

# The *BV* Photometry of the RR Lyrae Star, BH Ursae Majoris: Light Curves and Period Study

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The first presented *BV* light curves of BH UMa confirmed Krajci's (2005) result that BH UMa is an RR Lyr star that belongs to the RRc subgroup. The light curves showed a slight asymmetry of  $D = 0.453$  with an amplitude of about  $0.^m58$  in *B*,  $0.^m47$  in *V*, and  $0.^m11$  in *B-V* and with a small hump between  $0.^p82$  and  $0.^p86$ . We determined nine new times of minimum light and eight times of maximum light. We also analyzed all of the available unanalyzed minimum timings and found for the first time that the period of BH UMa has varied dramatically in at least three independent sinusoidal ways superposed on a secularly downward parabola over 66 years. The secular period decreasing rate was obtained as  $6.^d684 \times 10^{-8} \text{y}^{-1}$ , corresponding to  $-0.58 \text{ s/century}$ . The semi-amplitude and period for each of the three sinusoidal variations were ( $0.^d058$ ,  $14.^y44$ ), ( $0.^d044$ ,  $9.^y98$ ), and ( $0.^d005$ ,  $0.^y97$ ), respectively. It is uncertain whether the periodicity for the shortest period of  $0.^y97$  is real or spurious. The secular period decrease, well consistent with those of the other RRc stars, could be considered as a natural result of the evolution of the BH UMa system. The two possible sinusoidal terms were interpreted as both two light-time effects due to two additional bodies orbiting BH UMa and combinations of random fluctuations in the pulsation period of BH UMa. Two interpretations were shortly discussed with related parameters.

**Keywords:** RR Lyr star, BH UMa, period change, light-time effect, random fluctuations

## 1. INTRODUCTION

RR Lyr pulsating variables belong to a stellar group with short periods ranging from  $\sim 0.^d3$  to  $1.^d0$  and pulsation amplitudes smaller than about 2 mag in the *V* band. They are low mass central He burning Pop II stars, located in the Hertzsprung-Russell diagram on the horizontal branch evolutionary phase with an absolute magnitude in the *V* band ranging from  $\sim 0$  to  $\sim 1$  mag. There are two main subgroups: RRab, which oscillates in the fundamental mode, and RRc, which pulsates in the first overtone mode. The RRc stars have shorter periods, smaller amplitudes, and more symmetric light curves than the RRab stars. Both the light curves and the pulsation periods of RRc stars vary in time. The former is known as the Blazhko effect, and the latter is known to vary in diverse

patterns such as secular, sinusoidal, or erratic changes.

BH UMa (BV 36, CSV 6796, GSC 03449-00652), which was discovered as a variable star by von Geyer in 1955 (Meinunger 1965), has been known for a long time as a W UMa eclipsing binary star with a period of  $0.^d6987$ . Götz & Wenzel (1962) assigned the spectral type of BH UMa to B9. The first photographic light curve was published by Meinunger (1965), who obtained many plate times of minimum and determined a light elements as  $Min I = \text{HJD } 2430496.455 + 0.^d698685 E$ . He classified BH UMa as a W UMa binary star from the shape of its light curve. Since Meinunger's study, numerous times of minimum lights have been published (Krejner et al. 2001). Recently, Krajci (2005) observed the star without any filters and found that it was not a W UMa binary star but an RR Lyr star belonging to the RRc subtype. He determined a light

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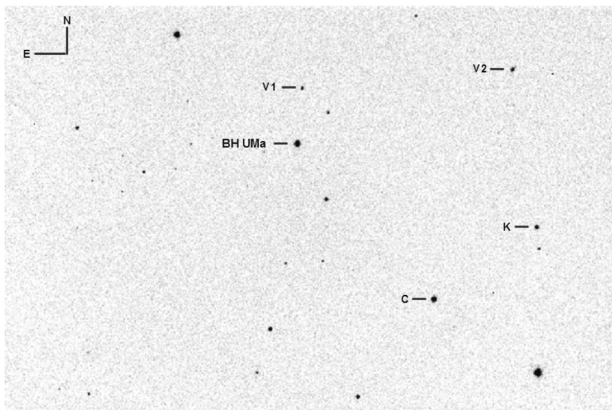
Tel: +82-43-261-3139 Fax: +82-43-274-2312

elements as  $Max I = HJD 2453053.6545 + 0.^d349350 E$ . As far as we know, no filtered light curves have been published. In this paper, we presented the *BV* light curves of BH UMa confirming the star to be the RRc subtype of RR Lyr stars, and we analyzed all published timings, including ours, to obtain a general understanding of the period behavior.

## 2. OBSERVATIONS AND LIGHT CURVES

### 2.1 Observations

The *BV* charge coupled device (CCD) observations of BH UMa were made on 17 nights during March and April 2008 with the 35-cm reflector of campus station of Chungbuk National University Observatory in Korea. An SBIG ST-8 CCD imaging system, which was electrically cooled with a  $19' \times 12'$  field of view, and a standard *BV* filter set were used. GSC 3449-707 and GSC 3449-726 were chosen as the comparison and check stars, respectively. Our



**Fig. 1.** The finding chart of BH UMa. The field of view is about  $19' \times 12'$ . "C" and "K" denote the comparison (GSC 3449-707) and check (GSC 3449-726) stars, respectively. "V1" is a new short period ( $0.^d5607$ )  $\beta$  Lyr eclipsing binary (GSC 3449-680) discovered during our observations. "V2" was recently discovered as a W UMa binary (GSC 3449-688) by Nelson et al. (2004).

