

## 상태-공간 모형에서의 주가의 가성 평균-회귀

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### Spurious Mean-Reversion of Stock Prices in the State-Space Model

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#### ■ Abstract ■

In order to explain the U-shaped pattern of autocorrelations of stock returns i.e., autocorrelations starting around 0 for short-term horizons and becoming negative and then moving toward 0 for long-term horizons, researchers suggested the use of a state-space model consisting of an I(1) permanent component and an AR(1) stationary component, where the two components are assumed to be independent. They concluded that auto-regression coefficients derived from the state-space model follow a U-shape pattern and thus there is mean-reversion in stock prices. In this paper, we show that only negative autocorrelations are feasible under the assumption that the permanent component and the stationary component are independent in the state-space model. When the two components are allowed to be correlated in the state-space model, we show that the sign of the auto-regression coefficients is not restricted as negative. Monthly return data for all NYSE stocks for the period from 1926 to 2007 support the state-space model with correlated noise processes. However, the auto-regression coefficients of the ARIMA process, equivalent to the state-space model with correlated noise processes, do not follow a U-shaped pattern, but are always positive.

Keyword : State-Space Model, ARIMA, Mean-Reversion, Correlated Noise Process

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## 1. Introduction

Financial economists have studied autocorrelations in short-term stock returns to test whether stock returns are predictable. Since early studies couldn't find significant evidence of autocorrelations in stock returns, they concluded that stock returns are unpredictable. That is, stock returns follow a random-walk model and the stock market is efficient (see Fama [5]; LeRoy [10]).

However, several studies have challenged this interpretation of short-horizon returns. Lo and MacKinlay [12], Conrad and Kaul [4], and Poterba and Summers [15] reported strong positive autocorrelations in short-term (e.g., daily, weekly, and monthly) stock returns. Summers [18] insisted on the phenomenon that prices take long temporary swings away from fundamental values, which means that prices have a slowly decaying stationary component. Fama and French [6], Lo and MacKinlay [12], and Poterba and Summers [15] showed that there is substantial mean-reversion in long-term stock returns that they attribute to the presence of a stationary component. By using regression tests for stock returns from 1926 to 1985, Fama and French [6] found a U-shaped pattern of autocorrelations in stock returns that was concluded to be due to both negative autocorrelations in returns beyond a year and substantial mean-reversion in stock market prices. In order to explain long-term mean-reversion due to negative autocorrelations, Fama and French [6] and Poterba and Summers [15] suggested the use of the state-space model (henceforth SS model) consisting of an I(1) permanent component and an AR(1) stationary com-

ponent where the two components are assumed to be independent.

Even though some skeptical studies followed the above results (e.g., Richardson and Stock [17]; Kim et al. [9]; Richardson [16]; Malliaropoulos [13]), many studies have showed the evidence of mean-reversion. Balvers et al. [1] found strong evidence of mean-reversion in relative stock index prices of 18 countries. Nam et al. [14] found the asymmetrical reverting behavior in US stock returns using asymmetric nonlinear smooth-transition GARCH approach. Balver et al. [1] and Nam et al. [14] interpreted that the detected mean-reversion leads to contrarian profits. Chaudhuri and Wu [2], [3] showed that a panel-based test that exploits cross-sectional information from 17 emerging equity markets rejects the null hypothesis of random-walk. Lim and Liew [11] demonstrated the random-walk assumption is rejected in Asian stock markets using nonlinear stationary tests.

In this study, we argue that the correlated structure between permanent and stationary components of stock prices has a crucial meaning to interpret short-term and long-term autocorrelations in stock prices. Lo and MacKinlay [12], Poterba and Summers [15], and Khil and Lee [8] pointed out that the SS model suggested by Fama and French [6] can explain the negative long-term autocorrelations, but not the positive short-term autocorrelations. In particular, Khil and Lee [8] suggested an SS model employing an AR(2) process instead of an AR(1) process as a stationary component in order to explain short and long-term autocorrelations at once. Khil and Lee [8] still adopted the independence assumption in their SS model.

The SS model suggested by previous studies is equivalent to the unobserved component model that economists often use for the decomposition of observable non-stationary time series into two unobservable components, a permanent trend and a transitory cycle. Joo and Jun [7] showed that the trend-cycle decomposition could be spurious if dependence between the permanent and the stationary component is not allowed in the SS model, and thus an erroneous interpretation could occur.

Based on Joo and Jun [7], it is shown that the previous SS model with an independence assumption is compatible only with negative autocorrelations in this study. It cannot describe positive autocorrelations unless dependence between the permanent and the stationary component is allowed. Furthermore, auto-regression coefficients for one month returns of all NYSE stocks for the period from 1926 to 2007 is estimated, and it is shown that the SS model displays positive autocorrelations that are compatible only with a dependence assumption.

