_\beta \sqrt{V_1} - z_{1-\alpha}\sqrt{V_0}}{\sigma_0 - \sigma_1} \right)^2, \tag{18}$$

and

$$c = \frac{z_{1-\alpha}\sqrt{V_0}\sigma_1 - z_\beta \sqrt{V_1}\sigma_0}{z_{1-\alpha}\sqrt{V_0} - z_\beta\sqrt{V_1}}, \tag{19}$$

where z_γ is the γ percentile of a standard normal distribution, V_0 is the value of V at ARL and V_1 is the value of V at LTRL.

In this paper, we consider the same cost function as proposed by Huang and Wu (2008) in order to determine the sample size and the acceptance number. Let C_a be the cost of installing all test items in the beginning of a life test (setup cost) and C_S be the cost of testing each item. Also let C_i be the cost of one inspection and C_0 be the operation cost per unit time. Then, the total cost required for the proposed sampling plan based on the progressive censoring scheme will be

$$TC(n,\ k,\ \tau) = C_a + nC_s + kC_i + k\tau C_0 \tag{20}$$

To obtain the design parameters (*n*, *c*, *k*, τ) of the proposed sampling plan we solve the following optimization problem:

$$\text{Minimize } TC(n,\ k,\ \tau) = C_a + nC_s + kC_i + k\tau C_0 \quad \text{(21a)}$$

$$\text{Subject to } n = \left(\frac{z_\beta \sqrt{V_1} - z_{1-\alpha}\sqrt{V_0}}{\sigma_0 - \sigma_1} \right)^2, \quad \text{(21b)}$$

Note that the acceptance number c is not involved in the above optimization problem. So the parameters n, k and τ will be obtained from the above problem and c will be derived from Eq. (19).

We find the design parameters as well as the total cost required for given values of p and σ_0 in Table 1~Table 4. Here we assume that $\sigma_1 = \xi\sigma_0$. As in Huang and Wu (2008), it is assumed that $C_a = 10C_s$, $C_i = 0.5C_s$ and $C_o = 0.1C_s$ and that $C_s = 1$.

From these tables, it is clear that as the value of ξ is increased for the same values of p and σ_0, the design parameters such as n, k and total cost are increased. Larger value of ξ indicates more strict reli

Table 1. Optimal acceptance number *c* and disposition of life test $(n,\ k,\ \tau)$ for $\alpha = 0.05$ and $\beta = 0.05$.

p	σ_0	ξ	c	n	k	τ	cost
0.05	10	0.2	3.4384	23	3	3.7927	35.6378
		0.4	5.7215	50	5	3.2758	64.1379
		0.6	7.4996	143	6	3.5693	158.1416
		0.8	8.8887	709	10	3.6369	727.6369
	100	0.2	33.0558	25	2	29.6761	41.9352
		0.4	56.6264	54	3	28.4736	74.0421
		0.6	74.8355	148	4	32.1946	172.8778
		0.8	88.8718	715	7	34.0912	752.3638
	1000	0.2	305.9638	39	1	214.6937	70.9694
		0.4	550.7234	72	2	165.1395	116.0279
		0.6	743.3508	176	2	269.0000	240.8000
		0.8	888.0912	760	4	282.7429	885.0971
0.10	10	0.2	3.4404	23	3	3.9401	35.6820
		0.4	5.7273	52	4	3.9062	65.5625
		0.6	7.5003	147	5	4.2044	161.6022
		0.8	8.8888	730	8	4.4530	747.5624
	100	0.2	33.0461	26	2	30.1397	43.0279
		0.4	56.6072	55	3	29.5066	75.3520
		0.6	74.8591	152	3	40.5499	175.6650
		0.8	88.8743	738	5	43.6636	772.3318
	1000	0.2	305.9638	39	1	214.6937	70.9694
		0.4	553.4252	75	1	313.5306	116.8531
		0.6	743.2990	177	2	273.2596	242.6519
		0.8	888.1331	779	3	367.6669	900.8001
0.25	10	0.2	3.4838	24	2	5.1840	36.0368
		0.4	5.7362	55	3	4.9470	67.9841
		0.6	7.5014	155	4	5.3214	169.1286
		0.8	8.8889	774	5	5.9360	789.4680
	100	0.2	33.0214	26	2	31.6344	43.3269
		0.4	56.7697	58	2	41.4984	77.2997
		0.6	74.8997	161	2	54.2004	182.8401
		0.8	88.8751	777	4	56.0377	811.4151
	1000	0.2	305.9638	39	1	214.6937	70.9694
		0.4	553.4252	75	1	313.5306	116.8531
		0.6	744.5997	189	1	486.5533	248.1553
		0.8	888.2246	817	2	513.9998	930.8000