**Table 1.** The characteristics of BH UMa, comparison and check stars.

Star	GSC 03449-	Position (J2000)		B		V	
		RA	DEC	Max	Min	Max	Min
BH UMa <sup>a</sup>	0652	$10^h45^m55.^s82$	$52^{\circ}14'50.^{\circ}8$	$11.^m28$	$11.^m36$	$11.^m03$	$11.^m50$
Comp <sup>b</sup>	0707	10 45 30.82	52 10 14.2	12.271		11.590	
Check <sup>b</sup>	0726	10 45 11.01	52 12 13.4	13.650		13.103	

<sup>a</sup>Standard magnitude estimated in this paper.

<sup>b</sup>Standard magnitude given by Nelson et al. (2004).

comparison and check stars were the same ones used by Nelson et al. (2004). The characteristics of the variable, comparison and check stars are listed in Table 1. Fig. 1 shows one of our CCD images in which our program stars are marked with two variable stars discovered by us in this paper (V1) and recently by Nelson et al. (2004) (V2). The camera exposure time ranged between 50 s and 140 s, depending on the quality of the night and the filter response.

The instrumentation and the reduction method used for the raw CCD frames have been described in detail by Kim et al. (2006). The resulting standard errors of our observations in terms of comparison minus check star were about  $\pm 0.^m02$  in blue and  $\pm 0.^m01$  in yellow. A total of 1,736 individual observations were obtained in two colors (866 in blue and 870 in yellow) and are available at the website<sup>1</sup>. From our *BV* observations, nine times of minimum light and eight times of maximum light were obtained by the conventional Kwee & van Woerden (1956) method. Each of these timings, as listed in Table 2, was a weighted-mean of two *BV* timings defining the same epoch.

### 2.2 New Light Curves

From our observations, the new *BV* light curves of BH UMa, as shown in the top of Fig. 2, were phased with the new light elements as:

$$Max I = HJD 2453053.650(1) + 0.^d349375(42) E \quad (1)$$

which was determined by a least-square scheme with the times of maximum lights in Table 2. The Krajci's (2005) white light curve was also drawn in the bottom of Fig. 2, which was phased with his light elements as:

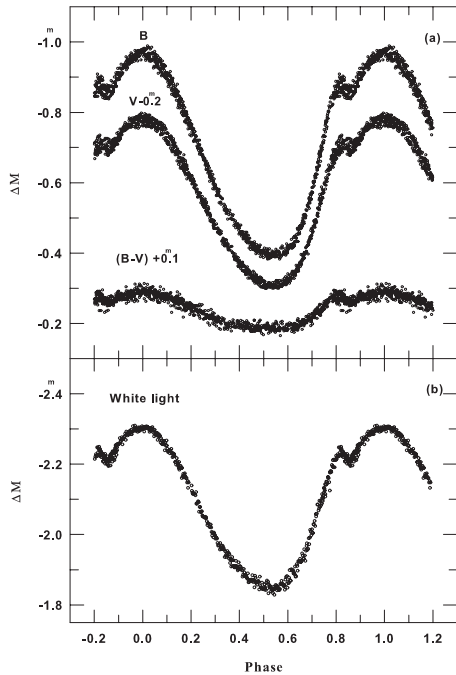
$$Max I = HJD 2453053.6545(5) + 0.^d349350(3) E. \quad (2)$$

Our light curves and Krajci's light curves are similar, both showing that BH UMa is really an RR Lyr star belonging to the RRc subgroup. Compared with Krajci's light curve, our light curves phased with Eq. (2) show a large phase shift of about  $0.^d3$ , which corresponds to about  $0.^d105$  (~2.5 hours). This phase shift may indicate a large period change even in the time interval of about  $4.^y2$  between two observing seasons (see also the periods in Eqs. (1) and (2)). We will discuss this change in more detail in the next section.

The light curves in Fig. 2a are slightly asymmetrical

<sup>1</sup>[http://binary.chungbuk.ac.kr/bbs/zboard.php?id=lab\\_photometry](http://binary.chungbuk.ac.kr/bbs/zboard.php?id=lab_photometry)

with the descending branch from the maximum (Max) to the minimum (Min) at about 0.<sup>p</sup>55 taking 4<sup>h</sup> 32<sup>m</sup> and with the ascending branch from the Min to the Max taking 3<sup>h</sup> 48<sup>m</sup>. The ratio *D* of the Min-to-Max time to the period is



**Fig. 2.** (a) The BV light and (B-V) color curves of BH UMa. (b) The Krajci's (2005) light curve.

**Table 2.** New times of minimum or maximum light of BH UMa.

Time (JD Hel 2450000+)	Type	Error	Note <sup>a</sup>
3053.4854	Min	0.0012	<sup>b</sup>
4530.2929	Min	0.0024	
4530.9975	Min	0.0008	
4533.0842	Min	0.0011	
4542.1675	Min	0.0008	
4546.0131	Min	0.0007	
4559.9936	Min	0.0004	<sup>b</sup>
4575.0132	Min	0.0006	<sup>c</sup>
4576.0553	Min	0.0013	
4584.0929	Min	0.0012	
3090.3352	Max	0.0004	
4447.6527	Max	0.0006	
4533.2549	Max	0.0004	
4543.0307	Max	0.0005	
4545.1260	Max	0.0004	
4560.1548	Max	0.0003	
4574.1299	Max	0.0004	
4576.2276	Max	0.0006	
4581.1183	Max	0.0005	

<sup>a</sup>All timings were weighted-mean values of two BV timings defining the same epoch.

