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Waste Disposal Models for Manufacturing Firm and Disposal Firm

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Abstract. This research considers a system containing a manufacturing firm who generates waste material during manufacturing process, and a disposal firm who collects and disposes the waste material. Identification of the optimal number of pick ups and the amount of waste to be disposed at certain period of time in terms of cost minimization is studied. Two types of waste accumulation rates, constant and linearly increasing, are discussed and mathematical models are developed. It can be shown that the results for these two different types of waste accumulation differ in a wide range because of the difference in the way of how waste is accumulated, which disturbs the storage cost. An integrated model is also developed and discussed in which both the manufacturing firm and the disposal firm benefit from the coordination between the two parties. It is shown that the optimal policy adopted by the integrated approach can provide a strong and consistent cost-minimizing effect for both the manufacturing firm and the disposal firm over the existing approach. Finally, all the models are verified by a numerical example and the results are compared.

Keywords: Waste Disposal, Inventory Control, Integrated Models

1. INTRODUCTION

Waste is an unwanted or undesired material or substance. Waste management is the human control of the collection, treatment and disposal of different wastes. This is in order to reduce the negative impacts waste has on environment and society. Waste may accumulate during the production process. For instance, the manufacturing processes in plastic injection molding, printed circuit board parts, offshore hydrocarbon production and other industries, the amount of waste produced may vary with time or remain constant. Waste disposal becomes necessary because of the costs of storing waste including costs of the storage area and necessary storage equipment. The manufacturing firm may enlist a disposal company to pick up the waste.

The costs of containing harmful emissions of the stored waste as well as those for integrating waste pick-ups into the operations planning of production companies are to be considered when determining the timing of pick-ups of waste by the disposal firm from the manufacturing firm. Two heuristic materials management strategies are common: pick-ups according to need and pick-ups according to a set time schedule. To cope with this issue, solutions for the number of pick-ups, the optimal quantity per pick-up and the related total cost are concerned. Thus, the waste-related problem of the manufacturing firm and the disposal firm comes forward.

The objective of this research is to find the optimal number of waste pick-ups for both the waste producing firm and the waste disposal firm. Here, in this research the inventory control of the waste products that are generated during the manufacturing process is concerned. From the point of view of the waste producing firm the main goal is to set-up the optimum number of pick-ups and the amount of waste for each pick-up so that the total cost is minimized. This research determines the optimal number of waste pick-ups with the help of a lot-sizing model, whereas the model presented here will use lot sizes that directly depend on the amount of waste to be disposed of.

The way waste is accumulated at the manufacturing firm is also of concern. It is common that waste accumu-

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lates at a constant rate as production proceeds. In some cases, the waste accumulation rate is found to be linearly increasing. The amount of waste produced increases linearly can be the result of the long time use of machines during production. In such case, machines should be shut-down to bring back to the normal state at certain point and then production should be restarted from the initial stage. Cost models with constant and linearly increasing waste accumulate rates will be developed and how waste accumulate rates affects decision making will be discussed.

The other aim of this research is to develop an integrated lot-splitting model. Focusing on the integrated total relevant costs for the manufacturing firm and the disposal firm, it will be investigated how the optimal policy adopted through a spirit of co-operation can be of economic benefit to both firms. Here the discussion is restricted and analyzed to a relatively simple manufacturing firm and the disposal firm scenario under deterministic conditions.

For examples of recent and earlier expansions on the classic lot-sizing model, see (Haase, 1994; Anwar and Nagi, 1997; Hofmann, 1998). Although there are a number of models of waste management systems (Hillier and Lieberman, 2002; Ljunggren, 2000), the analysis of the different components that go into these systems is affected by a lack of understanding of appropriate modeling tools and techniques. The economic production quantity (EPQ) model is often used in manufacturing sector to assist firms or factories in determining the optimal production lot-size that minimizes overall production-inventory costs (Hillier and Lieberman, 2002; Nahmias, 2001).