This paper is organized as follows. Section 2 discusses the SS model of stock prices considering dependence between the permanent and the stationary noise process. In section 2, the equivalent ARIMA from the SS model and auto-regression coefficients expressed as the parameter of the SS model are derived. In section 3, we analyze the NYSE stock return data and identify their ARIMA models. Based on these ARIMA estimates, we determine whether the noise processes are independent or dependent, and study the pattern of the auto-regression coefficients in order to try to explain mean-reversion in stock prices. Section 4 concludes the paper.

## 2. A State-Space Model for Stock Prices

### 2.1 Previous Studies with the State-Space Model for Long-Term Mean-Reverting Stock Prices

Many previous studies suggested the use of the SS model ignoring the dependence between noise processes for explaining the long-term mean-reversion of stock prices. By using regression tests with NYSE returns for the 1926 to 1985 period, Fama and French [6] reported the U-shaped pattern of stock returns beyond a year. They concluded this pattern is due to both large negative autocorrelations in returns beyond a year and substantial mean-reversion in stock market prices (see also Poterba and Summers [15]). The existence of mean-reversion in stock prices means that stock prices do not follow a random-walk but may contain a stationary component. In order to explain this phenomenon, Fama and French [6] and Poterba and Summers [15] suggest the SS model to describe the natural log of monthly stock prices,  $p_t$ , as the sum of a random walk component,  $q_t$ , and a stationary component,  $z_t$  :

$$\begin{aligned} p_t &= q_t + z_t \\ q_t &= u + q_{t-1} + w_t \\ z_t &= \phi z_{t-1} + v_t \end{aligned} \quad (1)$$

where  $u$  is the expected drift and  $w_t$  and  $v_t$  are normally distributed white noises as

$$\begin{bmatrix} w_t \\ v_t \end{bmatrix} \sim BVN\left(0, \sigma_w^2 \begin{bmatrix} 1 & 0 \\ 0 & \lambda \end{bmatrix}\right). \quad (2)$$

It was assumed that the noise processes,  $w_t$  and  $v_t$ , are independent. Since  $p_t$  is the natural log of the stock price, the continuously compounded return from  $t$  to  $t+T$  is

$$\begin{aligned} r(t, t+T) &= p_{t+T} - p_t \\ &= (q_{t+T} - q_t) + (z_{t+T} - z_t) \end{aligned} \quad (3)$$

Let  $\beta(T)$  be the slope of the following regression of the return  $r(t, t+T)$  on  $r(t-T, t)$  :

$$r(t, t+T) = \alpha + \beta(T) \cdot r(t-T, t) + e_t \quad (4)$$

Since the auto-regression coefficient  $\beta(T)$  is an OLS estimator and there is no correlation between the noise components,

$$\beta(T) = \frac{\text{cov}(z_{t+T} - z_t, z_t - z_{t-T})}{\text{var}(z_{t+T} - z_t) + \text{var}(q_{t+T} - q_t)} \quad (5)$$

If a stock price does not have a random-walk component, i.e.,  $\text{var}(q_{t+T} - q_t)$ , the slope  $\beta(T) = -(1 - \phi^T)/2$  approaches  $-0.5$  for large values of  $T$ . On the other hand, if a stock price does not have a temporary component, i.e.,  $\lambda = 0$ , the slope  $\beta(T)$  is zero for all values of  $T$ . Thus, Fama and French [6] insisted that if stock prices have both a random-walk and a stationary component, the slope  $\beta(T)$  might form the U-shaped pattern, starting around zero for short horizons, becoming more negative as  $T$  increases, and then moving back toward zero as the white noise variance begins to dominate in the long-term.

Other researchers also showed that the SS model consisting of an I(1) and an AR(1) component are compatible with negative long-term autocorrelations. Lo and MacKinray [12] and

Khil and Lee [8] showed that the SS model is compatible with negative long-term autocorrelations even if it has limitations for short-term autocorrelations. In other words, most studies suggested that the SS model with the independent assumption is appropriate for explaining negative long-term autocorrelations.

However, it is most likely that the above results are derived from the independence assumption in the SS model. In the following sections, we will show that when the independence assumption is introduced in the SS model, the parameter space of the equivalent ARIMA process is restricted. The restricted parameter space is identical to the area that induces the negative sign of the auto-regression coefficients from the above SS model.