<sup>b</sup>Determined in this paper from Krajci's (2005) observations.

<sup>c</sup>Determined in this paper from SuperWASP's archived observations.

0.453, a typical value of RRc stars. The variation amplitudes are about 0.<sup>m</sup>58 in *B* and 0.<sup>m</sup>47 in *V*. A small hump exists between 0.<sup>p</sup>82 and 0.<sup>p</sup>86 with the second Max and the second Min. The depths of the hump (the brightness difference between the second Max and the second Min) are about 0.<sup>m</sup>02 and 0.<sup>m</sup>01 in the *B* and *V* filters, respectively. We also noticed that the ascending branch from the Min to the second Max is steeper than that from the second Min to the Max. The *B-V* color curve also shows a variation pattern similar to those of the individual *BV* light curves. The difference of the *B-V* values between the Max and the Min is about 0.<sup>m</sup>11. The *BV* magnitudes of BH UMa measured at several characteristic phases (Max, Min, and Hump) are listed in Table 3. These magnitudes are differential relative to our comparison star (GSC 03449-0707).

Although our observations were not standardized, it would be possible to roughly estimate the *BV* standard magnitudes of BH UMa because the *BV* standard magnitudes for the comparison and check stars was given by Nelson et al. (2004), as given in Table 1, and the instrumental magnitudes for the comparison and check stars were measured by us. Our instrumental and standardized  $\Delta m$  (check-comparison) are 1.397 ( $\pm 0.024$ ) and 1.379 ( $\pm 0.022$ ) for the *B* filter and 1.491 ( $\pm 0.010$ ) and 1.513 ( $\pm 0.012$ ) for the *V* filter, respectively. Their differences relative to the standardized magnitude are 0.018 ( $\pm 0.023$ ) and -0.022 ( $\pm 0.011$ ) for the *B* and *V* filters, respectively. If these constant values are applied to the instrumental and standardized  $\Delta m$  (BH UMa-comparison), then the *BV* standard magnitudes for BH UMa could be estimated as those given in the first row of Table 1, which should be tentative magnitudes.

### 3. PERIOD STUDY

In general, the period changes of variable stars have been investigated with their (*O-C*) diagrams where the abscissa is usually time or cycle counted with an adopted ephemeris and where the ordinate is the difference between the observed times (*O*) of a particular phase

**Table 3.** The brightness and color of BH UMa at several characteristic phases.

Filter	Max		Min		Hump			
					Max		Min	
	Phase	Mag	Phase	Mag	Phase	Mag	Phase	Mag
<i>B</i>	0.000	-0.974	0.537	-0.396	0.811	-0.880	0.864	-0.858
<i>V</i>	0.000	-0.579	0.546	-0.109	0.823	-0.507	0.858	-0.494
<i>B-V</i>	0.000	-0.395	0.542	-0.287	0.817	-0.373	0.861	-0.364