Table 2. Optimal acceptance number c and disposition of life test (n, k, τ) for $\alpha = 0.05$ and $\beta = 0.1$.

p	σ_0	ξ	c	n	k	τ	cost
0.05	10	0.2	3.1761	21	3	3.9233	33.6770
		0.4	5.4407	44	4	3.7628	57.5051
		0.6	7.2745	120	6	3.6211	135.1726
		0.8	8.7678	576	10	3.6599	594.6599
	100	0.2	30.5573	24	2	29.7266	40.9453
		0.4	53.7769	48	3	28.0829	67.9249
		0.6	72.5772	125	4	31.5148	149.6059
		0.8	87.6629	585	6	35.9682	619.5809
	1000	0.2	284.7215	37	1	211.9999	68.7000
		0.4	523.7220	66	2	158.9999	108.8000
		0.6	720.8569	152	2	255.0002	214.0000
		0.8	875.9995	631	3	335.7142	743.2143
0.10	10	0.2	3.1781	21	3	4.0777	33.7233
		0.4	5.4410	45	4	4.0090	58.6036
		0.6	7.2755	124	5	4.2773	138.6386
		0.8	8.7679	594	8	4.4898	611.5918
	100	0.2	30.5492	24	2	30.1910	41.0382
		0.4	53.9332	50	2	39.2473	68.8495
		0.6	72.6026	129	3	40.0223	152.5067
		0.8	87.6639	601	5	43.1636	635.0818
	1000	0.2	284.7215	37	1	211.9999	68.7000
		0.4	526.0607	68	1	303.7287	108.8729
		0.6	720.8091	153	2	259.0001	215.8000
		0.8	875.9797	640	3	348.5631	756.0689
0.25	10	0.2	3.2184	23	2	5.3940	35.0788
		0.4	5.4507	48	3	5.1022	61.0307
		0.6	7.2770	130	4	5.4295	144.1718
		0.8	8.7681	630	5	5.9993	645.4997
	100	0.2	31.3544	26	1	52.2964	41.7296
		0.4	53.9173	51	2	41.3261	70.2652
		0.6	72.6475	136	2	54.0773	157.8155
		0.8	87.6693	637	3	59.7012	666.4104
	1000	0.2	284.7215	37	1	211.9999	68.7000
		0.4	526.0607	68	1	303.7287	108.8729
		0.6	722.0335	162	1	467.6105	219.2610
		0.8	876.0719	670	2	493.0002	779.6000

Table 3. Optimal acceptance number c and disposition of life test (n, k, τ) for $\alpha = 0.1$ and $\beta = 0.05$.