## 2.2 A State-Space Model Incorporating Dependence

For simplicity, previous studies assumed independence between a permanent component and a stationary component. But the assumption that the permanent and stationary components of stock prices are not correlated can be unrealistic. As Joo and Jun [7] showed, the unrealistic assumption of independence between the two components could cause spurious inferences. Consider the following dependent noise processes :

$$\begin{bmatrix} w_t \\ v_t \end{bmatrix} \sim BVN\left(0, \sigma_w^2 \begin{bmatrix} 1 & \rho\lambda^{1/2} \\ \rho\lambda^{1/2} & \lambda \end{bmatrix}\right), \quad (6)$$

where  $\lambda$  is the ratio of the variance of the noise process for the stationary component to that for the permanent component, i.e.,  $\sigma_v^2 = \lambda\sigma_w^2$ , and  $\rho$  is the correlation coefficient between the two

noises. In this case, the auto-regression coefficient  $\beta(T)$  is<sup>1)</sup>

$$\beta(T) = \begin{cases} \frac{-(1-\phi^T)}{2} & \text{if } \sigma_w^2 = 0 \\ \frac{-(1-\phi^T)\lambda + (1-\phi^T)(1+\phi)\rho\lambda^{1/2}}{2\lambda + T \cdot \frac{1-\phi^2}{1-\phi^T} + 2(1+\phi)\rho\lambda^{1/2}} & \text{if } \sigma_w^2 \neq 0 \end{cases} \quad (7)$$

### 2.3 The Parameter Space Restriction of the State-Space Model

Joo and Jun [7] showed that the usual assumption of independence between the noise process for the trend and that for the cycle may result in redundant AR and MA parameters in the equivalent ARIMA process. They proposed the SS trend-cycle decomposition of an ARIMA(1, 1, 1) process through the relationships between the model parameters. They showed that the independent noise assumption might restrict the parameter space of the SS model.

The SS model in equations (1) and (6) is equivalent to the following ARIMA(1, 1, 1) process :

$$(1 - \phi' B)(1 - B)p_t = (1 - \theta B)\epsilon_t, \quad (8)$$

where  $\epsilon_t$  is a normally distributed noise process with variance,  $\sigma_\epsilon^2$ . By equating the autocorrelations of the first difference of  $p_t$  in the ARIMA model to the autocorrelations in the SS model, the equivalence relationships for the model parameters can be obtained as follows :

$$\begin{aligned} \phi &= \phi', & (9-a) \\ \rho &= -\frac{(\phi' - \theta)(1 - \phi'\theta)}{(1 + \phi')(1 - \theta)^2} - \frac{1}{1 + \phi'}\lambda^{1/2}, \text{ and } & (9-b) \end{aligned}$$

$$\sigma_w^2 = \frac{\sigma_\epsilon^2}{(1 - \phi')^2}. \quad (9-c)$$

Since  $\phi'$  is always the same as  $\phi$  by equation (9-a), henceforth we will use the term  $\phi$  instead of  $\phi'$  for convenience. When the two noise processes are assumed independent, i.e.,  $\rho = 0$ , equation (9-b) is reduced to

$$\lambda = -\frac{(\phi - \theta)(1 - \phi\theta)}{(1 - \theta)^2} \quad (10)$$

and the restricted parameter space of the SS model in equation (10) is

$$\phi < \theta. \quad (11)$$

If the equivalent ARIMA(1, 1, 1) model has the same AR and MA parameters, i.e.,  $\phi = 0$ , a trend-cycle decomposition results in redundant parameter estimation. That is, the stock price  $p_t$  does not follow an ARIMA(1, 1, 1) process, but it follows a random-walk process. When the noise process of the random-walk trend and that of the AR(1) cycle in the SS model are assumed independent, the AR parameter is always less than the MA parameter, i.e.,  $\phi < \theta$ . When the AR parameter is greater than the MA parameter, i.e.,  $\phi > \theta$ , the SS model with independent noise processes never describes the ARIMA process correctly. Because of the parameter restriction, correlation between noise processes must be considered in the SS model.

### 2.4 The Sign of the Auto-Regression Coefficient

Replacing  $\rho$  in equation (7) with equation (9-b),

1) Appendix 1 shows the derivation of the auto-regression coefficient.

$$\beta(T) = \frac{(1-\phi^T) \cdot \frac{(\phi-\theta)(1-\phi\theta)}{(1-\theta)^2}}{\underbrace{T \cdot \frac{1-\phi^2}{1-\phi^T}}_{(a)} - 2 \cdot \underbrace{\frac{(\phi-\theta)(1-\phi\theta)}{(1-\theta)^2}}_{(b)}}. \quad (12)$$

From the stationarity and invertibility conditions of an ARMA(1, 1) model, absolute values of the AR and MA parameters should be smaller than 1, i.e.,  $|\phi| < 1$  and  $|\theta| < 1$ . Therefore, the sign of the auto-regression coefficient  $\beta(T)$  is determined by the sign of the term  $(\phi - \theta)$  since all other terms are positive.