**Table 4.** Times of minima of BH UMa.

Time of minima (HJD 2400000+)	<i>E</i>	$\Delta E$	<i>O-C</i> <sub>1</sub>	<i>O-C</i> <sub>2</sub>	Me <sup>a</sup>	Reference
30496.417	-41784.0	10.0	-3.495	0.054	P	Meinunger (1965)
30808.343	-40891.0	10.0	-3.533	-0.047	P	Meinunger (1965)
31194.406	-39786.0	10.0	-3.495	-0.086	P	Meinunger (1965)
31229.394	-39686.0	10.0	-3.441	-0.039	P	Meinunger (1965)
36626.384	-24237.0	7.0	-2.419	-0.025	P	Meinunger (1965)
36657.459	-24148.0	7.0	-2.435	-0.047	P	Meinunger (1965)
37761.398	-20988.0	6.0	-2.074	0.124	P	Meinunger (1965)
37777.366	-20942.0	6.0	-2.176	0.019	P	Meinunger (1965)
37783.412	-20925.0	6.0	-2.069	0.126	P	Meinunger (1965)
38140.438	-19903.0	6.0	-2.072	0.062	P	Meinunger (1965)
38162.413	-19840.0	6.0	-2.106	0.024	P	Meinunger (1965)
38255.376	-19574.0	6.0	-2.069	0.046	P	Meinunger (1965)
38386.700	-19198.0	6.0	-2.098	-0.005	P	Meinunger (1965)
38406.605	-19141.0	6.0	-2.106	-0.016	P	Meinunger (1965)
38407.645	-19138.0	6.0	-2.114	-0.025	P	Meinunger (1965)
38410.480	-19130.0	6.0	-2.073	0.015	P	Meinunger (1965)
38412.550	-19124.0	6.0	-2.099	-0.011	P	Meinunger (1965)
38413.605	-19121.0	6.0	-2.092	-0.004	P	Meinunger (1965)
38414.660	-19118.0	6.0	-2.085	0.002	P	Meinunger (1965)
38415.710	-19115.0	6.0	-2.084	0.004	P	Meinunger (1965)
38439.450	-19047.0	6.0	-2.099	-0.015	P	Meinunger (1965)
38440.520	-19044.0	6.0	-2.077	0.007	P	Meinunger (1965)
38465.320	-18973.0	6.0	-2.080	-0.001	P	Meinunger (1965)
38465.660	-18972.0	6.0	-2.090	-0.010	P	Meinunger (1965)
38466.340	-18970.0	6.0	-2.108	-0.029	P	Meinunger (1965)
38467.400	-18967.0	6.0	-2.096	-0.017	P	Meinunger (1965)
38525.410	-18801.0	6.0	-2.078	-0.008	P	Meinunger (1965)
42570.427	-7222.0	4.0	-1.416	0.013	VI	Diethelm (1975)
44299.318	-2273.0	3.0	-1.079	0.097	VI	Diethelm (1980)
45093.348	0.0	3.0	-1.108	-0.043	VI	German (1982)
45101.381	23.0	3.0	-1.110	-0.047	VI	German (1982)
45115.365	63.0	3.0	-1.100	-0.038	VI	German (1982)
46861.374	5061.0	2.0	-0.763	0.063	VI	Blättler (1987)
47208.276	6054.0	2.0	-0.759	0.021	VI	German (1988)
47595.359	7162.0	2.0	-0.749	-0.019	VI	Peter (1989)
47597.428	7168.0	2.0	-0.776	-0.046	VI	Peter (1989)
48361.343	9354.0	2.0	-0.527	0.106	VI	Peter (1991a)
48362.376	9357.0	2.0	-0.542	0.091	VI	Peter (1991a)
48385.426	9423.0	2.0	-0.549	0.081	VI	Peter (1991b)
48406.401	9483.0	2.0	-0.535	0.093	VI	Peter (1991b)
48689.375	10293.0	2.0	-0.529	0.063	VI	Peter (1992)
48733.379	10419.0	2.0	-0.543	0.045	VI	Peter (1992)
48763.421	10505.0	2.0	-0.544	0.039	VI	Peter (1992)
48770.403	10525.0	2.0	-0.549	0.034	VI	Peter (1992)
49090.393	11441.0	2.0	-0.558	-0.015	VI	Peter (1993)
49097.394	11461.0	2.0	-0.544	-0.001	VI	Peter (1993)
49163.427	11650.0	2.0	-0.537	-0.002	VI	Peter (1993)
50099.409	14330.0	1.0	-0.448	-0.023	VI	Kohl (1996)
50194.439	14602.0	1.0	-0.439	-0.028	VI	Kohl (1997)
51549.6962	18481.0	1.0	-0.2876	-0.0303	CCD	Kreiner et al. (2001)
51553.5327	18492.0	1.0	-0.2939	-0.0370	CCD	Kreiner et al. (2001)
51611.8896	18659.0	1.0	-0.2774	-0.0271	CCD	Kreiner et al. (2001)
51612.9423	18662.0	1.0	-0.2728	-0.0225	CCD	Kreiner et al. (2001)
51942.4493	19605.0	1.0	-0.1972	0.0168	CCD	Diethelm (2001)
52042.4269	19892.0	0.0	-0.1319	0.0711	VI	Brát et al. (2007)
52669.875	21688.0	0.0	-0.106	0.030	CCD	Dvorak (2004)
52704.8070	21788.0	0.0	-0.1087	0.0236	CCD	Nelson (2004)
53053.4854	22786.0	0.0	-0.0750	0.0199	CCD	Krajci (2005)
53515.0043	24107.0	0.0	-0.0396	0.0075	CCD	Nagai (2006)
53866.4736	25113.0	0.0	-0.0103	0.0009	CCD	Brát et al. (2007)
54530.2929	27013.0	0.0	0.0554	0.0004	CCD	This paper
54530.9975	27015.0	0.0	0.0613	0.0062	CCD	This paper
54533.0842	27021.0	0.0	0.0520	-0.0034	CCD	This paper
54542.1675	27047.0	0.0	0.0523	-0.0039	CCD	This paper
54546.0131	27058.0	0.0	0.0551	-0.0015	CCD	This paper
54559.9936	27098.0	0.0	0.0619	0.0039	CCD	This paper
54575.0132	27141.0	0.0	0.0597	0.0002	CCD	This paper
54576.0553	27144.0	0.0	0.0538	-0.0058	CCD	This paper
54584.0929	27167.0	0.0	0.0564	-0.0039	CCD	This paper

<sup>a</sup>P: plate, VI: visual, CCD: charge coupled device.

(e.g., the maximum, minimum, or the mid-point on the ascending branch of the light curve) and the predicted time of the same phase ( $C$ ), calculated according to the same ephemeris. In recent years, another method has often been used where the ( $O-C$ ) data are calculated as the time shift between the entire observed light curve and the normal light curve, rather than from one specific phase. This method is known to produce more stable results for period change investigation (Szeidl et al. 2011).

To investigate the period variation of BH UMa, we used the times of minimum light instead of those of maximum light because only nine maximum light timings were distributed locally in a short time interval of about 4 years, while the minimum light timings are numerous and widespread over 66 years. A total of 69 (22 visual, 27 sky patrol, and 20 CCD) times of minimum light, including ours in Table 2, have been collected from a modern database (Kreiner et al. 2001) and from the recent literature. These timings are listed in Table 4 where one time of HJD2453053.4854 was determined by us from the Krajci's (2005) observations.

