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GLOBAL EXPONENTIAL STABILITY OF BAM FUZZY CELLULAR NEURAL NETWORKS WITH DISTRIBUTED DELAYS AND IMPULSES†

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Abstract. In this paper, a class of bi-directional associative memory (BAM) fuzzy cellular neural networks with distributed delays and impulses is formulated and investigated. By employing an integro-differential inequality with impulsive initial conditions and the topological degree theory, some sufficient conditions ensuring the existence and global exponential stability of equilibrium point for impulsive BAM fuzzy cellular neural networks with distributed delays are obtained. In particular, the estimate of the exponential convergence rate is also provided, which depends on the delay kernel functions and system parameters. It is believed that these results are significant and useful for the design and applications of BAM fuzzy cellular neural networks. An example is given to show the effectiveness of the results obtained here.

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1. Introduction

The bi-directional associative memory (BAM) neural network was first introduced by Kosto [17]. It is an important model with the ability of information memory and information association, which is crucial for application in pattern recognition, solving optimization problems and automatic control engineering [17, 18]. In such applications, the stability of networks plays an important role, it is of significance and necessary to investigate the stability. In both biological and artificial neural networks, the delays arise because of the processing of information. Time delays may lead to oscillation, divergence, or instability which

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may be harmful to a system. Therefore, study of neural dynamics with consideration of the delayed problem becomes extremely important to manufacture high quality neural networks. Recently, BAM neural networks with various delays have been extensively studied both in theory and applications, for example, see [19-26] and references therein.

Since Yang et al. proposed the fuzzy cellular neural networks (FCNNs) [1], the dynamic analysis on FCNNs with various delays and BAM fuzzy neural networks with transmission delays has been the highlight in the neural network field, for example, see [2-16] and references therein. On the other hand, besides delay effect, impulsive effect likewise exists in a wide variety of evolutionary processes in which states are changed abruptly at certain moments of time, involving such fields as medicine and biology, economics, mechanics, electronics and telecommunications, etc. As artificial electronic systems, neural networks such as cellular neural networks, bidirectional neural networks and recurrent neural networks often are subject to impulsive perturbations which can affect dynamical behaviors of the systems just as time delays. Therefore, it is necessary to consider both impulsive effect and delay effect on the stability of neural networks, for example, see [6, 8, 9, 21-26]. To the best of our knowledge, few authors have considered BAM fuzzy cellular neural networks with distributed delays and impulses.

Motivated by the above discussions, the objective of this paper is to formulate and study BAM fuzzy cellular neural networks with distributed delays and impulses. Under quite general conditions, some sufficient conditions ensuring the existence and global exponential stability of equilibrium point are obtained by the topological degree theory and the integro-differential inequality with impulsive initial conditions and analysis technique.

The paper is organized as follows. In Section 2, the new neural network model is formulated, and the necessary knowledge is provided. Main results are presented in Section 3. In Section 4, an example is given to show the effectiveness of the results obtained here. Finally, we give the conclusion in Section 5.

2. Model description and Preliminaries

In this section, we will consider the model of BAM fuzzy neural networks with distributed delays and impulses, it is described by the following functional integro-differential equations:

$$
\begin{cases}
\dot{x}_i(t) = -a_i x_i(t) + \sum\limits_{j=1}^{m} a_{ij} g_j(y_j(t)) + \sum\limits_{j=1}^{m} \tilde{a}_{ij} v_j + I_i \\
\qquad + \bigwedge\limits_{j=1}^{m} \alpha_{ij} \int\limits_0^{+\infty} K_{ij}(s) g_j(y_j(t-s))ds \\
\qquad + \bigvee\limits_{j=1}^{m} \tilde{\alpha}_{ij} \int\limits_0^{+\infty} K_{ij}(s) g_j(y_j(t-s))ds + \bigwedge\limits_{j=1}^{m} T_{ij} v_j + \bigvee\limits_{j=1}^{m} H_{ij} v_j, \ t \neq t_k \\
x_i(t^+) = x_i(t^-) + P_{ik}(x_i(t^-)), \ t = t_k, \ k \in N =: \{1, 2, \cdots\},
\end{cases}
\qquad (1-1)
$$

$$\begin{cases} \dot{y}_j(t) = -b_j y_j(t) + \sum\limits_{i=1}^{n} b_{ji} f_i(x_i(t)) + \sum\limits_{i=1}^{n} \tilde{b}_{ji} u_i + J_j \\ \qquad + \bigwedge\limits_{i=1}^{n} \beta_{ji} \int\limits_{0}^{+\infty} \bar{K}_{ji}(s) f_i(x_i(t-s))ds \\ \qquad + \bigvee\limits_{i=1}^{n} \tilde{\beta}_{ji} \int\limits_{0}^{+\infty} \bar{K}_{ij}(s) f_i(x_i(t-s))ds + \bigwedge\limits_{i=1}^{n} \bar{T}_{ji} u_i + \bigvee\limits_{i=1}^{n} \bar{H}_{ji} u_i, \ t \neq t_k \\ y_j(t^+) = y_j(t^-) + Q_{jk}(y_j(t^-)), \ t = t_k, \ k \in N =: \{1, 2, \cdots\}, \end{cases} \tag{1 − 2}$$

for $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$, $t > 0$, where $x_i(t)$ and $y_j(t)$ are the states of the ith neuron and the jth neuron at time t, respectively; f_i and g_j denote the signal functions of the ith neuron and the jth neuron at time t, respectively; u_i, v_j and I_i, J_j denote inputs and bias of the ith neuron and the jth neuron, respectively; $a_i > 0, b_j > 0, a_{ij}, \tilde{a}_{ij}, \alpha_{ij}, \tilde{\alpha}_{ij}, b_{ji}, \tilde{b}_{ji}, \beta_{ji}, \tilde{\beta}_{ji}$ are constants, a_i and b_j represent the rate with which the ith neuron and the jth neuron will reset their potential to the resting state in isolation when disconnected from the networks and external inputs, respectively; a_{ij}, b_{ji} and $\tilde{a}_{ij}, \tilde{b}_{ji}$ denote connection weights of feedback template and feedforward template, respectively; α_{ij}, β_{ji} and $\tilde{\alpha}_{ij}, \tilde{\beta}_{ji}$ denote connection weights of the distributed fuzzy feedback MIN template and the distributed fuzzy feedback MAX template, respectively; $T_{ij}, \bar{T}_{ji}$ and $H_{ij}, \tilde{H}_{ji}$ are elements of fuzzy feedforward MIN template and fuzzy feedforward MAX template, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operations, respectively; $K_{ij}(s)$ and $\bar{K}_{ji}(s)$ correspond to the delay kernel functions, respectively. t_k is called impulsive moment, and satisfies $0 < t_1 < t_2 < \cdots$, $\lim\limits_{k \to +\infty} t_k = +\infty$; $x_i(t_k^-)$ and $x_i(t_k^+)$ denote the left-hand and right-hand limits at t_k, respectively; P_{ik} and Q_{jk} show impulsive perturbations of the ith neuron and jth neuron at time t_k, respectively. We always assume $x_i(t_k^+) = x_i(t_k)$ and $y_j(t_k^+) = y_j(t_k)$, $k \in N$. The initial conditions are given by

$$\begin{cases} x_i(t) = \phi_i(t), & -\infty \leq t \leq 0, \\ y_j(t) = \varphi_j(t), & -\infty \leq t \leq 0, \end{cases}$$

where $\phi_i(t), \varphi_j(t)$ $(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$ are bounded and continuous on $(-\infty, 0]$, respectively.