If the AR parameter is less than the MA parameter, i.e.,  $\phi < \theta$ , the numerator of  $\beta(T)$  in equation (12) is always negative and the denominator is always positive. Therefore,  $\beta(T)$  is always negative under the independence assumption between the noise processes. Also,  $\beta(T)$  is negative for the short-horizon and then moves toward zero as  $T$  increases since the denominator grows as  $T$  increases.

If the MA parameter is less than the AR parameter, i.e.,  $\phi > \theta$ , the sign of the numerator of  $\beta(T)$  is always positive, but that of the denominator is somewhat complicated. The sign of the denominator is determined by the relative magnitude between (a) and (b) in equation (12). Part (a) keeps on increasing, but part (b) is constant as  $T$  increases.<sup>2)</sup> If  $\phi > -0.9$ , the denominator is always positive after one year, i.e.,  $T > 12$ . Moreover, the denominator is always positive after a month if  $\phi > -0.5$ . That is, in most cases,  $\beta(T)$  has both a positive and negative sign for the short-horizon but eventually converges to zero for the long-horizon regar-

dless of the magnitude of (a) and (b).<sup>3)</sup> Hence, the parameter space that always makes the auto-regression coefficient negative is equal to the restricted parameter space by the independence assumption, and the SS model cannot describe the positive auto-regression coefficient without dependence between noise processes.

### 3. Empirical Analysis

#### 3.1 Data

We analyzed the stock market data to determine whether correlations between the permanent component and the stationary component matter in the SS model of stock prices. We collected monthly returns for all New York Stock Exchange (NYSE) stocks for the period from July 1926 to December 2007 provided by the Center for Research in Security Prices. At the end of each year, stocks are ranked on the basis of size and grouped into ten deciles. Monthly portfolio returns, with equal weighting of securities, are calculated and transformed into continuously compounded returns.<sup>4)</sup> All portfolio returns are adjusted for the U.S. Consumer Price Index (CPI) to get the real returns.

#### 3.2 ARIMA Estimation and Diagnostic Check

We performed ARMA identifications of monthly stock returns. Since the SS model consisting of an I(1) and an AR(1) process is equivalent to

2) The maximum of part (b) is 2 for  $|\phi| < 1$  and  $|\theta| < 1$ . The detailed derivation is shown in appendix 2.

3) Details for relationships of the sign of  $\beta(T)$  with  $T$  and  $\phi$  are derived in appendix 2.

4) We also analyzed portfolio returns with value weighting of securities. Overall results are very similar to those of a portfolio with equal weighting of securities.

an ARIMA(1, 1, 1) model, the stock return series follows an ARMA(1, 1) model. When the stock return data is fitted to an ARMA(1, 1) model, the AR and MA parameters appear to be insignificant even at the 90% confidence level. The insignificance of the parameters of the ARMA(1, 1) model is also reported by Khil and Lee [8]. However, the return series of each decile portfolio is identified as an ARMA(1, 0) or an ARMA(0, 1) model at the 99% confidence level. Hence, we perform diagnostic check for these three models : ARMA(1, 1), ARMA(1, 0), and ARMA(0, 1). We compare these models using AIC (Akaike Information Criterion) and BSC (Bayesian Schwartz Criterion), which are generally employed in diagnostic checking. The results are shown in <Table 1>.

From the results, the ARMA(1, 0) model is selected as the best model for decile 1, 2, 4, 5,

and 7 portfolios and the ARMA(0, 1) model is selected for decile 3, 6, 8, 9, and 10 portfolios. Thus, the ARMA(1, 0) or ARMA(0, 1) model fits the data better than the ARMA(1, 1) model.

### 3.3 The Parameter Space Restriction

From section 2, it is derived that the independence assumption in the SS model restricts the parameter space of the equivalent ARIMA(1, 1, 1) model as  $\phi < \theta$ . In [Figure 1], the dark colored area of the upper-left part indicates the restricted parameter space of the ARIMA(1, 1, 1) model due to the independence assumption. However, all estimates for the real data are located in the uncolored area of the lower-right part, which cannot be expressed by previous studies. It means that the correlation between noise processes must be considered to construct the appropriate SS model.