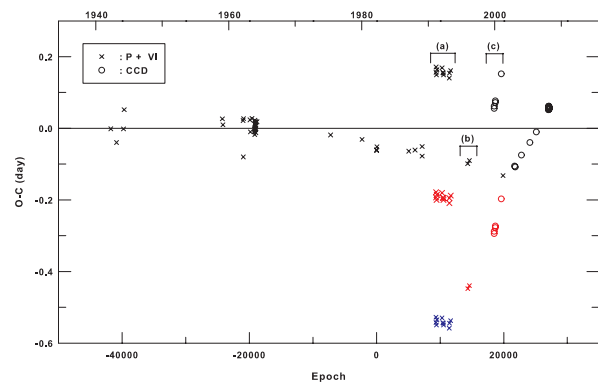
The ( $O-C$ ) residuals of all timings in Table 4 were calculated with the following linear ephemeris:

$$C_1 = \text{HJD } 2445093.408 + 0.^{\text{d}}349344 E. \quad (3)$$

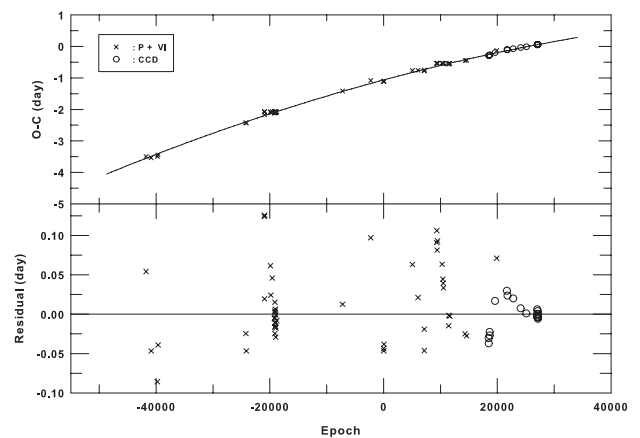
The ephemeris was adopted from the o-c gateway<sup>2</sup> managed by the Czech Astronomical Society. The ( $O-C$ ) diagram is shown in Fig. 3 where crosses and open circles represent non-CCD and CCD minima, respectively. The three groups of ( $O-C$ ) residuals during the early, mid-, and late 1990s (denoted as (a), (b) and (c) in Fig. 3, respectively) show dramatic jumps (or discontinuities) when compared with the neighboring residuals, which indicates the occurrence of a 'cycle slip' effect (or cycle-count uncertainties) rather than a series of a sudden real change during the time interval. The cycle slip effects may occur frequently in the  $O-C$  diagrams of RR Lyrae variables (Szeidl et al. 2011), cataclysmic variables (Kim et al. 2005, Andronov et al. 2006), and some eclipsing variables (Kang et al. 2004), showing very fast period changes in a short time interval. This effect could be solved by properly adjusting the cycles (Kim et al. 2005). Thus, the ( $O-C$ ) values (plotted in red) for the timings corresponding to such jumping residuals were recalculated with Eq. (3) by adding one cycle to their cycle numbers. After adding one cycle, the residuals for group (a) are still discontinuous from the neighboring residuals, while the

residuals for groups (b) and (c) show beautiful continuity with those measured since 2000. After adding two cycles, the ( $O-C$ ) values of the timings of group (a) were recalculated with the same equation and were plotted in blue in Fig. 3, showing that the discontinuity has disappeared.

This procedure was applied to all of the timings prior to 1990<sup>y</sup> to form continuous ( $O-C$ ) residuals with their neighboring ones. The resultant ( $O-C$ ) residuals were shown in the upper panel of Fig. 4 and were listed in the fourth column of Table 4. In the calculation, the cycle correction for each timing was made with the cycle slip number ( $\Delta E$ ) given in the third column of Table 4. The diagram shows a clear long-term parabolic trend with small disturbances. A least-square fit of the trend to a quadratic



**Fig. 3.** The ( $O-C$ ) diagram of BH UMa. The ( $O-C$ ) residuals during the early, mid and late 1990s, denoted as (a), (b), and (c), respectively, show dramatic jumps compared with the neighboring residuals, indicating that a 'cycle slip' effect in the ( $O-C$ ) calculation had occurred. The residuals in red color were recalculated with Eq. (3) by adding one cycle to their cycle numbers. The blue ones were obtained by adding two cycles to their cycle numbers (see the text for more detail).



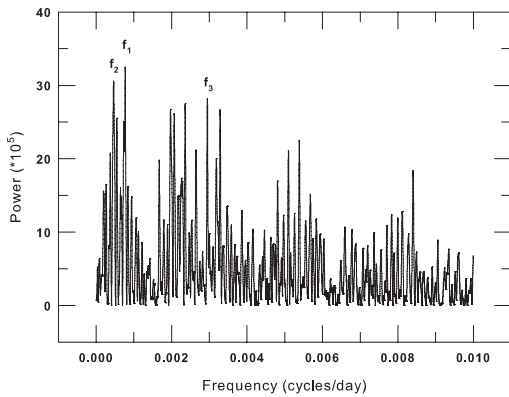
**Fig. 4.** The ( $O-C$ ) diagram of BH UMa after the cycle corrections. The continuous line represents the quadratic term of Eq. (4). The residuals from Eq. (4) were drawn in the bottom panel.

