Efficiency Improvement of the Fixed-complexity Sphere Decoder

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Abstract

In this paper, we propose two schemes to reduce the complexity of fixed-complexity sphere decoder (FSD) algorithm in the ordering and tree-search stages, respectively, while achieving quasi-ML performance. In the ordering stage, we propose a QR-decomposition-based FSD signal ordering based on the zero-forcing criterion (FSD-ZF-SQRD) that requires only a few number of additional complex flops compared to the unsorted QRD. Also, the proposed ordering algorithm is extended using the minimum mean square error (MMSE) criterion to achieve better performance. In the tree-search stage, we introduce a threshold-based complexity reduction approach for the FSD depending on the reliability of the signal with the largest noise amplification. Numerical results show that in 8×8 MIMO system, the proposed FSD-ZF-SQRD and FSD-MMSE-SQRD only require 19.5% and 26.3% of the computational efforts required by Hassibi's scheme, respectively. Moreover, a third threshold vector is outlined which can be used for high order modulation schemes. In 4×4 MIMO system using 16-QAM and 64-QAM, simulation results show that when the proposed threshold-based approach is employed, FSD requires only 62.86% and 53.67% of its full complexity, respectively.

Keywords: MIMO detection, fixed-complexity sphere decoder, signal ordering, tree search.

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1. Introduction

Multiple-input multiple-output (MIMO) spatial multiplexing techniques linearly increase the channel capacity without requiring additional spectral resources which are not only expensive but also scarce [1]. At the transmitter side, signals are transmitted simultaneously from sufficiently-separated antennas, whereas at the receiver side signals are recovered by means of detection algorithms.

The maximum-likelihood detector (MLD) is the optimum receiver for MIMO spatial multiplexing systems [2]. Nevertheless, MLD becomes infeasible for systems employing large number of transmit antennas and high order modulations due to its exponential complexity increase. Therefore, suboptimal detection schemes were proposed in the literature to achieve a tradeoff between performance and complexity as below.

Linear detection schemes including zero-forcing (ZF) and minimum mean square error (MMSE) treat the received signal by the pseudo-inverse and a regularized pseudo-inverse of the channel matrix, respectively. Although linear detection schemes are simple in terms of computational complexity, they lead to degradation in the performance due to the independent processing of the transmitted signals [3].

Successive interference cancellation (SIC) schemes detect signals iteratively, where already-detected signals are subtracted out from the received vector leading to a system with fewer interferers. ZF and MMSE VBLAST (vertical Bell Laboratories layered space-time) detection schemes [2], which are the first proposed SIC schemes, outperform linear algorithms. Nonetheless, VBLAST detection schemes have high computational complexity because of requiring a matrix pseudo-inversion at each iterative detection stage. Low complexity SIC detection algorithms based on the QR decomposition (QRD) of the channel matrix were proposed in the literature [4][5][6]. Although QRD-based SIC schemes are favorable in terms of computational complexity, their performance is still far from the optimum performance of the MLD.

Several detection algorithms have been proposed in the literature achieving quasi-ML performance while requiring lower computational complexity. Among these suboptimal algorithms, sphere decoding (SD) achieves quasi-ML performance with polynomial average computational complexity [7]. It is shown; however, that SD has variable complexity which depends on the channel condition and the instantaneous noise power, where the worst-case complexity of SD is consequently comparable with that of MLD [8][9]. That is, the extreme-case complexity of SD is exponential. Also, in terms of implementation complexity, SD is inefficient due to its sequential nature in the tree search stage that limits the possibility of pipelining, where consequently the detection latency is increased [10][11].

To overcome the two aforementioned drawbacks of SD, fixed-complexity sphere decoder (FSD) was proposed in [10][11][12]. Analysis on the error performance of the FSE was introduced in [13]. The main idea of the FSD is to perform the search over a fixed number of hypotheses of the transmitted vector independently of the noise power and the channel condition. The complexity of FSD is, therefore, fixed and known prior to the tree search phase of the detection algorithm. Moreover, the hypotheses are tested in parallel leading to reduction in the algorithm latency, i.e., increase in the detector throughput. To achieve a quasi-ML performance, FSD requires a specific signal ordering in which signals suffering from the highest noise amplification are detected in levels where all signal candidates are retained. As a consequence, weak signals don't affect the performance of the detection algorithm. The

conventional VBLAST algorithm is, therefore, used to obtain accurate signal ordering [14]. Despite that VBLAST ordering algorithm obtains a precise signal ordering, its complexity is shown to be $O(N^4)$, with N as the number of transmit antennas. This is computationally heavy when signal ordering is frequently performed [15].