<Table 1> Estimation Results of the Stock Return from July 1926 to December 2007

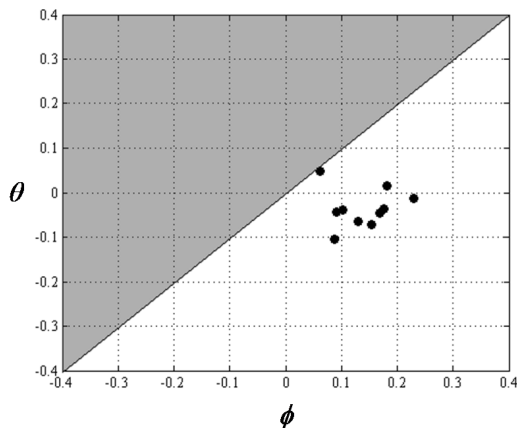
Decile Portfolio	ARMA(1, 1) $(1-\phi B)r_t = (1-\theta B)\epsilon_t$				ARMA(1, 0) $(1-\phi B)r_t = \epsilon_t$			ARMA(0, 1) $r_t = (1-\theta B)\epsilon_t$			Model Selection
	$\phi^*$	$\theta^*$	AIC	BSC	$\phi^{**}$	AIC	BSC	$\theta^{**}$	AIC	BSC	
Decile 1	0.2347	-0.0099	7.6311	7.6411	0.2441	<b>7.6291</b>	<b>7.6341</b>	-0.2263	7.6324	7.6374	ARMA(1, 0)
Decile 2	0.2058	-0.0096	7.2906	7.3006	0.2149	<b>7.2886</b>	<b>7.2936</b>	-0.2029	7.2901	7.2951	ARMA(1, 0)
Decile 3	0.1465	-0.0970	7.0707	7.0807	0.2371	7.0696	7.0746	-0.2303	<b>7.0695</b>	<b>7.0745</b>	ARMA(0, 1)
Decile 4	0.1890	-0.0289	6.9305	6.9405	0.2164	<b>6.9285</b>	<b>6.9335</b>	-0.2040	6.9300	6.9350	ARMA(1, 0)
Decile 5	0.1395	-0.0536	6.8376	6.8476	0.1908	<b>6.83580</b>	<b>6.84080</b>	-0.1832	6.83582	6.84082	ARMA(1, 0)
Decile 6	0.1039	-0.0881	6.7477	6.7577	0.1882	6.7462	6.7512	-0.1852	<b>6.7453</b>	<b>6.7503</b>	ARMA(0, 1)
Decile 7	0.1630	-0.0031	6.6316	6.6416	0.1660	<b>6.6295</b>	<b>6.6345</b>	-0.1569	6.6298	6.6348	ARMA(1, 0)
Decile 8	0.1048	-0.0403	6.5424	6.5524	0.1441	6.5404	6.5454	-0.1397	<b>6.5399</b>	<b>6.5449</b>	ARMA(0, 1)
Decile 9	0.0937	-0.0366	6.4584	6.4684	0.1295	6.4564	6.4614	-0.1257	<b>6.4559</b>	<b>6.4609</b>	ARMA(0, 1)
Decile 10	0.0497	-0.0662	6.2479	6.2579	0.1145	6.2460	6.2510	-0.1144	<b>6.2451</b>	<b>6.2501</b>	ARMA(0, 1)

Note) \* All estimates of the ARMA(1, 1) model are insignificant at the 95% confidence level.

(Except  $\phi$  for decile 1, all estimates of the ARMA(1, 1) model are insignificant at the 90% confidence level).

\*\* All estimates of the ARMA(1, 0) and ARMA(0, 1) models are significant at the 99% confidence level.

The bold number indicates the lowest value in each criterion for each decile portfolio.



Note) The dark colored area indicates the restricted parameter space by the independence assumption.

[Figure 1] Plot of the Estimates of the ARIMA(1, 1, 1) Model

Since an ARIMA(1, 1, 0) model is equivalent to the ARIMA(1, 1, 1) model with a zero MA parameter, the parameter space is restricted as  $\phi < 0$ . In the same manner, the restricted parameter space of an ARIMA(0, 1, 1) model is  $\theta > 0$ . From <Table 1>, it can be easily checked whether the estimates belong to these restricted parameter space for the ARIMA(1, 1, 0) and ARIMA(0, 1, 1) models. Since all estimates of the ARIMA(1, 1, 0) model are positive and all estimates of the ARIMA(0, 1, 1) model are negative, they are out of the restricted parameter space. It suggests that the independence assumption is not compatible with the estimated parameters, therefore, the dependence between noise processes must be considered for building the correct SS model.

### 3.4 The Auto-Regression Coefficient

The auto-regression coefficient for an ARIMA(1, 1, 0) and ARIMA(0, 1, 1) models can be easily derived from equation (12). Setting  $\phi = 0$ , the model is equivalent to the ARIMA(1, 1, 0) model

and

$$\beta(T) = \frac{-\frac{\theta}{(1-\theta)^2}}{T+2 \cdot \frac{\theta}{(1-\theta)^2}}. \quad (13)$$

Similarly, setting  $\theta = 0$ , the model is equivalent to the ARIMA(0, 1, 1) model and