p	σ_0	ξ	c	n	k	τ	cost
0.05	10	0.2	3.7176	15	3	3.4046	27.5214
		0.4	6.0364	36	4	3.3969	49.3587
		0.6	7.7371	106	6	3.3988	121.0393
		0.8	9.0129	546	9	3.6386	563.7747
	100	0.2	35.3198	18	2	24.8153	33.9631
		0.4	59.5100	39	3	24.7401	57.9220
		0.6	77.1632	111	4	29.2464	134.6986
		0.8	90.1109	553	6	34.8597	586.9158
	1000	0.2	323.5116	29	1	173.1642	56.8164
		0.4	576.9250	55	2	137.7145	93.5429
		0.6	765.7411	136	2	234.3090	193.8618
		0.8	900.4147	598	3	323.8596	706.6579
0.10	10	0.2	3.7185	16	3	3.5318	28.5595
		0.4	6.0435	38	3	4.1232	50.7369
		0.6	7.7375	109	5	4.0160	123.5080
		0.8	9.0129	563	7	4.4484	579.6138
	100	0.2	35.3030	18	2	25.1894	34.0379
		0.4	59.6639	41	2	34.7155	58.9431
		0.6	77.1848	114	3	37.2190	136.6657
		0.8	90.1114	569	5	41.8568	602.4284
	1000	0.2	323.5116	29	1	173.1642	56.8164
		0.4	579.3679	57	1	263.9999	93.9000
		0.6	765.6887	137	2	237.9998	195.6000
		0.8	900.3934	607	3	336.2160	719.3648
0.25	10	0.2	3.7661	17	2	4.7029	28.9406
		0.4	6.0435	39	3	4.6038	51.8811
		0.6	7.7382	115	4	5.0995	129.0398
		0.8	9.0129	596	5	5.8348	611.4174
	100	0.2	36.2779	20	1	44.1552	34.9155
		0.4	59.6360	42	2	36.5255	60.3051
		0.6	77.2250	120	2	50.4399	141.0880
		0.8	90.1157	603	3	57.9713	631.8914
	1000	0.2	323.5116	29	1	173.1642	56.8164
		0.4	579.3679	57	1	263.9999	93.9000
		0.6	766.9385	144	1	431.0000	197.6000
		0.8	900.4815	635	2	476.3695	741.2739

Table 4. Optimal acceptance number c and disposition of life test (n, k, τ) for $\alpha = 0.1$ and $\beta = 0.1$.

p	σ_0	ξ	c	n	k	τ	cost
0.05	10	0.2	3.4146	14	3	3.5139	26.5542
		0.4	5.7185	31	4	3.4698	44.3879
		0.6	7.4985	88	5	3.7180	102.3590
		0.8	8.8886	431	9	3.6551	448.7896
	100	0.2	32.4474	16	2	24.6939	31.9388
		0.4	56.4934	35	2	33.6457	52.7291
		0.6	74.7492	91	4	28.2447	114.2979
		0.8	88.8647	437	6	33.9325	470.3595
	1000	0.2	299.1226	27	1	170.0000	54.5000
		0.4	548.7136	51	1	252.7058	86.7706
		0.6	741.5568	115	2	218.9025	169.7805
		0.8	887.8703	479	3	302.3580	581.2074
0.10	10	0.2	3.4585	15	2	4.6419	26.9284
		0.4	5.7268	32	3	4.2390	44.7717
		0.6	7.4982	89	5	4.0698	103.5349
		0.8	8.8886	444	7	4.4802	460.6361
	100	0.2	32.4329	17	2	25.0653	33.0131
		0.4	56.4845	35	2	34.2003	52.8401
		0.6	74.7731	94	3	36.2773	116.3832
		0.8	88.8654	449	5	40.9857	481.9929
	1000	0.2	299.1226	27	1	170.0000	54.5000
		0.4	548.7136	51	1	252.7058	86.7706
		0.6	741.5042	116	2	222.2136	171.4427
		0.8	887.9672	495	2	426.9999	591.4000
0.25	10	0.2	3.4618	15	2	4.8953	26.9791
		0.4	5.7274	33	3	4.7361	45.9208
		0.6	7.5010	95	3	5.5153	108.1546
		0.8	8.8887	470	5	5.8908	485.4454
	100	0.2	33.3131	18	1	44.5089	32.9509
		0.4	56.4580	36	2	35.9718	54.1944
		0.6	74.8192	99	2	49.8122	119.9624
		0.8	88.8712	476	3	57.5479	504.7644
	1000	0.2	299.1226	27	1	170.0000	54.5000
		0.4	548.7136	51	1	252.7058	86.7706
		0.6	742.6893	121	1	407.9999	172.3000
		0.8	887.9371	506	2	450.0002	607.0000

ability requirement by consumers, so it is agreed with intuition. The design parameters do not vary much according to different values of p when the other conditions remain unchanged. When the value of σ_0 increases, the sample size increases but the number of inspections decreases. When the consumer's risk or the producer's risk increases, the total cost tends to decrease.