<sup>2</sup><http://var.astro.cz/ocgate/>

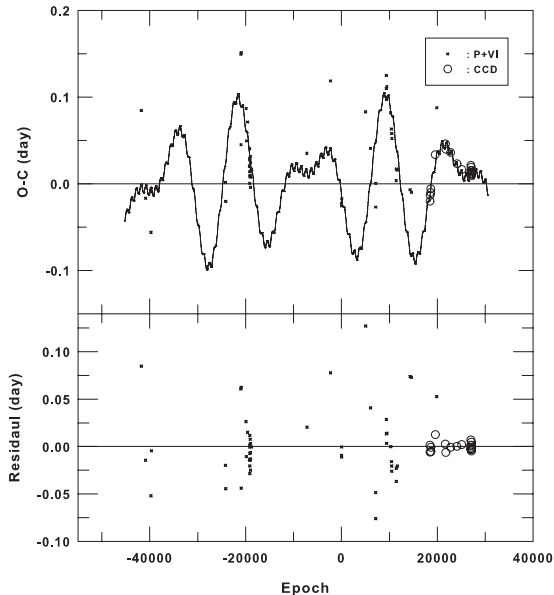
ephemeris was made to find the following ephemeris:

$$C_2 = \text{HJD } 2445093.3915(7) + 0.434939253(2) - 2.619(11) \times 10^{-10} E \quad (4)$$

where the parenthesized value to the right of each term is its standard error. In this and subsequent calculations, the weight of the CCD minima was given ten times that of each of the non-CCD minima, according to the ratio of the inversely squared values of each scatter for non-CCD and CCD minima. The continuous line in Fig. 4, which fits quite well to the residuals, was drawn by using the quadratic term in Eq. (4). The secular period decreasing rate was deduced as  $-0.58$  s/century from the quadratic coef-



**Fig. 5.** The power spectra of the residuals in the bottom panel of Fig. 4. The frequencies are:  $f_1 = 0.000772$  cycle/d,  $f_2 = 0.000465$  cycle/d, and  $f_3 = 0.002949$  cycle/d.



**Fig. 6.** Top: the (O-C) diagram of BH UMa constructed with the linear term of Eq. (5). The continuous line is the contribution from the cyclic terms of Eq. (5). Bottom: the residual from Eq. (5).

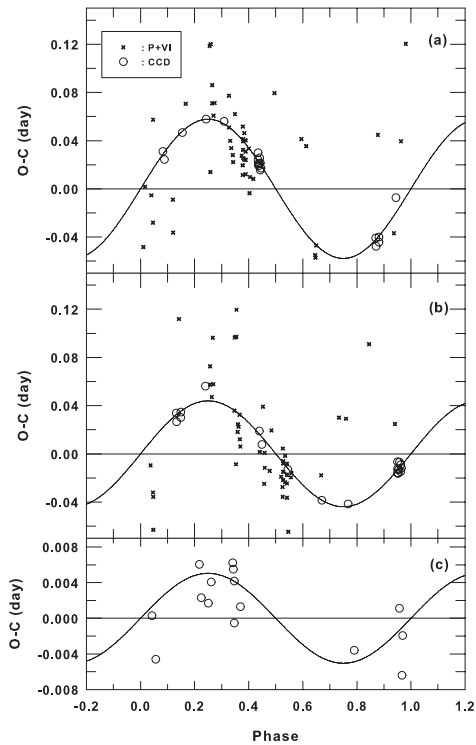
icient. In the bottom panel of Fig. 4, there were plotted the residuals from Eq. (4) which show large short-term changes of period, especially in the recent CCD minima. The residuals for non-CCD minima seemed to have large scatters, but a careful investigation suggests that the scatters could result from very fast period changes in a short-time interval (several years). Therefore, period searches for all the residuals in the bottom panel of Fig. 4 were made with the Lomb-Scargle periodogram (Lomb 1976, Scargle 1982). The resulting power spectrum shown in Fig. 5 reveals at least three meaningful peaks at  $f_1 = 0.000772$  cycle/d,  $f_2 = 0.000465$  cycle/d, and  $f_3 = 0.002949$  cycle/d in the order of their powers, corresponding to 3.756, 5.795, and 0.793, respectively. These three periodicities were then compared with three independent sine curves; namely, all of the residuals were fitted to the ephemeris with three sine functions as follows:

$$C_3 = T_0 + PE + \sum_{i=1}^3 A_i \sin(B_i E + C_i) \quad (5)$$

where  $A_i$ ,  $B_i$ , and  $C_i$  are the semi-amplitude, angular speed ( $=2\pi/P_i$ ), and initial phase of each sine curve, respectively. The eleven fitting parameters in Eq. (5) were iteratively searched for using the Levenberg-Maquardt method (Press et al. 1992). The three periods obtained with the above periodogram analysis were used as the initial parameters for  $B_i$ . The initial parameters for  $A_i$  and  $C_i$  were estimated from Fig. 4. The final solution converged quickly, and the results are listed in Table 5, wherein the parenthesized values give the standard deviations of the tabulated quantities. Fig. 6 shows our final solution, where the top panel shows the cyclical components of the fitting to the residuals and where the lower panel gives the residuals from all terms in Eq. (5). Interestingly, the two longer periods among the three periods in Table 5 were quite different from those by the periodogram analysis while the shortest ones were consistent with each other. The ratios of the two longer periods ( $P_1$  and  $P_2$ ) relative to the