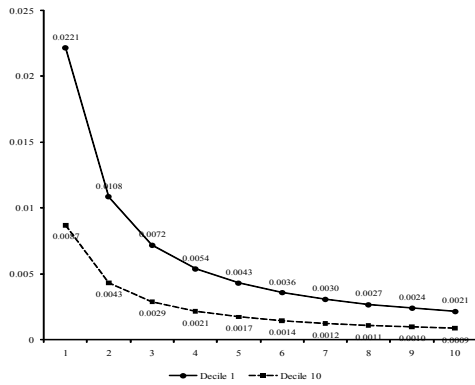
$$\beta(T) = \frac{(1-\phi^T) \cdot \phi}{T \cdot \frac{1-\phi^2}{1-\phi^T} - 2\phi}. \quad (14)$$

In each case, the parameter space that always makes the auto-regression coefficient negative is the same as the restricted parameter space by the independence assumption, i.e.,  $\phi < 0$  for the ARIMA(1, 1, 0) model, and  $\theta > 0$  for the ARIMA(0, 1, 1) model. Hence, the SS model without dependence cannot be suitable for positive auto-correlations of the stock return data.

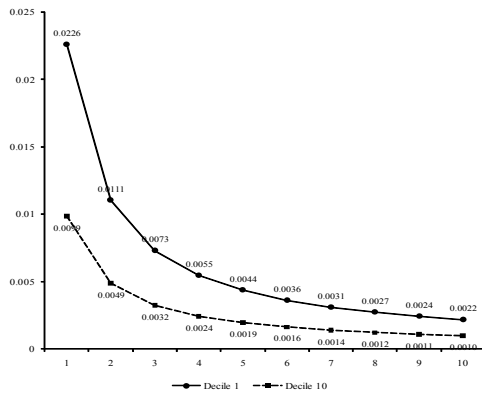
[Figure 2] shows the patterns of the auto-regression coefficients of decile 1 and 10 for each ARIMA model. We look into the decile 1 and 10 to compare extreme cases, the smallest-sized portfolio and the biggest-sized portfolio. The values of the remaining portfolios are located between those of decile 1 and 10.

From [Figure 2], it can be observed whether the firm size affects the autocorrelations. For short-sized horizon returns, a small-sized portfolio (e.g., decile 1 portfolio) has stronger positive autocorrelations, while a big-sized portfolio (e.g., decile 10 portfolio) has relatively weaker positive autocorrelations. That is, prices of the small-sized firm take a longer swing away from fundamental values than those of the big-sized firm. These results related to firm size are also

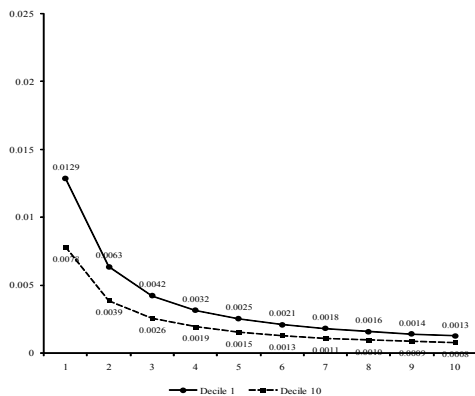




[Figure 2-a] The Auto-Regression Coefficient based on the Estimates of the ARIMA(1, 1, 1) Model



[Figure 2-b] The Auto-Regression Coefficient based on the Estimates of the ARIMA(1, 1, 0) Model



[Figure 2-c] The Auto-Regression Coefficient based on the Estimates of the ARIMA(0, 1, 1) Model

supported by previous studies.<sup>5)</sup>

From these results, the stock return data does not support the previous SS model with the independence assumption, in order to explain the U-shaped pattern. Furthermore, the estimation result of the equivalent ARIMA model suggests that positive autocorrelations can be supported only by the revised SS model to allow dependence between the permanent and the stationary components. However, even with dependence, the parameter estimates shows that the stock return data has only positive autocorrelations that converge to zero. Therefore, the U-shaped mean-reverting pattern cannot be described by the SS model consisting of an I(1) and an AR(1) process: It is suggested that the higher order ARIMA model with dependence should be studied.

#### 4. Conclusion

In order to explain negative autocorrelations in returns of long holding periods, previous studies used an SS model consisting of an I(1) permanent component, and an AR(1) stationary component. In the SS model, they assumed independence between the permanent component and the stationary component. In this paper, based on Joo and Jun [7]-if the correlations between noise processes are not considered properly in the SS model, the parameter space can be restricted and the decomposition of an integrated time series into two components using the SS model can be spurious-we show that auto-regression coefficients are negative when the noise processes are independent by comparing the parameters of the

5) See also Fama and French [6] and Khil and Lee [8].

equivalent ARIMA process with those of the SS model. On the other hand, the auto-regression coefficients can be either positive or negative when the noise processes are dependent.