If the impulsive operators $P_{ik}(x_i) = 0$, $Q_{jk}(y_j) = 0$, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m, k \in N$, then system (1) may reduce to the following model:

$$\begin{aligned} \dot{x}_i(t) = -a_i x_i(t) &+ \sum_{j=1}^{m} a_{ij} g_j(y_j(t)) + \sum_{j=1}^{m} \tilde{a}_{ij} v_j \\ &+ I_i + \bigwedge_{j=1}^{m} \alpha_{ij} \int_{0}^{+\infty} K_{ij}(s) g_j(y_j(t-s))ds \\ &+ \bigvee_{j=1}^{m} \tilde{\alpha}_{ij} \int_{0}^{+\infty} K_{ij}(s) g_j(y_j(t-s))ds + \bigwedge_{j=1}^{m} T_{ij} v_j + \bigvee_{j=1}^{m} H_{ij} v_j, \end{aligned} \tag{2 − 1}$$

$$\begin{aligned}\dot{y}_j(t) = &-b_j y_j(t) + \sum_{i=1}^{n} b_{ji} f_i(x_i(t)) + \sum_{i=1}^{n} \tilde{b}_{ji} u_i \\ &+J_j + \bigwedge_{i=1}^{n} \beta_{ji} \int_0^{+\infty} \bar{K}_{ji}(s) f_i(x_i(t-s))ds \\ &+ \bigvee_{i=1}^{n} \tilde{\beta}_{ji} \int_0^{+\infty} \bar{K}_{ij}(s) f_i(x_i(t-s))ds + \bigwedge_{i=1}^{n} \bar{T}_{ji} u_i + \bigvee_{i=1}^{n} \bar{H}_{ji} u_i.\end{aligned} \qquad (2-2)$$

System (2) is called the continuous system of model (1).

Throughout this paper, we make the following assumptions:

(H1) For neuron activation functions f_i and g_j $(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$, there exist two positive diagonal matrices $F = diag(F_1, F_2, \cdots, F_n)$ and $G = diag(G_1, G_2, \cdots, G_m)$ such that

$$F_i = \sup_{x \neq y} \left| \frac{f_i(x) - f_i(y)}{x - y} \right|, \quad G_j = \sup_{x \neq y} \left| \frac{g_j(x) - g_j(y)}{x - y} \right|$$

for all $x, y \in R$ $(x \neq y)$.

(H2) The delay kernels $K_{ij} : [0, +\infty) \to R$ and $\bar{K}_{ji} : [0, +\infty) \to R$ are real-valued piecewise continuous, and there exists $\delta > 0$ such that

$$k_{ij}(\lambda) = \int_0^{+\infty} e^{\lambda s} |K_{ij}(s)| ds, \qquad \bar{k}_{ji}(\lambda) = \int_0^{+\infty} e^{\lambda s} |\bar{K}_{ji}(s)| ds$$

are continuous for $\lambda \in [0, \delta)$, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$.

(H3) Let $\bar{P}_k(x) = x + P_k(x)$ and $\bar{Q}_k(y) = y + Q_k(y)$ be Lipschitz continuous in R^n and R^m, respectively, that is, there exist nonnegative diagnose matrices $\Gamma_k = diag(\gamma_{1k}, \gamma_{2k}, \cdots, \gamma_{nk})$ and $\bar{\Gamma}_k = diag(\bar{\gamma}_{1k}, \bar{\gamma}_{2k}, \cdots, \bar{\gamma}_{mk})$ such that

$$|\bar{P}_k(x) - \bar{P}_k(y)| \leq \Gamma_k |x - y|, \quad for\ all\ x, y \in R^n, \quad k \in N,$$

$$|\bar{Q}_k(u) - \bar{Q}_k(v)| \leq \bar{\Gamma}_k |u - v|, \quad for\ all\ u, v \in R^m, \quad k \in N,$$

where

$$\begin{aligned}&\bar{P}_k(x) = (\bar{P}_{1k}(x_1), \bar{P}_{2k}(x_2), \cdots, \bar{P}_{nk}(x_n))^T, \\ &\bar{Q}_k(x) = (\bar{Q}_{1k}(y_1), \bar{Q}_{2k}(y_2), \cdots, \bar{Q}_{mk}(y_m))^T, \\ &P_k(x) = (P_{1k}(x_1), P_{2k}(x_2), \cdots, P_{nk}(x_n))^T, \\ &Q_k(y) = (Q_{1k}(y_1), Q_{2k}(y_2), \cdots, Q_{mk}(y_m))^T.\end{aligned}$$

To begin with, we introduce some notation and recall some basic definitions.

$PC[J, R^l] =: \{z(t) : J \to R^l | z(t)$ is continuous at $t \neq t_k$, $z(t_k^+) = z(t_k)$ and $z(t_k^-)$ exists for $t,\ t_k \in J,\ k \in N\}$, where $J \subset R$ is an interval, $l \in N$.

$PC_l =: \{\psi : (-\infty, 0] \to R^l |\ \psi(s)$ is bounded, and $\psi(s^+) = \psi(s)$ for $s \in (-\infty, 0)$, $\psi(s^-)$ exists for $s \in (-\infty, 0]$, $\phi(s^-) = \phi(s)$ for all but at most a finite number of points $s \in (-\infty, 0]\}$.

Definition 1. *A function* $(x, y)^T : (-\infty, +\infty) \to R^{n+m}$ *is said to be the special solution of system* (1) *with initial condition*

$$x(s) = \phi(s), \ \ y(s) = \varphi(s) \quad s \in (-\infty, 0],$$

if the following two conditions are satisfied

(i) $(x, y)^T$ *is piecewise continuous with first kind discontinuity at the points* $t_k,\ k = 1, 2, \cdots$. *Moreover,* $(x, y)^T$ *is right continuous at each discontinuity point.*

(ii) $(x, y)^T$ *satisfies model* (1) *for* $t \geq 0$, *and* $x(s) = \phi(s)$, $y(s) = \varphi(s)$ *for* $s \in (-\infty, 0]$.

Especially, a point $(x^*, y^*)^T \in R^{n+m}$ *is called an equilibrium point of model* (1), *if* $(x(t), y(t))^T = (x^*, y^*)^T$ *is a solution of* (1).