Seeking an optimal solution from only the manufacturer's point of view would be neither effective nor feasible in the long run. Given a long-term contract, implementing the integrated approach in favor of the manufacturer only means a cost shifting from the disposal firm to the manufacturing firm. Integration the disposal firm into the manufacturing firm effective integration approach requires that the disposal firm compensate the manufacturer's long-term commitment for him. Ramasesh (1990) referred to this as a buyer's fixed investment which must be determined by the policy-level decision at the contract stage. Hoque and Goyal (2000) proposed an optimal policy for a single-vendor single-buyer integrated production-inventory system with a limited capacity of transport equipment. This study shows that cooperation between both parties at the outset of the long-term contract can provide them a better opportunity of increasing their mutual benefit.

The rest of the paper is organized as follows. The next section presents a basic model for the manufacturing firm, in which two different types of waste accumulation rates (constant and linearly increasing waste accumulation) are discussed. Section 3 discuses the integrated model of the manufacturing firm and the disposal firm for both the constant and linearly increasing waste accumulation rates. The paper concludes in Section 4.

2. BASIC MODEL

The basic model contains a manufacturing firm and a disposal firm. The manufacturing firm generates waste during its stable and continuous manufacturing process. The waste accumulates in the storage facility at the manufacturing firm and is picked up and shipped away by the disposal firm periodically. The following assumptions are made.

- For a given period of time T, there are n pickups at regular interval of time.
- There is no self-decomposition.
- All kinds of wastes are considered.
- No capacity limit on the storage facility.

The objective is to find the optimal number of pick-ups in a given time period T such that total costs of waste storage and waste pick-ups are minimized. Two cases with different types of waste accumulation rate are presented as follows:

2.1 Case: 1 (Constant *Waste* Accumulation Rate)

In this case, it is considered that the waste accumulation rate is constant, which is derived in waste disposal and waste avoidance (Wiese and Zelewski, 2002). The optimal number of pick-ups and the optimum quantity for each pick-up which minimizes the total cost are derived.

In this case, for a given period of time $[0, T]$, it is assumed that an amount of waste Q is accumulated. The waste accumulation rate, α, is constant, where $\alpha = Q/T$, with an equal time intervals of waste collection, T/n. The waste amount function, $x(t, n)$, indicates the amount of stored waste that has accumulated up to time t when a total of n number of pick-ups. Under these premises, the waste amount function is given by

$$x(t,n) = \alpha t - (i-1)\frac{Q}{n} \tag{1}$$

where $i \in N$.

Figure 1 illustrates the waste amount curve over time.

In this model, for the total amount of waste, Q, the waste disposal costs, C_D, is divided into waste pick-up costs, C_P, and storage costs, C_S. The waste pick-up costs and the waste storage costs are derived as follows.

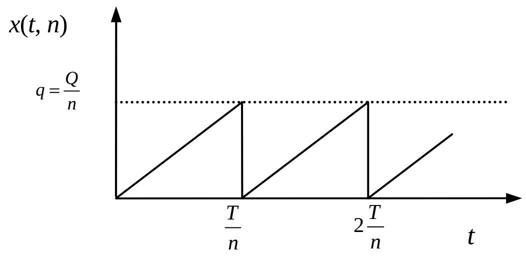

Figure 1. Waste amount curve over time (constant waste accumulation rate).

The waste pick-up costs include a fixed cost for each pick-up, c_n, and a constant cost per unit of picked-up waste (waste unit costs), c_Q. By this, the pick-up cost. C_P, is then given by, $C_P(n) = c_n n + c_Q Q$.

Assuming a constant waste storage costs rate, c_S, and n pick-ups spread out evenly over T, the waste storage cost, C_S, can be written as

$$C_S(n) = n\int_0^{\frac{T}{n}} c_S \alpha t dt = \frac{1}{2} c_S \alpha \frac{T^2}{n} \tag{2}$$

Note that any discount factors are omitted. If the planning horizon is relatively short, this omission will not be very serious.