Our original contributions: In this paper, we propose two schemes to reduce the complexity of FSD algorithm in the ordering and tree-search phases as follows [16]:

- **QRD-based FSD signal ordering:** In the ordering stage, we propose to embed the signal ordering required by the FSD in the QRD. Thus, the proposed ordering scheme requires a few additional complex multiplications and additions compared to the unsorted QRD algorithm with negligible degradation in the performance. Therefore, the proposed ordering scheme requires only a small fraction of the computational effort required by the conventional FSD-*VBLAST* sorting algorithm. ZF and MMSE performance-based criteria are used in the proposed FSD-*ZF-SQRD* and FSD-*MMSE-SQRD*, respectively, where SQRD refers to sorted QR decomposition.
- **Analysis of the computational complexity:** We give analytical formulas for the computational complexities of the conventional and proposed ordering schemes.
- **Threshold-based complexity reduction:** In the tree-search stage, based on the reliability of the weakest received signal, i.e., signal with the lowest received signal to noise ratio (SNR), the number of retained symbol replicas is controlled. Thus, when the received SNR of the weakest signal is larger than a pre-defined threshold, unnecessary computations by the FSD are avoided while achieving a quasi-ML performance.

The rest of this paper is organized as follows. In section II, we introduce the system model and review the FSD. In section III, we introduce the proposed FSD-ZF-SQRD and FSD-MMSE-SQRD schemes, and in section IV we derive the formulas for the computational complexities of the introduced sorting algorithms. In section V, threshold-based complexity reduction scheme is proposed. Simulation results are shown in section VI, and conclusions are drawn in section VII.

2. System Description and Review of the FSD

We consider a MIMO multiplexing system employing N transmit and M receive antennas, where $M \geq N$. Under the assumption of narrowband flat-fading channel, the received vector $\mathbf{r} \in \mathbf{C}^{M \times 1}$ is given by:

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{x} \in \mathbf{C}^{N \times 1}$ is the transmitted vector whose elements are drawn from the modulation set Ω satisfying $E[\mathbf{x}\mathbf{x}^*] = \mathbf{I}_N$, where $\mathbf{I}_N$ is the $N \times N$ identity matrix. $\mathbf{n} \in \mathbf{C}^{M \times 1}$ is the additive white Gaussian noise with zero mean and unity variance. $\mathbf{H} \in \mathbf{C}^{M \times N}$ is the channel matrix whose row-column element $h_{i,j}$, the complex channel coefficient between the j-th transmit and i-th receive antennas, is modeled as circularly symmetric complex Gaussian random variable with zero mean and unity variance.

Working on $\mathbf{x}$, the matrix $\mathbf{H}$ generates the lattice

$$\Delta(\mathbf{H}) = \left\{ \mathbf{H}\mathbf{x} : \mathbf{x} \in \Omega^N \right\}, \tag{2}$$

where the columns of $\mathbf{H}$, i.e., ($\mathbf{h}_1$, $\mathbf{h}_2$, …, $\mathbf{h}_N$), are the bases of the lattice. The received vector $\mathbf{r}$ is then represented as the lattice point $\mathbf{H}\mathbf{x}$ perturbed by the noise vector $\mathbf{n}$. As a consequence, the MLD finds the closest lattice point $\mathbf{H}\hat{\mathbf{x}}$ to the received vector $\mathbf{r}$, where $\hat{\mathbf{x}}$ is the estimate of the transmitted vector $\mathbf{x}$. That is,

$$\hat{\mathbf{x}}_{ML} = \arg\min_{\mathbf{x} \in \Omega^N} \left\| \mathbf{r} - \mathbf{H}\mathbf{x} \right\|^2. \tag{3}$$

SD restricts the search in (3) to the lattice points that reside in the hyper-sphere of radius d and centred at the received vector $\mathbf{r}$. Therefore,

$$\hat{\mathbf{x}}_{SD} = \arg\min_{\mathbf{x} \in \Omega^N} \left(\left\| \mathbf{r} - \mathbf{H}\mathbf{x} \right\|^2 \leq d^2 \right). \tag{4}$$