Throughout this paper, we always assume that the impulsive jumps P_k and Q_k satisfy (referring to [22-26])

$$P_k(x^*) = 0 \quad and \quad Q_k(y^*) = 0, \quad k \in N,$$

i.e.,

$$\bar{P}_k(x^*) = x^* \quad and \quad \bar{Q}(y^*) = y^*, \quad k \in N, \tag{3}$$

where $(x^*, y^*)^T$ is the equilibrium point of continuous systems (2). That is, if $(x^*, y^*)^T$ is an equilibrium point of continuous system (2), then $(x^*, y^*)^T$ is also the equilibrium of impulsive system (1).

Definition 2. *The equilibrium point* $(x^*, y^*)^T$ *of model* (1) *is said to be globally exponentially stable, if there exist constants* $\lambda > 0$ *and* $M \geq 1$ *such that*

$$\|x(t) - x^*\| + \|y(t) - y^*\| \leq M(\| \phi - x^* \| + \| \varphi - y^* \|)e^{-\lambda t}$$

for all $t \geq 0$, *where* $(x(t), y(t))^T$ *is any solution of system* (1) *with initial value* $(\phi(s), \varphi(s))^T$ *and*

$$\|x(t) - x^*\| = \sum_{i=1}^{n} |x_i(t) - x_i^*|, \quad \|y(t) - y^*\| = \sum_{j=1}^{m} |y_j(t) - y_j^*|,$$

$$\|\phi - x^*\| = \sup_{-\infty < s \leq 0} \sum_{i=1}^{n} |\phi_i(s) - x_i^*|, \quad \|\varphi - y^*\| = \sup_{-\infty < s \leq 0} \sum_{j=1}^{m} |\varphi_j(s) - y_j^*|.$$

Lemma 1. ([2]) *For any positive integer* n, *let* $h_j : R \to R$ *be a function* $(j = 1, 2, \cdots, n)$, *then we have*

$$\Big| \bigwedge_{j=1}^{n} \alpha_j h_j(u_j) - \bigwedge_{j=1}^{n} \alpha_j h_j(v_j) \Big| \leq \sum_{j=1}^{n} \Big|\alpha_j\Big| \cdot \Big|h_j(u_j) - h_j(v_j)\Big|,$$

$$\Big| \bigvee_{j=1}^{n} \alpha_j h_j(u_j) - \bigvee_{j=1}^{n} \alpha_j h_j(v_j) \Big| \leq \sum_{j=1}^{n} \Big|\alpha_j\Big| \cdot \Big|h_j(u_j) - h_j(v_j)\Big|$$

for all $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n)^T$, $u = (u_1, u_2, \cdots, u_n)^T$, $v = (v_1, v_2, \cdots, v_n)^T \in R^n$.

Lemma 2. *Let* $a < b \le +\infty$, *and* $(u(t), v(t))^T$ $(u(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T \in PC[[a,b), R^n]$, $v(t) = (v_1(t), v_2(t), \cdots, v_m(t))^T \in PC[[a,b), R^m])$ *satisfies the following integro-differential inequalities with the initial condition* $u(a+s) \in PC_n$ *and* $v(a+s) \in PC_m$:

$$\begin{cases} D^+u_i(t) \le -r_iu_i(t) + \sum\limits_{j=1}^{m} p_{ij}v_j(t) + \sum\limits_{j=1}^{m} q_{ij}\int_0^{+\infty} |K_{ij}(s)|v_j(t-s)ds, \\ D^+v_j(t) \le -\bar{r}_jv_j(t) + \sum\limits_{i=1}^{n} \bar{p}_{ji}u_i(t) + \sum\limits_{i=1}^{n} \bar{q}_{ji}\int_0^{+\infty} |\bar{K}_{ji}(s)|u_i(t-s)ds, \end{cases}$$

for $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$, *where* $r_i > 0$, $p_{ij} > 0$, $q_{ij} > 0$, $\bar{r}_j > 0$, $\bar{p}_{ji} > 0$, $\bar{q}_{ji} > 0$, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$. *If the initial conditions satisfies*

$$\begin{cases} u(s) \le \kappa\xi e^{-\lambda(s-a)}, & s \in (-\infty, a], \\ v(s) \le \kappa\eta e^{-\lambda(s-a)}, & s \in (-\infty, a], \end{cases}$$

where $\lambda > 0$, $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0$ *and* $\eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ *satisfy*

$$\begin{cases} (\lambda - r_i)\xi_i + \sum\limits_{j=1}^{m} (p_{ij} + q_{ij}k_{ij}(\lambda))\eta_j < 0, & i = 1, 2, \cdots, n, \\ (\lambda - \bar{r}_j)\eta_j + \sum\limits_{i=1}^{n} (\bar{p}_{ji} + \bar{q}_{ji}\bar{k}_{ji}(\lambda))\xi_i < 0, & j = 1, 2, \cdots, m. \end{cases}$$

Then

$$\begin{cases} u(t) \le \kappa\xi e^{-\lambda(t-a)}, & t \in [a, b), \\ v(t) \le \kappa\eta e^{-\lambda(t-a)}, & t \in [a, b). \end{cases}$$

The proof of Lemma 2 is perfectly similar to the proof of Lemma 2 in [26]. Here, we omit it.

3. Main results

Theorem 1. *Under assumptions* (H1)-(H3), *if the following conditions hold:*

(C1) *There exist vectors* $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0, \eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ *and positive number* $\lambda > 0$ *such that*

$$\begin{cases} (\lambda - a_i)\xi_i + \sum\limits_{j=1}^{m} \Big[|a_{ij}| + (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|)k_{ij}(\lambda)\Big]G_j\eta_j < 0, & i = 1, 2, \cdots, n, \\ (\lambda - b_j)\eta_j + \sum\limits_{i=1}^{n} \Big[|b_{ji}| + (|\beta_{ji}| + |\tilde{\beta}_{ji}|)\bar{k}_{ji}(\lambda)\Big]F_i\xi_i < 0. & j = 1, 2, \cdots, m; \end{cases}$$

(C2) $\mu = \sup\limits_{k\in N} \left\{\frac{\ln \mu_k}{t_k - t_{k-1}}\right\} < \lambda$, *where* $\mu_k = \max\limits_{1\le i\le n, 1\le j\le m}\{1, \gamma_{ik}, \bar{\gamma}_{jk}\}$, $k \in N$;

then system (1) *has exactly one globally exponentially stable equilibrium point, and its exponential convergence rate equals* $\lambda - \mu$.