Summing up, waste disposal costs, C_D, is given by,

$$C_D(n) = c_n n + c_Q Q + \frac{1}{2} c_S \alpha \frac{T^2}{n} \tag{3}$$

The optimal number of pick-up then can be written as

$$n^* = \sqrt{\frac{c_S QT}{2c_n}} \tag{4}$$

And the optimal amount of waste per pick-up can easily be confirmed to be

$$q^{opt} = \frac{Q}{n^*} = \sqrt{\frac{2Qc_n}{Tc_S}} \tag{5}$$

Substituting n^* into (3) yields,

$$C_D(n^*) = \sqrt{2c_n c_S TQ} + c_Q Q \tag{6}$$

2.2 Case: 2 (Linearly Increasing Waste Accumulation Rate)

Now it is considered that the waste accumulation rate is no more a constant. It increases linearly with time. Two types of linearly increasing waste accumulation rate are discussed below.

2.2.1 $\alpha = bt$

It is assumed that the waste accumulation rate increases linearly in time, that is, $\alpha = bt$. Under these premises, the waste accumulation rate over time is shown in Figure 2.

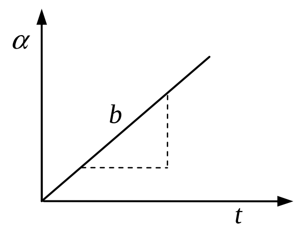

Figure 2. Waste accumulation rate over time ($\alpha = bt$).

The waste amount function $x(t)$ indicates the amount of stored waste that has accumulated up to time t. The waste amount function at time t_1 is then

$$x(t) = \frac{bt^2}{2} - (i-1)q \tag{7}$$

where $i \in N$.

Figure 3 shows the number of pick-ups and amount of waste per pick-up.

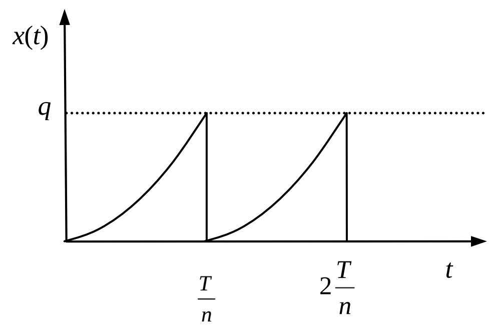

Figure 3. Number of pick-ups and amount of waste per pick-up.

The waste amount function, $x(t)$, is given by

$$x(t) = \frac{bt^2}{2} \tag{8}$$

When $t = T/n$, which is the time when there is a pick-up between the given period with n pick-ups, the quantity of waste accumulated at time t is obtained by substituting $t = T/n$ in the waste amount function equation.

$$q = \frac{bT^2}{2n^2} \tag{9}$$

The total quantity of waste accumulated at n pick-ups is thus,

$$Q = nq = \frac{bT^2}{2n} \tag{10}$$

The average inventory level for time period T, with

n pick-ups is then

$$\text{Total inventory at } T \quad = n\int_0^{T/n}\left(\frac{bt^2}{2}\right)dt \qquad (11)$$

Now, the manufacturing firm needs to consider about the costs that are acquired by these waste products. Thus, the manufacturing firm needs to dispose the accumulated wastes. For the quantity of waste accumulated, the manufacturing firm's total cost for disposing the waste accumulated must be minimized to run the firm profitable.

In this model the cost factors are set to be the same as that of the constant waste accumulation rate. Thus, the waste disposal costs, C_D, are divided into waste pick-up costs, C_P, and storage costs, C_S.

The waste pick-up costs are assumed to be a fixed cost for a pick-up fee which is $c_n > 0$, and a constant cost per unit of picked-up $c_Q > 0$. By this, the pick-up costs, C_P, is given by,

$$C_p(n) = c_n n + c_Q \frac{bT^2}{2n} \qquad (12)$$

The waste storage cost, C_S, is also the same as previous model. Assuming a constant waste storage costs rate, c_S, and n pick-ups spread out evenly over T;

$$C_S(n) = c_S \frac{bT^3}{6n^2} \qquad (13)$$

Summing up, waste disposal costs, C_D, are given by,

$$C_D(n) = c_n n + c_Q \frac{bT^2}{2n} + \frac{c_S bT^3}{6n^2} \qquad (14)$$

Since the second derivative of the total cost function is positive, the total cost function is convex and there lies an optimal solution which could be found easily by any optimization technique.