We estimated the parameters of three models, the ARIMA(1, 1, 1), ARIMA(1, 1, 0), and ARIMA(0, 1, 1) models, fitting the monthly returns of the NYSE for the period from 1926 to 2007. The estimated parameters suggest that the correlation between noise processes in the SS model must be considered. It implies that the decomposition of stock prices using the SS model suggested by previous studies, which do not consider correlations, is likely to be spurious.

When we relax the independence assumption in the SS model, the auto-regression coefficients computed using the parameter estimates are never negative but always positive. Based on the estimated parameters of the stock return series, we conclude that the auto-regression coefficients from the SS models equivalent to the ARIMA(1, 1, 1), ARIMA(1, 1, 0), and ARIMA(0, 1, 1) models do not follow a U-shaped pattern but are always positive. It suggests that the higher order ARMA model with dependence should be studied to explain the observed U-shaped pattern of autocorrelations.

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## 〈Appendix 1〉 The Derivation of the Auto-Regression Coefficient

First, let  $\sigma_w^2$  be zero. It means a stock price does not have a random-walk component and  $\beta(T) = -(1 - \phi^T)/2$ . Now let  $\sigma_w^2$  not be zero. Based on equation (1), the following characteristics can be easily obtained.

$$\text{cov}(q_{t+T} - q_t, q_t - q_{t-T}) = \text{cov}\left(\sum_{i=t+1}^{t+T} w_i, \sum_{j=t-T+1}^t w_j\right) = 0, \quad (\text{A1-a})$$

$$\text{cov}(q_{t+T} - q_t, z_t - z_{t-T}) = \text{cov}\left(\sum_{i=t+1}^{t+T} w_i, (\phi^T - 1)z_{t-T} + \sum_{j=t-T+1}^t \phi^{t-j} v_j\right) = 0, \quad (\text{A1-b})$$

$$\begin{aligned} \text{cov}(z_{t+T} - z_t, q_t - q_{t-T}) &= \text{cov}\left((\phi^T - 1)z_t + \sum_{i=t+1}^{t+T} \phi^{t+T-i} v_i, \sum_{j=t-T+1}^t w_j\right) \\ &= (\phi^T - 1) \text{cov}\left(z_t, \sum_{j=t-T+1}^t w_j\right) \\ &= (\phi^T - 1) \text{cov}\left((\phi^T - 1)z_{t-T} + \sum_{i=t-T+1}^t \phi^{t-i} v_i, \sum_{j=t-T+1}^t w_j\right) \\ &= (\phi^T - 1)(1 + \phi + \phi^2 + \dots + \phi^{T-1}) \text{cov}(v_t, w_t) \\ &= (\phi^T - 1)(1 + \phi + \phi^2 + \dots + \phi^{T-1}) \sigma_w^2 \rho \lambda^{1/2} \end{aligned} \quad (\text{A1-c})$$

$$\text{cov}(z_{t+T} - z_t, z_t - z_{t-T}) = -(1 - 2\phi^T + \phi^{2T}) \text{var}(z_t) = -\frac{(1 - \phi^T)^2}{1 - \phi^2} \cdot \sigma_w^2 \lambda, \quad (\text{A1-d})$$

$$\begin{aligned} \text{cov}(r(t, t+T), r(t-T, t)) &= \text{cov}(q_{t+T} - q_t, q_t - q_{t-T}) + \text{cov}(z_{t+T} - z_t, z_t - z_{t-T}) \\ &\quad + \text{cov}(z_{t+T} - z_t, q_t - q_{t-T}) + \text{cov}(q_{t+T} - q_t, z_t - z_{t-T}) \\ &= \text{cov}(z_{t+T} - z_t, z_t - z_{t-T}) + \text{cov}(z_{t+T} - z_t, q_t - q_{t-T}) \end{aligned} \quad (\text{A1-e})$$

$$\text{var}(q_{t+T} - q_t) = \text{var}\left(Tu + \sum_{i=t+1}^{t+T} w_i\right) = T\sigma_w^2, \quad (\text{A1-f})$$

$$\text{var}(z_{t+T} - z_t) = \text{var}\left((\phi^T - 1)z_t + \sum_{i=t+1}^{t+T} \phi^{t+T-i} v_i\right) = 2 \cdot \frac{1 - \phi^T}{1 - \phi^2} \sigma_w^2 \lambda, \quad (\text{A1-g})$$

$$\begin{aligned} \text{var}(r(t-T, t)) &= \text{var}(z_t - z_{t-T}) + \text{var}(q_t - q_{t-T}) \\ &\quad + 2 \text{cov}(z_t - z_{t-T}, q_t - q_{t-T}) \\ &= \text{var}(z_{t+T} - z_t) + \text{var}(q_{t+T} - q_t) \\ &\quad + 2(1 + \phi + \phi^2 + \dots + \phi^{T-1}) \text{cov}(v_t, w_t) \\ &= \text{var}(z_{t+T} - z_t) + \text{var}(q_{t+T} - q_t) \\ &\quad + 2(1 + \phi + \phi^2 + \dots + \phi^{T-1}) \sigma_w^2 \rho \lambda^{1/2} \end{aligned} \quad (\text{A1-h})$$