Proof. Let $h(x_1, \cdots, x_n, y_1, \cdots, y_m) = (h_1, \cdots, h_n, \overline{h}_1, \cdots, \overline{h}_m)^T$, where

$$\begin{cases} h_i = a_ix_i - \sum\limits_{j=1}^{m} a_{ij}g_j(y_j) - \bigwedge\limits_{j=1}^{m} \alpha_{ij}k_{ij}(0)g_j(y_j) - \bigvee\limits_{j=1}^{m} \tilde{\alpha}_{ij}k_{ij}(0)g_j(y_j) - \tilde{I}_i, \\ \overline{h}_j = b_jy_j - \sum\limits_{i=1}^{n} b_{ji}f_i(x_i) - \bigwedge\limits_{i=1}^{n} \beta_{ji}\bar{k}_{ji}(0)f_i(x_i) - \bigvee\limits_{i=1}^{n} \tilde{\beta}_{ji}\bar{k}_{ji}(0)f_i(x_i) - \tilde{J}_j \end{cases}$$

for $i = 1, 2, \cdots, n$; $j = 1, 2, \cdots, m$, and

$$\begin{cases} \tilde{I}_i = \sum_{j=1}^{m} \tilde{a}_{ij} v_j + I_i + \bigwedge_{j=1}^{m} T_{ij} v_j + \bigvee_{j=1}^{m} H_{ij} v_j, \quad i = 1, 2, \cdots, n, \\ \tilde{J}_j = \sum_{i=1}^{n} \tilde{b}_{ji} u_i + J_j + \bigwedge_{i=1}^{n} \bar{T}_{ji} u_i + \bigvee_{i=1}^{n} \bar{H}_{ji} u_i, \quad j = 1, 2, \cdots, m. \end{cases}$$

Obviously, from assumption (H2), the equilibrium points of model (2) are the solutions of system of equations:

$$\begin{cases} h_i = 0, \quad i = 1, 2, \cdots, n, \\ \bar{h}_j = 0, \quad j = 1, 2, \cdots, m. \end{cases} \tag{4}$$

Define the following homotopic mapping:

$$H(x_1, \cdots, x_n, y_1, \cdots, y_m) = \theta h(x_1, \cdots, x_n, y_1, \cdots, y_m) + (1 - \theta)(x_1, \cdots, x_n, y_1, \cdots, y_m)^T,$$

where $\theta \in [0, 1]$. Let $H_k(k = 1, 2, \cdots, n + m)$ denote the kth component of $H(x_1, \cdots, x_n, y_1, \cdots, y_m)$, then from assumption (H1) and Lemma 2, we have

$$\begin{cases} |H_i| \geq [1 + \theta(a_i - 1)]|x_i| - \theta \sum_{j=1}^{m} \Big[|a_{ij}| + (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|)k_{ij}(0)\Big] G_j |y_j| \\ \qquad - \theta \sum_{j=1}^{m} \Big[|a_{ij}| + (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|)k_{ij}(0)\Big] |g_j(0)| - \theta |\tilde{I}_i|, \\ |H_{n+j}| \geq [1 + \theta(b_j - 1)]|y_j| - \theta \sum_{i=1}^{n} \Big[|b_{ji}| + (|\beta_{ji}| + |\tilde{\beta}_{ji}|)\bar{k}_{ji}(0)\Big] F_i |x_i| \\ \qquad - \theta \sum_{i=1}^{n} \Big[|b_{ji}| + (|\beta_{ji}| + |\tilde{\beta}_{ji}|)\bar{k}_{ji}(0)\Big] |f_i(0)| - \theta |\tilde{J}_j| \end{cases} \tag{5}$$

for $i \in 1, 2, \cdots, n, j \in 1, 2, \cdots, m$. Denote

$\bar{H} = (|H_1|, |H_2|, \cdots, |H_{n+m}|)^T, \qquad z = (|x_1|, \cdots, |x_n|, |y_1|, \cdots, |y_m|)^T$
$C = diag(a_1, \cdots, a_n, b_1, \cdots, b_m), \qquad L = diag(F_1, \cdots, F_n, G_1, \cdots, G_m),$
$P = (|\tilde{I}_1|, \cdots, |\tilde{I}_n, |, |\tilde{J}_1|, \cdots, |\tilde{J}_m|)^T, \qquad Q = (|f_1(0)|, \cdots, |f_n(0)|, |g_1(0)|, \cdots, |g_m(0)|)^T,$
$A = \Big(|a_{ij}| + (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|)k_{ij}(0)\Big)_{n\times m}, \quad B = \Big(|b_{ji}| + (|\beta_{ji}| + |\tilde{\beta}_{ji}|)\bar{k}_{ji}(0)\Big)_{m\times n},$
$T = \begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}, \quad \omega = (\xi_1, \cdots, \xi_n, \eta_1, \cdots, \eta_m)^T > 0.$

Then the matrix form of (5) is

$$\bar{H} \geq [E + \theta(C - E)]z - \theta T L z - \theta(P + TQ) = (1 - \theta)z + \theta[(C - TL)z - (P + TQ)].$$

Since condition (C1) holds, and $k_{ij}(\lambda)$, $\bar{k}_{ji}(\lambda)$ are continuous on $[0,\delta)$, when $\lambda=0$ in (C1), we obtain

$$\begin{cases} -a_i\xi_i + \sum\limits_{j=1}^{m}\Big[|a_{ij}| + (|\alpha_{ij}|+|\tilde{\alpha}_{ij}|)k_{ij}(0)\Big]G_j\eta_j < 0,\ i=1,2,\cdots,n, \\ -b_j\eta_j + \sum\limits_{i=1}^{n}\Big[|b_{ji}| + (|\beta_{ji}|+|\tilde{\beta}_{ji}|)\bar{k}_{ji}(0)\Big]F_i\xi_i < 0,\ j=1,2,\cdots,m. \end{cases}$$

or in matrix form,

$$(-C+TL)\omega < 0. \tag{6}$$

It is easy to know from (6) that $C-TL$ is a non-singular M-matrix, so $(C-TL)^{-1}$ is a non-negative matrix. Let

$$\Gamma = \Big\{z=(x_1,\cdots,x_n,y_1,\cdots,y_m)^T \ \Big|\ z \le \omega + (C-TL)^{-1}(P+TQ)\Big\},$$

then Γ is non-empty, and from (5), we know for any $z=(x_1,\cdots,x_n,y_1,\cdots,y_m)^T \in \partial\Gamma$,

$$\begin{aligned} \bar{H} &\ge (1-\theta)z + \theta(C-TL)[z-(C-TL)^{-1}(P+TQ)] \\ &= (1-\theta)[\omega+(C-TL)^{-1}(P+TQ)] + \theta(C-TL)\omega > 0, \quad \theta\in[0,1]. \end{aligned}$$

Therefore, for any $(x_1,\cdots,x_n,y_1,\cdots,y_m)^T \in \partial\Gamma$ and $\theta\in[0,1]$, we have $H \ne 0$. From homotopy invariance theorem [27], we get

$$deg(h,\Gamma,0) = deg(H,\Gamma,0) = 1,$$

by topological degree theory, we know that (4) has at least one solution in Γ. That is, system (2) has at least an equilibrium point. This implies that system (1) has also at least an equilibrium point.