2.2.2 $\alpha = a + bt$

The waste accumulation rate over time can be expressed as $\alpha = a + bt$, as depicted in Figure 4.

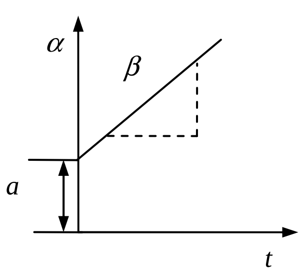

Figure 4. Waste accumulation rate over time ($\alpha = a + bt$).

The waste amount function at time t_1 is then

$$x(t_1) = \int_0^{t_1} a + bt dt \qquad (15)$$

Derivation of cost functions is similar to that of the previous case. The pick-up costs, waste storage costs and waste disposal costs are as follows.

$$C_p(n) = c_n n + c_Q\left[aT + \frac{bT^2}{2n}\right] \qquad (16)$$

$$C_S(n) = c_S\left[\frac{T^2}{2n}\left(a + \frac{bT}{3n}\right)\right] \qquad (17)$$

$$C_D(n) = c_n n + c_Q\left[aT + \frac{bT^2}{2n}\right] + c_S\frac{T^2}{2n}\left[a + \frac{bT}{3n}\right] \qquad (18)$$

Since the second derivative of the total cost function is positive, the total cost function is convex as same as in the case of above model and there lies an optimal solution which could be found easily by any optimization technique.

2.2.3 Comparison Between Waste Accumulation Rates

Consider an example assuming a time period of 1 year. Waste storage cost rate is $15 per unit, fixed cost per pick-up is $150, and cost per unit of picked-up waste is $60 per unit. For Case 1, the constant rate is set at 2,000 units per year. For the case with the rate, $\alpha = bt$, b is set at 24,164.46 per year. For the case with the rate, $\alpha = a + bt$, a is 150 units per year and b is 22,352.01 per year. The waste accumulation rates are set as above such that the three lead to the same amount of total costs when the number of pick-ups is 6. Total costs for different number of pick-ups are listed in Table 1. It is found the optimal number of pick-ups is 10 for Case 1, 70 for the case with $\alpha = bt$, and 67 for the case with $\alpha = a + bt$.

On comparing the two types of waste accumulation rates, since the total quantity of waste accumulated (Q) is constant for the constant waste accumulation rate and which varies depending on the waste accumulation rate for the linearly increasing waste accumulation rate, the results of both the case differ in a wide range. When n is increased gradually, the total cost decreases deeply for the linearly increasing waste accumulation rate than that of the constant waste accumulation rate and vise-versa. Also, the total cost attains the minimum value at smaller n for the constant waste accumulation rate and which is precisely higher for the linearly increasing waste accumulation rate. This is because of the exponentially increasing accumulation of waste which captures a large storage cost when the waste is stored for a long time and the cost per unit of waste.

Also on comparing the two different models of linearly increasing waste accumulation rates, waste accumulates faster in the case with $\alpha = bt$ since the value of b is larger. As a result, the preferred number of pickups is larger. It leads to more frequent pick-ups, thus reducing

Table 1. Total costs for different number of pick-ups (basic models).