**Table 5.** Final solution of Eq. (5).

Parameter	Value			Unit
$T_0$	2445092.3223(17)			HJD
$P$	0.34939275(7)			Day
	Sine curve			
	$i = 1$	$i = 2$	$i = 3$	
$A_i$	0.058(2)	0.044(1)	0.005(1)	Day
$B_i$	0.02385(7)	0.03452(12)	0.35560(76)	Degree/P
$C_i$	232.3(1.7)	129.6(2.4)	193(19)	Degree
$P_i$	14.439(71)	9.978(57)	0.968(3)	Year



**Fig. 7.** Top to bottom: the  $O-C$  diagrams phased with the periods of 14.<sup>7</sup>44, 9.<sup>9</sup>98, and 0.<sup>9</sup>97, respectively.

shorter one ( $P_3 = 0.^{\circ}968$ ) are 14.92 and 10.31; no commensurability between them was found. From top to bottom in Fig. 7, three different ( $O-C$ ) diagrams were phased with the three periods of 14.<sup>7</sup>41, 9.<sup>9</sup>98, and 0.<sup>9</sup>97, respectively. As seen in the figure, the residuals in the bottom diagram were not represented by the sine curve with the shortest period of 0.<sup>9</sup>97, which may be spurious, while the two sine curves with longer periods seemed to fit successfully to the observed timings, especially to the CCD ones.

#### 4. SUMMARY AND DISCUSSION

We presented the first  $BV$  light curves of BH UMa and confirmed Krajci's (2005) result that BH UMa is an RR Lyra star that belongs to the RRc subgroup. The light curves showed a slight asymmetry of  $D = 0.453$  with the amplitude of about 0.<sup>m</sup>58 in  $B$ , 0.<sup>m</sup>47 in  $V$ , and 0.11 in  $B-V$  and with a small hump between 0.<sup>p</sup>82 and 0.<sup>p</sup>86. Nine times of minimum light and eight times of maximum light were obtained from our  $BV$  observations. A total of 69 minimum timings available to us were intensively analyzed for the first time. The period of BH UMa was found to have varied dramatically in at least three independent sinusoidal ways superposed on a secularly downward pa-

rabola over 66 years. The secular period decreasing rate was obtained as  $6.^{\circ}684 \times 10^{-8} \text{ y}^{-1}$ , corresponding to  $-0.58 \text{ s/century}$ . The semi-amplitude and the period for each of the three sinusoidal variations were (0.<sup>d</sup>058, 14.<sup>7</sup>44), (0.<sup>d</sup>044, 9.<sup>9</sup>98), and (0.<sup>d</sup>005, 0.<sup>9</sup>97), respectively. It is currently uncertain whether the periodicity for the shortest period of 0.<sup>9</sup>97 is real or spurious. More accurate timings are needed to resolve the ambiguity.

The secular period decrease would be a natural result of evolution of the BH UMa system, as many investigators have argued (Smith 1997, Marconi 2009, Szeidl et al. 2011). As Smith (1997) discussed, the quantity  $\alpha (= 1/P)(dP/dt)$  for BH UMa, where the time interval  $dt$  is taken to be  $10^6$  years, is about 0.191, which is consistent with those of other RRc stars (Fig. 2 of Smith's (1997) paper). The two sinusoidal period variations may be due to two light-time effects caused by two hypothetical tertiaries in the BH UMa system. With the parameters in Table 5, the mass functions were obtained to be  $4.44 \pm 0.31 M_{\odot}$  for the 9.<sup>9</sup>98 period and  $4.86 \pm 0.31 M_{\odot}$  for the 14.<sup>7</sup>44 one. Such large mass functions imply that the companions have relatively high masses and that the orbit is likely to be second edge-on if the second and third companions exist. If the mass of BH UMa was assumed to be  $0.55 M_{\odot}$ , a mean mass typical of RR Lyrae stars, then the implied minimum masses for the second and the third bodies would be  $5.3 \pm 0.3 M_{\odot}$  and  $11.3 \pm 0.8 M_{\odot}$ , respectively, which are about 10 and 20 times larger than that of BH UMa. Alternatively, if two sinusoidal trends of the ( $O-C$ ) residuals in Fig. 7 could be assumed to occur by chance, then they might arise from random fluctuations (or erratic changes) in the pulsation period of BH UMa, which have been attributed not to a star's secular evolution but to various causes such as mixing events in the stellar core (Sweigart & Renzini 1979), hydromagnetic events (Stothers 1980), convection (Stothers 2010), or passage through the RR Lyrae instability strip (Silva Aguirre et al. 2008). At the moment, no method is available to discern which mechanism among the existing theories is responsible for the observed changes in the pulsation period of BH UMa. However, the seemingly sinusoidal period changes could be produced if two or more mechanisms are concurrently active.

With the continuous monitoring of timings, high-precision observations of radial velocity curve and light curves are needed to understand this intriguing system.