Let $(x_1^*,\cdots,x_n^*,y_1^*,\cdots,y_m^*)^T$ be an equilibrium point of system (1), $(x_1(t),\cdots, x_n(t),y_1(t),\cdots,y_m(t))^T$ is any solution of system (1) with the initial conditions $(\phi(s),\varphi(s))^T$. Now let $\bar{x}_i(t)=x_i(t)-x_i^*$, $i=1,2,\cdots,n$, $\bar{y}_j(t)=y_j(t)-y_j^*$, $j=1,2,\cdots,m$. It is easy to see that system (1) can be transformed into the following system

$$\begin{cases} \dot{\bar{x}}_i(t) = -a_i\bar{x}_i(t) + \sum\limits_{j=1}^{m} a_{ij}\Big(g_j(\bar{y}_j(t)+y_j^*) - g_j(y_j^*)\Big) - \bigwedge\limits_{j=1}^{m}\alpha_{ij}\int\limits_0^{+\infty}K_{ji}(s)g_j(y_j^*)ds \\ \qquad + \bigwedge\limits_{j=1}^{m}\alpha_{ij}\int\limits_0^{+\infty}K_{ij}(s)g_j(\bar{y}_j(t-s)+y_j^*)ds - \bigvee\limits_{j=1}^{m}\tilde{\alpha}_{ij}\int\limits_0^{+\infty}K_{ij}(s)g_j(y_j^*)ds \\ \qquad + \bigvee\limits_{j=1}^{m}\tilde{\alpha}_{ij}\int\limits_0^{+\infty}K_{ij}(s)g_j(\bar{y}_j(t-s)+y_j^*)ds,\ t\ne t_k, \\ \bar{x}_i(t_k^+) = \tilde{P}_{ik}(\bar{x}_i(t_k^-)),\ k\in N \end{cases} \tag{7-1}$$

$$\begin{cases} \dot{\bar{y}}_j(t) = -b_j\bar{y}_j(t) + \sum\limits_{i=1}^{n} b_{ji}\Big(f_i(\bar{x}_i(t)+x_i^*) - f_i(x_i^*)\Big) - \bigwedge\limits_{i=1}^{n} \beta_{ji} \int\limits_0^{+\infty} \bar{K}_{ji}(s)f_i(x_i^*)ds \\ \qquad + \bigwedge\limits_{i=1}^{n} \beta_{ji} \int\limits_0^{+\infty} \bar{K}_{ji}(s)f_i(\bar{x}_i(t-s)+x_i^*)ds - \bigvee\limits_{i=1}^{n} \tilde{\beta}_{ji} \int\limits_0^{+\infty} \bar{K}_{ij}(s)f_i(x_i^*)ds \\ \qquad + \bigvee\limits_{i=1}^{n} \tilde{\beta}_{ji} \int\limits_0^{+\infty} \bar{K}_{ij}(s)f_i(\bar{x}_i(t-s)+x_i^*)ds,\ t \neq t_k \\ \bar{y}_j(t_k^+) = \tilde{Q}_{jk}(y_j(t_k^-)),\ k \in N, \end{cases} \tag{7 − 2}$$

where $\tilde{P}_{ik}(\bar{x}_i(t)) = \bar{P}_{ik}(\bar{x}_i(t)+x_i^*) - \bar{P}_{ik}(x_i^*)$, $\tilde{Q}_{jk}(\bar{y}_j(t)) = \bar{Q}_{jk}(\bar{y}_j(t)+y_j^*) - \bar{Q}_{jk}(y_j^*)$, and the initial conditions of (7) are

$$\begin{cases} \tilde{\phi}(s) = x(s) - x^* = \phi(s) - x^*, & s \in (-\infty, 0], \\ \tilde{\varphi}(s) = y(s) - y^* = \varphi(s) - y^*, & s \in (-\infty, 0]. \end{cases}$$

From (H1) and Lemma 2, we calculate the upper right derivative along the solutions of first equation and third equation of (7), we can obtain

$$\begin{cases} D^+|\bar{x}_i(t)| \le -a_i|\bar{x}_i(t)| + \sum\limits_{j=1}^{m} |a_{ij}|G_j|\bar{y}_j(t)| \\ \qquad\qquad + \sum\limits_{j=1}^{m} (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|)G_j \int_0^{+\infty} |K_{ij}(s)||\bar{y}_j(t-s)|ds, \\ D^+|\bar{y}_j(t)| \le -b_j|\bar{y}_j(t)| + \sum\limits_{i=1}^{n} |b_{ji}|F_i|\bar{x}_i(t)| \\ \qquad\qquad + \sum\limits_{i=1}^{n} (|\beta_{ji}| + |\tilde{\beta}_{ji}|)F_i \int_0^{+\infty} |\bar{K}_{ji}(s)||\bar{x}_i(t-s)|ds \end{cases}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.

Let $u_i(t) = |\bar{x}_i(t)|$, $v_j(t) = |\bar{y}_j(t)|$, $r_i = a_i$, $p_{ij} = |a_{ij}|G_j$, $q_{ij} = (|\alpha_{ij}| + |\tilde{\alpha}_{ij}|)G_j$, $\bar{r}_j = b_j$, $\bar{p}_{ji} = |b_{ji}|F_i$, $\bar{q}_{ji} = (|\beta_{ji}| + |\tilde{\beta}_{ji}|)F_i\,(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$, then we have

$$\begin{cases} D^+u_i(t) \le -r_iu_i(t) + \sum\limits_{j=1}^{m} p_{ij}v_j(t) + \sum\limits_{j=1}^{m} q_{ij} \int_0^{+\infty} |K_{ij}(s)|v_j(t-s)ds, \\ D^+v_j(t) \le -\bar{r}_jv_j(t) + \sum\limits_{i=1}^{n} \bar{p}_{ji}u_i(t) + \sum\limits_{i=1}^{n} \bar{q}_{ji} \int_0^{+\infty} |\bar{K}_{ji}(s)|u_i(t-s)ds, \end{cases} \tag{8}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, and from (C1), there exist vectors $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0, \eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ and positive number $\lambda > 0$ such that

$$\begin{cases} (\lambda - r_i)\xi_i + \sum\limits_{j=1}^{m} \Big[p_{ij} + q_{ij}k_{ij}(\lambda)\Big]G_j\eta_j < 0, & i = 1, 2, \cdots, n, \\ (\lambda - \bar{r}_j)\eta_j + \sum\limits_{i=1}^{n} \Big[\bar{p}_{ji} + \bar{q}_{ji}\bar{k}_{ji}(\lambda)\Big]F_i\xi_i < 0. & j = 1, 2, \cdots, m. \end{cases} \tag{9}$$