Number of pick-ups	Case 1	Case 2	
	constant rate	$\alpha = bt$	$\alpha = a + bt$
1	135,150.00	785,494.95	736,715.51
2	127,800.00	377,869.68	359,112.75
3	125,450.00	248,806.95	239,554.05
4	124,350.00	185,609.14	181,013.87
5	123,750.00	148,153.21	146,322.29
6	**123,400.00**	**123,400.39**	**123,400.00**
7	123,192.86	105,844.85	10,7145.48
8	123,075.00	92,760.65	95,033.81
9	123,016.67	82,644.02	85,671.59
10	**123,000.00**	74,597.49	78,227.35
11	123,013.00	68,052.34	72,174.13
12	123,050.00	62,630.67	67,161.84
⋮	⋮	⋮	⋮
64	129,834.38	20,941.84	29,108.72
65	129,980.77	20,917.13	29,096.85
66	130,127.27	20,897.31	29,089.88
67	130,273.88	20,883.37	**29,087.60**
68	130,420.59	20,873.85	29,089.81
69	130,576.39	20,868.98	29,096.31
70	130,714.29	**20,868.53**	29,106.91
71	130,861.27	20,872.32	29,121.44

amount of waste accumulation. In addition, it is found that the cost for the case with $\alpha = bt$ is increasing more higher than that of the case with $\alpha = a + bt$ above the equal total cost (where the number of pick-ups is 6) and decreasing more lower than that of the case with $\alpha = a + bt$ below the equal total cost. However, with the same value of b in both linearly increasing rates and a positive value of a in the case with $\alpha = a + bt$, waste accumulates relatively slower, in the case with $\alpha = bt$, and a smaller number of pick-ups is preferred.

3. INTEGRATED MODEL

This section introduces an integrated model in which the manufacturing firm and the disposal firm are integrated in order to lower the total cost of the whole system. In this model, the disposal firm's total cost consists of the cost for each unit of waste to be disposed of and the pick-up cost which are incurred as a result of multiple deliveries. The revenue of the disposal firm comes from the manufacturing firm. Hence the cost per pick-up and the cost per unit of waste to be disposed of for the manufacturing firm should be always greater than or equal to that of the disposal firm. The manuacturing firm's total cost function includes holding cost, cost for each unit of waste to be transferred to the disposal firm and the pick-up cost for each pick-up.

Denote c_n^d as the fixed costs for a pick-up for disposal firm and c_Q^d as the costs per unit of waste to be disposed of for disposal firm. It is assumed that $c_n \geq c_n^d$ and $c_Q \geq c_Q^d$. The assumptions are to ensure that it is possible for the disposal firm to gain profits.

3.1 Case: 1 (Constant Waste Accumulation Rate)

The total cost of the disposal firm can be expressed as (19).

$$TC(n)_D = -\left[(c_n - c_n^d)n + (c_Q - c_Q^d)Q\right] \tag{19}$$

Since the total cost function for the disposal firm is linearly increasing, the upper limit of the number of pick-ups will be the largest number of pick-ups that can be accepted by the manufacturing firm.

The total cost of the manufacturing firm can be expressed as (20).

$$TC(n)_M = c_n n + c_Q Q + \frac{Qc_S}{2n}T \tag{20}$$

Solving for n, the optimal number of pick-ups for the manufacturing firm is obtained as follows.

$$n_M^* = \sqrt{\frac{Qc_S T}{2c_n}} \tag{21}$$

Finally, the integrated total cost can be written as (22).

$$TC(n)_I = c_n^d n + c_Q^d Q + \frac{Qc_S}{2n}T \tag{22}$$

Solving for n, the optimal number of pick-ups for the manufacturing firm is obtained as follows.

$$n_I^* = \sqrt{\frac{Qc_S T}{2c_n^d}} \tag{23}$$

3.2 Case: 2 (linearly Increasing Waste Accumulation Rate, $\alpha = a + bt$)

With a linearly increasing waste accumulation rate, $\alpha = a + bt$, the total cost of the disposal firm can be written as (24).

$$TC(n)_D = -\left[(c_n - c_n^d)n + (c_Q - c_Q^d)\left[aT + \frac{bT^2}{2n}\right]\right] \tag{24}$$

It can be shown that (24) is concave. Thus, the lower limit of the optimal number of pick-ups for the disposal firm is 1 and the upper limit of the number of pick-ups will be the largest number of pick-ups that can be accepted by the manufacturing firm.