Using the above characteristics and some calculations, the auto-regression coefficient  $\beta(T)$  is derived as

$$\beta(T) = \frac{\text{cov}(r(t, t+T), r(t-T, t))}{\text{var}(r(t-T, t))} = \frac{-(1-\phi^T)\lambda + (1-\phi^T)(1+\phi)\rho\lambda^{1/2}}{2\lambda + T \cdot \frac{1-\phi^2}{1-\phi^T} + 2(1+\phi)\rho\lambda^{1/2}}. \quad (\text{A2})$$

## 〈Appendix 2〉 Relationship of the Sign of $\beta(T)$ with $T$ and $\phi$ when $\phi > \theta$

As we mentioned in section 2, when  $\phi > \theta$ , the sign of  $\beta(T)$  depends on the relative magnitude between (a) and (b) in equation (12). Part (b) can be written as the following quadratic equation with respect to  $\phi$  :

$$2 \cdot \frac{(\phi-\theta)(1-\phi\theta)}{(1-\theta)^2} = \frac{2}{(1-\theta)^2} [\theta\phi^2 - (1+\theta^2)\phi + \theta]. \quad (\text{A3})$$

In this equation, the maximum can be easily derived using the characteristic of a quadratic equation. Let  $\theta$  be zero. Then this equation becomes  $2\phi$  and the maximum converges to two as  $\phi \rightarrow 1$ . Now let  $\theta$  not be zero. Although the extreme point of this equation is  $\phi = (1+\theta^2)/2\theta$ , this point is located out of the condition,  $-1 < \theta < \phi < 1$ . If  $\theta > 0$ , this extreme point is the global maximum and always located on the right side of  $\phi$  since  $(1+\theta^2)/2\theta > 1$ . Therefore, the maximum of part (b) converges to two as  $\phi \rightarrow 1$ . In the same manner, the maximum of part (b) converges to two as  $\phi \rightarrow 1$  if  $\theta < 0$ . Hence, the maximum of part (b) converges to two regardless of the sign of  $\theta$ .

From this result, the minimum period to always make  $\beta(T)$  positive can be obtained. Since the maximum of part (b) is two, the denominator of equation (12) satisfies the following inequality :

$$T \cdot \frac{1-\phi^2}{1-\phi^T} - 2 \cdot \frac{(\phi-\theta)(1-\phi\theta)}{(1-\theta)^2} > T \cdot \frac{1-\phi^2}{1-\phi^T} - 2. \quad (\text{A4})$$

Thus, the minimum period to always make  $\beta(T)$  positive is given as follows :

$$T \cdot \frac{1-\phi^2}{1-\phi^T} - 2 > 0. \quad (\text{A5})$$

However, since the closed solution of equation (A5) is complicated, we compute the minimum period numerically and report the results in table A1.

〈Table A1〉 Relationships of the Sign of  $\beta(T)$  with  $T$  and  $\phi$  if  $\phi > \theta$ 

$\phi$	$T^*$	The Sign of $\beta(T)$ when $T$ is odd	The Sign of $\beta(T)$ when $T$ is even
$\geq -0.5$	$\leq 1$	either (+) or (-)	(+)
	$> 1$	(+)	(+)
$= -0.6$	$\leq 4$	either (+) or (-)	(+)
	$> 4$	(+)	(+)
$= -0.7$	$\leq 4$	either (+) or (-)	(+)
	$> 4$	(+)	(+)
$= -0.8$	$\leq 6$	either (+) or (-)	(+)
	$> 6$	(+)	(+)
$= -0.9$	$\leq 14$	either (+) or (-)	(+)
	$> 14$	(+)	(+)
$= -0.95$	$\leq 26$	either (+) or (-)	(+)
	$> 26$	(+)	(+)
$= -0.96$	$\leq 32$	either (+) or (-)	(+)
	$> 32$	(+)	(+)
$= -0.97$	$\leq 42$	either (+) or (-)	(+)
	$> 42$	(+)	(+)
$= -0.98$	$\leq 64$	either (+) or (-)	(+)
	$> 64$	(+)	(+)
$= -0.99$	$\leq 128$	either (+) or (-)	(+)
	$> 128$	(+)	(+)
$= -0.999$	$\leq 1278$	either (+) or (-)	(+)
	$> 1278$	(+)	(+)

Note) \* These results mean that the minimum value of  $T$  to always make  $\beta(T)$  positive cannot exceed the reported value.