Taking $\frac{\|\tilde{\phi}\|+\|\tilde{\varphi}\|}{\min\limits_{1\le i\le n,1\le j\le m}\{\xi_i,\eta_j\}}$, it is easy to prove that

$$\begin{cases} u(t)\le \kappa\xi e^{-\lambda t}, & -\infty\le t\le 0=t_0,\\ v(t)\le \kappa\eta e^{-\lambda t}, & -\infty\le t\le 0=t_0. \end{cases} \tag{10}$$

From Lemma 2, we obtain that

$$\begin{cases} u(t)\le \kappa\xi e^{-\lambda t}, & t_0\le t<t_1,\\ v(t)\le \kappa\eta e^{-\lambda t}, & t_0\le t<t_1. \end{cases} \tag{11}$$

Suppose that for $l\le k$, the inequalities

$$\begin{cases} u(t)\le \kappa\mu_0\mu_1\cdots\mu_{l-1}\xi e^{-\lambda t}, & t_{l-1}\le t<t_l,\\ v(t)\le \kappa\mu_0\mu_1\cdots\mu_{l-1}\eta e^{-\lambda t}, & t_{l-1}\le t<t_l. \end{cases} \tag{12}$$

hold, where $\mu_0=1$. When $l=k+1$, we note that

$$\begin{aligned} u(t_k)=|\tilde{P}_k(u(t_k^-))|\le \Gamma_k u(t_k^-)\le \kappa\mu_0\mu_1\cdots\mu_{k-1}\Gamma_k\xi\lim_{t\to t_k^-}e^{-\lambda t}\\ \le \kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\xi e^{-\lambda t_k}, \end{aligned} \tag{13}$$

and

$$\begin{aligned} v(t_k)=|\tilde{Q}_k(v(t_k^-))|\le \bar{\Gamma}_k v(t_k^-)\le \kappa\mu_0\mu_1\cdots\mu_{k-1}\bar{\Gamma}_k\eta\lim_{t\to t_k^-}e^{-\lambda t}\\ \le \kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\eta e^{-\lambda t_k}. \end{aligned} \tag{14}$$

From (13), (14) and $\mu_k\ge 1$, we have

$$\begin{cases} u(t)\le \kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\xi e^{-\lambda t}, & -\infty\le t\le t_k,\\ v(t)\le \kappa\mu_0\mu_1\cdots\mu_{k-1}\mu_k\eta e^{-\lambda t}, & -\infty\le t\le t_k. \end{cases} \tag{15}$$

Combining (8),(9),(15) and Lemma 2, we obtain that

$$\begin{cases} u(t)\le \kappa\mu_0\mu_1\cdots\mu_k\xi e^{-\lambda t}, & t_k\le t<t_{k+1},\\ v(t)\le \kappa\mu_0\mu_1\cdots\mu_k\eta e^{-\lambda t}, & t_k\le t<t_{k+1}. \end{cases} \tag{16}$$

Applying the mathematical induction, we can obtain the following inequalities

$$\begin{cases} u(t)\le \kappa\mu_0\mu_1\cdots\mu_k\xi e^{-\lambda t}, & t\in[t_k,t_{k+1}), \quad k\in N,\\ v(t)\le \kappa\mu_0\mu_1\cdots\mu_k\eta e^{-\lambda t}, & t\in[t_k,t_{k+1}), \quad k\in N. \end{cases} \tag{17}$$

According to (C2), we have $\mu_k\le e^{\mu(t_k-t_{k-1})}<e^{\lambda(t_k-t_{k-1})}$, so we have

$$u(t)\le \kappa e^{\mu t_1}e^{\mu(t_2-t_1)}\cdots e^{\mu(t_{k-1}-t_{k-2})}\xi e^{-\lambda t}=\kappa\xi e^{\mu t_{k-1}}e^{-\lambda t}\le \kappa\xi e^{-(\lambda-\mu)t},$$

and

$$v(t)\le \kappa e^{\mu t_1}e^{\mu(t_2-t_1)}\cdots e^{\mu(t_{k-1}-t_{k-2})}\eta e^{-\lambda t}=\kappa\eta e^{\mu t_{k-1}}e^{-\lambda t}\le \kappa\eta e^{-(\lambda-\mu)t},$$

for $t\in[t_{k-1},t_k)$, $k\in N$. That is

$$\begin{cases} u(t)\le \kappa\xi e^{-(\lambda-\mu)t}, & t\in(-\infty,t_k), \quad k\in N,\\ v(t)\le \kappa\eta e^{-(\lambda-\mu)t}, & t\in(-\infty,t_k), \quad k\in N. \end{cases} \tag{18}$$

It follows that

$$\begin{aligned}\sum_{i=1}^{n}|x_i(t)-x_i^*|+\sum_{j=1}^{m}|y_j(t)-y_j^*| &= \sum_{i=1}^{n}u_i(t)+\sum_{j=1}^{m}v_j(t)\\ &\le \sum_{i=1}^{n}\kappa\xi_i e^{-(\lambda-\mu)t}+\sum_{j=1}^{m}\kappa\eta_j e^{-(\lambda-\mu)t}\\ &= \frac{\sum_{i=1}^{n}\xi_i+\sum_{j=1}^{m}\eta_j}{\min\limits_{1\le i\le n,1\le j\le m}\{\xi_i,\eta_j\}}(\|\tilde{\phi}\|+\|\tilde{\varphi}\|)e^{-(\lambda-\mu)t}\\ &= \frac{\sum_{i=1}^{n}\xi_i+\sum_{j=1}^{m}\eta_j}{\min\limits_{1\le i\le n,1\le j\le m}\{\xi_i,\eta_j\}}\Big(\|\phi-x^*\|+\|\varphi-y^*\|\Big)e^{-(\lambda-\mu)t}.\end{aligned}$$

Let $M=\frac{\sum_{i=1}^{n}\xi_i+\sum_{j=1}^{m}\eta_j}{\min\limits_{1\le i\le n,1\le j\le m}\{\xi_i,\eta_j\}}$, then we have

$$\|x(t)-x^*\|+\|y(t)-y^*\|\le M\Big(\|\phi-x^*\|+\|\varphi-y^*\|\Big)e^{-(\lambda-\mu)t}.$$

□

Remark 1. In Theorem 1, the parameters μ_k and μ depend on the impulsive disturbance of system (1), and λ is actually an estimate of exponential convergence rate of continuous system (2), which depends on the delay kernel functions and system parameters. In order to obtain more precise estimate of the exponential convergence rate of system (1) (or system (2)), we suggest the following optimization problem.