The total cost of the manufacturing firm can be expressed as (25).

$$TC(n)_M = c_n n + c_Q \left[aT + \frac{bT^2}{2n} \right] + c_S \frac{T^2}{2n} \left[a + \frac{bT}{3n} \right] \quad (25)$$

Equation (25) can be shown to be convex. The optimal number of pick-ups for the manufacturing firm can easily be found by a simple search method.

Finally, the integrated total cost can be written as (26).

$$TC(n)_I = c_n^d n + c_Q^d \left[aT + \frac{bT^2}{2n} \right] + c_S \frac{T^2}{2n} \left[a + \frac{bT}{3n} \right] \quad (26)$$

Again, it can be shown that (26) is convex. Thus, the optimal number of pick-ups for the integrated model can easily be found by a simple search method.

3.3 Numerical Examples

Numerical examples with a time period of 1 year are designed. For the manufacturing firm, waste storage cost rate is \$15 per unit, fixed cost per pick-up is \$150, and cost per unit of picked-up waste is \$60 per unit. For the disposal firm, a fixed costs of \$90 occurs for a pick-up, and costs waste to be disposed off are \$20 per unit. For Case 1, the constant rate is again set at 2,000 units per year. Table 2 shows the total costs of the disposal firm, the manufacturing firm, and integrated total costs.

It can be seen how an increase in the number of pick-ups can affect both parties' total costs. For the disposal firm, larger numbers of pick-ups are preferred. As the number of shipments, n, increases, the manufacturing firm's total cost sharply decreases. Its total cost is the lowest with 10 pick-ups, and then increases due to a high transportation cost incurred by the large number of shipments.

Table 2. Total costs for different number of pick-ups (Case 1 of integrated model).

Number of pick-ups	Disposal firm's cost	Manufacturing firm's cost	Integrated cost
1	-80,060.00	135,150.00	55,090.00
2	-80,120.00	127,800.00	47,680.00
3	-80,180.00	125,450.00	45,270.00
4	-80,240.00	124,350.00	44,110.00
5	-80,300.00	123,750.00	43,450.00
6	-80,360.00	123,400.00	43,040.00
7	-80,420.00	123,192.86	42,980.86
8	-80,480.00	123,075.00	42,595.00
9	-80,540.00	123,016.67	42,476.67
10	-80,600.00	**123,000.00**	42,400.00
11	-80,660.00	123,013.00	42,353.00
12	-80,720.00	123,050.00	42,330.00
13	-80,780.00	123,103.85	**42,323.85**
14	-80,840.00	123,171.43	42,331.34
15	-80,900.00	123,250.00	42,350.00

Table 3. Total costs for different number of pick-ups (Case 2 with $\alpha = a + bt$ of integrated model).

Number of pick-ups	Disposal firm's cost	Manufacturing firm's cost	Integrated cost
1	-453,100.32	736,715.51	283,615.19
2	-229,640.16	359,112.75	129,472.59
3	-155,193.44	239,554.05	84,360.31
4	-118,000.08	181,013.75	63,013.79
5	-95,708.06	146,322.29	50,614.23
6	-80,866.72	123,400.00	42,533.28
⋮	⋮	⋮	⋮
49	-18,063.27	30,081.13	12,017.86
50	-17,940.81	29,956.05	**12,015.24**
51	-17,825.50	29,841.78	12,016.28
52	-17,716.93	29,737.69	12,020.76
53	-17,614.72	29,643.19	12,028.47
54	-17,518.52	29,557.78	12,039.26
55	-17,428.01	29,480.93	12,052.92
56	-17,342.86	29,412.19	12,069.33
57	-17,262.81	29,351.16	12,088.35
58	-17,187.59	29,297.38	12,109.79
59	-17,116.95	29,250.56	12,133.61
60	-17,050.67	29,210.28	12,159.61
61	-16,988.52	29,176.25	12,187.72
62	-16,930.32	29,148.17	12,217.84
63	-16,875.88	29,125.74	12,249.86
64	-16,825.01	29,108.72	12,283.72
65	-16,777.54	29,096.85	12,319.31
66	-16,733.34	29,089.88	12,354.26
67	-16,692.24	**29,087.60**	12,395.36
68	-16,654.12	29,089.81	12,435.69

From the point of view of the integrated approach, 13 shipments for each setup should be selected as an optimal decision at the both parties' total cost of \$42,323.85 which in turn consists of \$-80,780 from the disposal firm and \$123,103.85 for the manufacturing firm. On the other hand, if an integrated approach is implemented from the manufacturing firm's initiation only, the optimal decision is 10 shipments, leading to a higher integrated total cost.