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## REFERENCES

- Andronov IL, Baklanov AV, Burwitz V, The unique magnetic cataclysmic variable V1432 Aql (Research Note) Third type of minima and synchronization, *A&A*, 452, 941-944 (2006). doi: 10.1051/0004-6361:20054117
- Blättler E, 117rd List of minima of eclipsing binaries, *BBSAG Bull.*, 84, 5 (1987).
- Brát L, Zejda M, Svoboda P, B.R.N.O. contributions # 34, *OEJV*, 74, 1 (2007).
- Diethelm R, 56th List of minima of eclipsing binaries, *BBSAG Bull.*, 23, 6 (1975).
- Diethelm R, 79rd List of minima of eclipsing binaries, *BBSAG Bull.*, 46, 4 (1980).
- Diethelm R, 157. List of minima of eclipsing binaries, *BBSAG Bull.*, 124, 9 (2001).
- Dvorak SW, Times of minima for neglected eclipsing binaries in 2003, *IBVS*, 5502, 1 (2004).
- German R, 93rd List of minima of eclipsing binaries, *BBSAG Bull.*, 60, 4 (1982).
- German R, 120th List of minima of eclipsing binaries, *BBSAG Bull.*, 87, 5 (1988).
- Götz W, Wenzel W, Spektraltypen von Veränderlichen (Fortsetzung), *Mitt. Veränderliche Sterne, Sonneberg*, 2, 702 (1962).
- Kang YW, Lee HW, Hong KS, Kim C-H, Guinan EE, The chromospherically active contact binary CE Leonis, *AJ*, 128, 846-857 (2004). doi: 10.1086/422706
- Kim C-H, Lee C-U, Yoon Y-N, Park, S-S, Kim D-H, Cha S-M, Won J-H, New CCD times of minima of eclipsing binary systems, *IBVS*, 5694, 1 (2006).
- Kim YG, Andronov IL, Park SS, Jeon YB, Orbital and spin variability of the intermediate polar BG CMi, *A&A*, 441, 663-674 (2005) doi: 10.1051/0004-6361:20052995.
- Kohl M, 144. List of minima of eclipsing binaries, *BBSAG Bull.*, 111, 9 (1996).
- Kohl M, 147. List of minima of eclipsing binaries, *BBSAG Bull.*, 114, 13 (1997).
- Krajci T, Observations of variable stars, *IBVS*, 5599, 1 (2005)
- Kreiner JM, Kim C-H, Nha I-S, An atlas of (O-C) diagrams of eclipsing binary stars (Wydawn. Nauk. Akad. Pedagogicznej, Krakow, 2001).
- Kwee KK, van Woerden H, A method for computing accurately the epoch of minimum of an eclipsing variable, *BAN*, 12, 327-330 (1956).
- Lomb NR, Least-squares frequency analysis of unequally spaced data, *Ap&SS*, 39, 447-462 (1976). doi: 10.1007/BF00648343
- Marconi, M, RR Lyrae pulsation theory, *AIP Conf. Proc.*, 1170, 223-234 (2009). doi: 10.1063/1.3246450
- Meinunger L, 21 wenig bekannte oder falsch klassifizierte veränderliche, *Mitt. Veränderliche Sterne, Sonneberg*, 3, 9 (1965).
- Nagai K, Visual and CCD minima of eclipsing binaries during 2005, *Var. Star Bull. Japan*, 44, 1 (2006).
- Nelson RH, CCD minima for selected eclipsing binaries in 2003, *IBVS*, 5493, 1 (2004).
- Nelson RH, Henden AA, Krajci T, A new solar-type overcontact binary, *IBVS*, 5546, 1 (2004).
- Peter H, 124. List of minima of eclipsing binaries, *BBSAG Bull.*, 91, 13 (1989).
- Peter H, 130. List of minima of eclipsing binaries, *BBSAG Bull.*, 97, 10 (1991a).
- Peter H, 131. List of minima of eclipsing binaries, *BBSAG Bull.*, 98, 10 (1991b).
- Peter H, 134. List of minima of eclipsing binaries, *BBSAG Bull.*, 101, 7 (1992).
- Peter H, 137. List of minima of eclipsing binaries, *BBSAG Bull.*, 104, 6 (1993).
- Press WH, Teukolsky SA, Vetterling WT, Flannery BP, *Numerical recipes* (Cambridge University Press, Cambridge, 1992), Chap. 15.
- Scargle JD, Studies astronomical time series analysis. II. Statistical aspects of spectral analysis of unevenly spaced data, *ApJ*, 263, 835-853 (1982). doi: 10.1086/160554
- Silva Aguirre V, Catelan M, Weiss A, Valcarce AAR, Stellar evolution and variability in the pre-ZAHB phase, *A&A*, 489, 1201-1208 (2008). doi: 10.1051/0004-6361:200810047
- Smith H, RR Lyrae period changes: still puzzling after all these years, *BaltA*, 6, 89-95 (1997).
- Stothers R, Hydromagnetics and period changes in RR Lyrae stars, *PASP*, 92, 475-478 (1980). doi: 10.1086/130696
- Stothers RB, Observational evidence of convective cycles as the cause of the Blazhko effect in RR Lyrae stars, *PASP*, 122, 536-540 (2010). doi: 10.1086/652909
- Sweigart AV, Renzini A, Semiconvection and period changes in RR Lyrae stars, *A&A*, 71, 66-78 (1979).
- Szeidl B, Hurta Zs, Jurcsik J, Clement C, Lovas M, Long-term photometric monitoring of Messier 5 variables I. Period changes of RR Lyrae stars, *MNRAS*, 411, 1744-1762 (2011). doi: 10.1111/j.1365-2966.2010.17815.x