$$(OP)\quad \begin{cases}\max\lambda,\\ s.t.\,(C1)\,holds,\end{cases}$$

Remark 2. Note that Lemma 2 transforms the fuzzy AND ($\bigwedge$) and the fuzzy OR ($\bigvee$) operation into the SUM operation ($\sum$). So above results can be applied to the following classical BAM neural networks with distributed delays and impulses:

$$\begin{cases}\dot{x}_i(t)=-a_ix_i(t)+\sum\limits_{j=1}^{m}a_{ij}g_j(y_j(t))\\ \qquad +\sum\limits_{j=1}^{m}\alpha_{ij}\int\limits_{0}^{+\infty}K_{ij}(s)g_j(y_j(t-s))ds+I_i,\quad t\ne t_k\\ x_i(t^+)=x_i(t^-)+P_{ik}(x_i(t^-)),\qquad t=t_k,\quad k\in N,\\ \dot{y}_j(t)=-b_jy_j(t)+\sum\limits_{i=1}^{n}b_{ji}f_i(x_i(t))\\ \qquad +\sum\limits_{i=1}^{n}\beta_{ji}\int\limits_{0}^{+\infty}\bar{K}_{ji}(s)f_i(x_i(t-s))ds+J_j,\quad t\ne t_k\\ y_j(t^+)=y_j(t^-)+Q_{jk}(y_j(t^-)),\qquad t=t_k,\quad k\in N\end{cases}\tag{19}$$

for $i=1,2,\cdots,n$; $j=1,2,\cdots,m$.

For system (19), it is easy to obtain the following result:

Theorem 2. *Under assumptions* (H1)-(H3), *if the following conditions hold:*

(C1′) *There exist vectors* $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0, \eta = (\eta_1, \eta_2, \cdots, \eta_m)^T > 0$ *and positive number* $\lambda > 0$ *such that*

$$\begin{cases} (\lambda - a_i)\xi_i + \sum\limits_{j=1}^{m} \Big(|a_{ij}| + |\alpha_{ij}| k_{ij}(\lambda)\Big) G_j \eta_j < 0, & i = 1, 2, \cdots, n, \\ (\lambda - b_j)\eta_j + \sum\limits_{i=1}^{n} \Big(|b_{ji}| + |\beta_{ji}| \bar{k}_{ji}(\lambda)\Big) F_i \xi_i < 0. & j = 1, 2, \cdots, m; \end{cases}$$

(C2) $\mu = \sup\limits_{k \in N} \left\{ \frac{\ln \mu_k}{t_k - t_{k-1}} \right\} < \lambda$, *where* $\mu_k = \max\limits_{1 \le i \le n, 1 \le j \le m} \{1, \gamma_{ik}, \bar{\gamma}_{jk}\}$, $k \in N$;

then system (19) *has exactly one globally exponentially stable equilibrium point, and its exponential convergence rate equals* $\lambda - \mu$.

Remark 3. If we assume that

(C2′) the impulsive operators $I_{ik}(x_i(t_k^-))$ and $\bar{I}_{jk}(y_j(t_k^-))$ satisfy [21-24]

$$\begin{cases} I_{ik}(x_i(t_k^-)) = -\delta_{ik}(x_i(t_k^-) - x_i^*), & 0 < \delta_{ik} < 2, \quad i = 1, 2, \cdots, n, \ k \in N, \\ \bar{I}_{ik}(y_j(t_k^-)) = -\bar{\delta}_{jk}(y_j(t_k^-) - y_j^*), & 0 < \bar{\delta}_{jk} < 2, \quad j = 1, 2, \cdots, m, \ k \in N. \end{cases}$$

Then from (H3), we easily obtain $\gamma_{ik} = |1 - \delta_{ik}| < 1$ and $\bar{\gamma}_{jk} = |1 - \bar{\delta}_{jk}| < 1$. It follows that $\mu_k = \max\limits_{1 \le i \le n, 1 \le j \le m} \{1, \gamma_{ik}, \bar{\gamma}_{jk}\} = 1$, $k \in N$, which implies $\mu = \frac{\ln \mu_k}{t_k - t_{k-1}} = 0 < \lambda$. From Remark 1, we know that the stability of the continuous system can guarantee the stability of the corresponding impulsive system when the impulsive operators satisfy (C2′). Hence, the assumptions about the impulsive operator in [21-24] are conservative and restrictive. In addition, all results in [22, 23] are involved in Theorem 2.

4. An illustrative example

In order to illustrate the feasibility of our above-established criteria in the preceding sections, we provide a concrete example. Although the selection of the coefficients and functions in the example is somewhat artificial, the possible application of our theoretical theory is clearly expressed.

Example 1. *Consider the following BAM fuzzy neural networks with distributed delays and impulses:*

$$\begin{cases} \dot{x}_i(t) = -a_i x_i(t) + \sum\limits_{j=1}^{2} a_{ij} g_j(y_j(t)) + \sum\limits_{j=1}^{2} \tilde{a}_{ij} v_j + I_i \\ \qquad + \bigwedge\limits_{j=1}^{2} \alpha_{ij} \int\limits_0^{+\infty} K_{ij}(s) g_j(y_j(t-s)) ds \\ \qquad + \bigvee\limits_{j=1}^{2} \tilde{\alpha}_{ij} \int\limits_0^{+\infty} K_{ij}(s) g_j(y_j(t-s)) ds \\ \qquad + \bigwedge\limits_{j=1}^{2} T_{ij} v_j + \bigvee\limits_{j=1}^{2} H_{ij} v_j, \quad t \neq t_k \\ x_i(t^+) = x_i(t^-) - \Big(1 + e^{0.125k}\Big)(x_i(t^-) - 1), \quad t = t_k, \quad k \in N, \end{cases} \tag{20 - 1}$$

$$\begin{cases} \dot{y}_j(t) = -b_j y_j(t) + \sum\limits_{i=1}^{2} b_{ji} f_i(x_i(t)) + \sum\limits_{i=1}^{2} \tilde{b}_{ji} u_i + J_j \\ \qquad + \bigwedge\limits_{i=1}^{2} \beta_{ji} \int\limits_0^{+\infty} \bar{K}_{ji}(s) f_i(x_i(t-s))ds \\ \qquad + \bigvee\limits_{i=1}^{2} \tilde{\beta}_{ji} \int\limits_0^{+\infty} \bar{K}_{ij}(s) f_i(x_i(t-s))ds \\ \qquad + \bigwedge\limits_{i=1}^{2} \bar{T}_{ji} u_i + \bigvee\limits_{i=1}^{2} \bar{H}_{ji} u_i, \quad t \neq t_k \\ y_j(t^+) = y_j(t^-) - \Big(1 + e^{0.125k}\Big)(y_j(t^-) - 1), \quad t = t_k, \quad k \in N, \end{cases} \tag{20-2}$$

for $i = 1,2$*,* $j = 1,2$*,* $t > 0$*,* $t_0 = 0$*,* $t_k = t_{k-1} + 0.5k$*,* $k \in N$*, where*