For the case with the rate, $\alpha = a + bt$, set a as 150 units per year and b as 22,352.01 per year. All other parameter values are the same as the previous example. Table 3 lists the total costs of the disposal firm, the manufacturing firm, and integrated total costs.

From the point of view of the integrated approach, 50 shipments should be selected as an optimal decision at the both parties' total cost of $12,015.24 which in turn consists of $-17,940.81 from the disposal firm and $29,956.05 for the manufacturing firm. On the other hand, if an integrated approach is implemented from the manufacturing firm's initiation only, the optimal decision is 67 shipments at the total cost of $12,395.36 which indicates $344.12 additional cost to both parties in aggregate.

3.4 Comparison Between the Integrated Models

After comparing the two integrated models presented above, several remarks can be made.

(1) For the disposal firm in the case of constant waste accumulation rate, the optimum numbers of pick-ups is as large as possible and there is no lower limit. In the case of linearly increasing waste accumulation rate, there is an upper limit and a lower limit. The lower limit is one and the upper limit is same as that for the constant waste accumulation rate which is the acceptable number of pick-ups for both the firms.

(2) For the manufacturing firm, the total cost curve is convex with respect to the number of pick-ups, for both the constant and linearly increasing waste accumulation rates. Furthermore, for the case of constant waste accumulation rate, the optimal number of pick-ups for the manufacturing firm always lies above the optimal number of pick-ups for the integrated system. It can be seen by comparing (21) and (23) and that $c_n \geq c_n^d$.

(3) For the case of constant waste accumulation rate, the optimal number of pick-ups for both the manufacturing firm and the integrated system are the same if $c_n = c_n^d$. Thus, to integrate with the manufacturing firm, an obvious decision for the disposal firm is to set the cost c_n equal to c_n^d.

4. CONCLUSION AND FUTURE RESEARCH

In this study an inventory control model was proposed for solving the problem of waste inventory due to the waste accumulated in industries. The primary goal of this research is to provide managers an easy and reliable way for their decision making process. The managers can make the necessary decision by using the result obtained from the model and keep the decisions making process in good performance.

The optimal number of pick-ups and the total cost associated for a known period of time with different waste accumulation rates are considered as the primary problem and solved. It is found that the constant and linearly increasing waste accumulation rates differ in a wide range. But the total cost function for both the cases is convex. Also an integrated model of the manufacturing firm and the disposal firm is discussed. It is concluded that the integrated model gives a global optimum solution. Thus, when the manufacturing and the disposal firm are integrated the total cost is less than that of the separated low total cost and the extra savings could be used as a part of their revenue which gives a best result.

In the outlook of management, it is better to find which case of waste accumulation is existing and to find a better solution. In this research, it is found that both the cases have different expenditure. Hence in a management where the waste accumulation rate is not a constant the decision making should be done with more awareness. Also it is exposed that the decision made by the integration of the manufacturing firm and the disposal firm is better than that of the decision made by the firms separately. Thus in a consciousness, this research provide a way to know that difference in the waste accumulation rate will affect the management's profit.

The future research could be continued to the perspective of the waste disposal firm to set the cost parameters so that the disposal firm can earn maximum revenue without any matter pertaining to the manufacturing firm. In this model, the setup cost for each pick-up. This could be considered in the future research for a more complicated situation when there come about such costs. The research could also be further extended to the waste avoidance ways in different waste avoidance cost. This area of study will be interesting.

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