Further, we compare the design parameters and cost for fixed value of $\alpha = 0.05$ and two values of $\beta = 0.05$ and $\beta = 0.01$ for $p = 0.1$ and $\sigma_0 = 100$ from Table 1 and Table 2. We presented these design parameters along with the cost in Table 5. From Table 5 we can see that for other fixed values as β increases from 0.05 to 0.1, the values of c, n and cost decreases.

We noted the same trend in design parameters and cost when α decreasing from 0.05 to 0.01 (Table 3~ Table 4) and same values of β.

Example 1: Suppose that the quality engineer wants to use the proposed reliability sampling plan for a particular lot of products. The lifetime of the products follows a Pareto distribution of second kind with shape parameter $\lambda = 1$ and scale parameter σ. Because σ is the median life, large value of σ is desirable to the engineer (or producer) as well as the consumer. The producer's risk is specified by 0.05 if the true median life is as large as $\sigma_0 = 100$ and the consumer's risk is specified by 0.05 if the true median life is as low as $\sigma_1 = 60$. Consider a progressive type-I interval censoring with removal probability $p = 0.1$ and cost parameters $C_a =$ \$10, $C_s =$ \$1, $C_i =$ \$0.5, $C_o =$ \$0.1. From Table 1, the parameters are obtained by: $n = 152$, $k = 3$, $\tau = 40.5499$ and $c = 74.8591$ Therefore, the engineer needs to draw a random sample with size $n = 152$ from the lot and put them on a 3-stage progressive type-I interval censored life test with constant inspection length $t = 40.5499$. Suppose now that we obtain the failure data as $n_1 = 46$, $n_2 = 17$, $n_3 = 14$. Then, the number of items removed at each inspection is determined by: $r_1 = 0.1(152-46) \approx 1$, $r_2 = 0.1(95-17) \approx 8$ and $r_3 = 70-14 = 56$. The MLE $\hat{\sigma} = 100.5548$ is obtained from (12). As it is greater than the acceptance number $c = 74.8591$, therefore the engineer should accept the lot. The total cost required for this test is 175.665.

Table 5. Comparison of Plan Parameters and Cost when $\alpha = 0.05$, $p = 0.1$ and $\sigma_0 = 100$.

	$\beta = 0.05$					$\beta = 0.1$				
ξ	c	n	k	τ	cost	c	n	k	τ	cost
0.2	33.0461	26	2	30.1397	43.0279	30.5492	24	2	30.1910	41.0382
0.4	56.6072	55	3	29.5066	75.3520	53.9332	50	2	39.2473	68.8495
0.6	74.8591	152	3	40.5499	175.6650	72.6026	129	3	40.0223	152.5067
0.8	88.8743	738	5	43.6636	772.3318	87.6639	601	5	43.1636	635.0818

4. CONCLUDING REMARKS

In the paper, the Pareto distribution of the second kind is considered as a life distribution when designing the acceptance sampling plan based on the progressively type-1 interval censoring scheme. As the Pareto distribution of second kind is skewed distribution, we use the median life as reliability measure. The decision upon the acceptance of lots is based on the MLE of the median life. The cost model is considered and the cost minimization is formulated so as to determine the design parameters such as the sample size, number of inspections and the inspection interval.

As stated in Huang and Wu (2008), one should be cautious in using this sampling plan when the sample size is small because design parameters are derived from the asymptotic distribution. The extension of the present study to some other distributions such as the gamma or the generalized Rayleigh distribution may be possible area for future research.

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