$$\begin{array}{cccccc} a_1 = 3, & a_2 = 3, & a_{11} = \frac{4}{3}, & a_{12} = -\frac{1}{2}, & a_{21} = \frac{1}{2}, & a_{22} = \frac{2}{3}, \\ \tilde{a}_{11} = 1, & \tilde{a}_{12} = -2, & \tilde{a}_{21} = -2, & \tilde{a}_{22} = 1, & I_1 = \frac{49}{12}, & I_2 = -\frac{37}{12}, \\ \alpha_{11} = \frac{1}{3}, & \alpha_{12} = -\frac{1}{4}, & \alpha_{21} = \frac{1}{4}, & \alpha_{22} = \frac{2}{3}, & \tilde{\alpha}_{11} = \frac{1}{3}, & \tilde{\alpha}_{12} = \frac{1}{4}, \\ \tilde{\alpha}_{21} = -\frac{1}{4}, & \tilde{\alpha}_{22} = \frac{2}{3}, & T_{11} = 1, & T_{12} = 0, & T_{21} = 0, & T_{22} = 1, \\ H_{11} = 1, & H_{12} = 0, & H_{21} = 0, & H_{22} = 1, & v_1 = 1, & v_2 = 2; \\ b_1 = 3, & b_2 = 3, & b_{11} = \frac{1}{3}, & b_{12} = -\frac{2}{3}, & b_{21} = \frac{4}{3}, & b_{22} = \frac{1}{3}, \\ \tilde{b}_{11} = 1, & \tilde{b}_{12} = 3, & \tilde{b}_{21} = 2, & \tilde{b}_{22} = -2, & J_1 = -\frac{7}{4}, & J_2 = 0, \\ \beta_{11} = \frac{1}{3}, & \beta_{12} = -\frac{1}{6}, & \beta_{21} = \frac{1}{3}, & \beta_{22} = \frac{1}{3}, & \tilde{\beta}_{11} = \frac{1}{3}, & \tilde{\beta}_{12} = \frac{1}{6} \\ \tilde{\beta}_{21} = \frac{1}{3} & \tilde{\beta}_{22} = \frac{1}{3}, & \tilde{T}_{11} = 1, & \tilde{T}_{12} = 0, & \tilde{T}_{21} = 0, & \tilde{T}_{22} = 1, \\ \tilde{H}_{11} = 1, & \tilde{H}_{12} = 0, & \tilde{H}_{21} = 0, & \tilde{H}_{22} = 1, & u_1 = 1, & u_2 = 1; \end{array}$$

$$K_{ij}(s) = e^{-s},\ \bar{K}_{ij}(s) = e^{-2s},\ f_i(s) = g_j(s) = \tfrac{|s+1|-|s-1|}{2},\ i,j = 1,2.$$

From above parameters, we have $F_1 = F_2 = 1$*,* $G_1 = G_2 = 1$*,*

$$(k_{ij}(\lambda))_{2\times 2} = \begin{pmatrix} \frac{1}{1-\lambda} & \frac{1}{1-\lambda} \\ \frac{1}{1-\lambda} & \frac{1}{1-\lambda} \end{pmatrix}, \qquad (\bar{k}_{ji}(\lambda))_{2\times 2} = \begin{pmatrix} \frac{1}{2-\lambda} & \frac{1}{2-\lambda} \\ \frac{1}{2-\lambda} & \frac{1}{2-\lambda} \end{pmatrix},$$

$$\Gamma_k = \begin{pmatrix} e^{0.125k} & \\ & e^{0.125k} \end{pmatrix}, \qquad \bar{\Gamma}_k = \begin{pmatrix} e^{0.125k} & \\ & e^{0.125k} \end{pmatrix}$$

Solving the following optimization problem

$$\begin{cases} \max \lambda \\ (\lambda - a_1)\xi_1 + \Big(|a_{11}| + (|\alpha_{11}| + |\tilde{\alpha}_{11}|)k_{11}(\lambda)\Big)G_1\eta_1 + \Big(|a_{12}| + (|\alpha_{12}| + |\tilde{\alpha}_{12}|)k_{12}(\lambda)\Big)G_2\eta_2 < 0, \\ (\lambda - a_2)\xi_1 + \Big(|a_{21}| + (|\alpha_{21}| + |\tilde{\alpha}_{21}|)k_{21}(\lambda)\Big)G_1\eta_1 + \Big(|a_{22}| + (|\alpha_{22}| + |\tilde{\alpha}_{22}|)k_{22}(\lambda)\Big)G_2\eta_2 < 0, \\ (\lambda - b_1)\eta_1 + \Big(|b_{11}| + (|\beta_{11}| + |\tilde{\beta}_{11}|)\bar{k}_{11}(\lambda)\Big)F_1\xi_1 + \Big(|b_{12}| + (|\beta_{12}| + |\tilde{\beta}_{12}|)\bar{k}_{12}(\lambda)\Big)F_2\xi_2 < 0, \\ (\lambda - b_2)\eta_2 + \Big(|b_{21}| + (|\beta_{21}| + |\tilde{\beta}_{21}|)\bar{k}_{21}(\lambda)\Big)F_1\xi_1 + \Big(|b_{22}| + (|\beta_{22}| + |\tilde{\beta}_{22}|)\bar{k}_{22}(\lambda)\Big)F_2\xi_2 < 0, \\ \lambda > 0, \quad \xi = (\xi_1, \xi_2)^T > 0, \quad \eta = (\eta_1, \eta_2)^T > 0, \end{cases}$$

we get $\lambda \approx 0.303 > 0$*,* $\xi = (1082041, 1327618)^T > 0$ and $\eta = (716212, 1050021)^T > 0$*, so (C1) holds. From Theorem 1, we know system (20) has a unique equilibrium point, this equilibrium point is* $(1,1,1,1)^T$*. Also,*

$$\mu_k = \max_{1\le i\le 2, 1\le j\le 2}\{1, \gamma_{ik}, \bar{\gamma}_{jk}\} = e^{0.125k},$$

$$\mu = \sup_{k\in N} \frac{\ln \mu_k}{t_k - t_{k-1}} = \frac{0.125k}{0.5k} = 0.25 < 0.303 = \lambda.$$

That is, (C2) holds. From Theorem 1, the unique equilibrium point $(1, 1, 1, 1)^T$ *of system (20) is globally exponentially stable, and its exponential convergence rate is about 0.053.*

5. Conclusions

In this paper, a class of BAM fuzzy cellular neural networks with distributed delays and impulses has been formulated and investigated. Some new criteria on the existence and global exponential stability of equilibrium point for the formulated networks have been derived by using the topological degree theory and the impulsive delay integro-differential inequality. The obtained stability criteria are delay-dependent and impulse-dependent. The neuronal output activation functions and the impulsive operators only need to is Lipschitz continuous, but need not to be bounded and monotonically increasing. Some restrictions of delay kernel functions are also removed. It is worthwhile to mention that our technical methods are practical, in the sense that all new stability conditions are stated in simple algebraic forms and provided a more precise estimate of the exponential convergence rate, so their verification and applications are straightforward and convenient. The effectiveness of our results has been demonstrated by the convenient numerical example.

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