To perform the search in (4) successively, the channel matrix $\mathbf{H}$ is factorized into the multiplication of a unitary matrix $\mathbf{Q}$ and an upper triangular matrix $\mathbf{R}$.
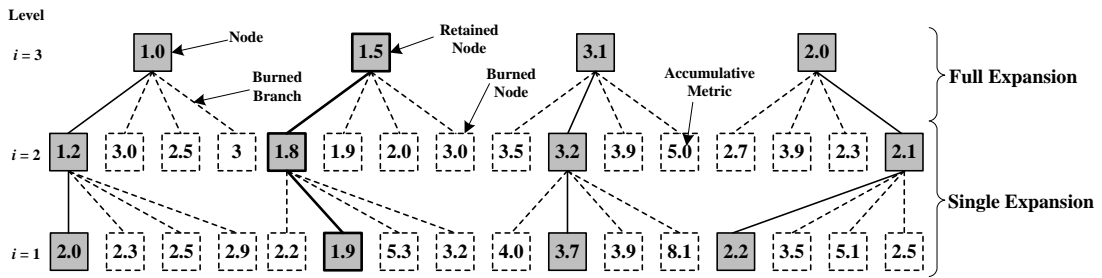

Fig. 1. An example of the fixed-complexity sphere decoder (FSD) for QPSK modulation, N = 3 and p = 1.

Therefore, the search problem in (4) is simplified to:

$$\hat{\mathbf{x}}_{SD} = \arg\min_{\mathbf{x} \in \Omega^N} \left(\left\| \mathbf{y} - \mathbf{R}\mathbf{x} \right\|^2 \leq d^2 \right), \tag{5}$$

where $\mathbf{y} = \mathbf{Q}^H \mathbf{r}$. The accumulative metric in (5) is then calculated successively, where the metric at the N-th detection level is given by:

$$E_N = \left(y_N - R_{N,N} \hat{x}_N \right)^2, \tag{6}$$

and the accumulative metric at the (N-1)-th detection level is given as follows:

$$E_{N-1} = E_N + \left(y_{N-1} - R_{N-1,N-1} \hat{\hat{x}}_{N-1} - R_{N-1,N} x_N \right)^2, \tag{7}$$

and so on.

Although the average complexity of SD is known to be polynomial, its extreme-case complexity is believed to be exponential, particularly for high noise power and when the channel matrix is ill-conditioned. Also, due to the sequential nature of the SD, it can't be pipelined leading to high latency in the detection algorithm.

To overcome the aforementioned problems of the SD, FSD was proposed by Barbero *et al.* FSD achieves a quasi-ML performance by performing the following two-stage tree search:

- **Fixed expansion stage:** In the first p detection levels, a full expansion of the retained nodes is performed, where all the resulting branches are retained for the next levels.
- **Single expansion stage:** All retained branches in the precedent level are extended independently to all possible nodes. The accumulative metrics of the resulting branches are calculated as in (5), and only the branch with the smallest accumulative metric is retained for the next level. This can be done using Schnorr-Euchner strategy [17], where only the metric of the node with the smallest accumulative metric is calculated.

Fig. 1 shows an example of the FSD for quadratic phase shift keying (QPSK) signalling, $N = 3$, and $p = 1$. Each node represents a constellation point belonging to the QPSK constellation set $\Omega =$ {-1-j, -1+j, 1+j, 1-j}. At the 3rd level, all the symbol replica candidates for x_3 are retained for the next detection levels, and the metric of each node, given inside the node in **Fig. 1**, is calculated as in (6). At the 2nd level, each retained node at the precedent level is extended independently from others, and the best child node is selected such that the accumulative metric is minimized. This strategy is repeated up to the 1st level, where the node with the smallest accumulative metric and its connected nodes are considered as the final estimate of the transmitted vector. For the example in **Fig. 1**, $\hat{\mathbf{x}} =$ {-1+j, -1-j, -1-j} and is indicated by the thick branches and nodes.

Because all possible symbol candidates are retained in the first p levels, the reliability of signals does not affect the final detection performance. Therefore, signals with the least robustness are detected in the full expansion stage. On the other hand, in the remaining $(N-p)$ levels, signals are sorted based on their reliability, where signals with the least noise amplification are detected first.

In the conventional FSD, VBLAST signal sorting is iteratively used to decide the order in which signals are detected. Although reduced complexity VBLAST ordering schemes were proposed in the literature [15][18], it is still complex compared to other QRD-based signal sorting schemes.

In the following section, we propose a low-complexity ordering scheme that achieves the required performance while reducing the computational complexity. Also, computational complexity comparison among different ordering schemes is performed.

3. Proposed QRD-based FSD Signal Ordering Schemes

In the pre-detection stage, the channel matrix is decomposed into the multiplication of a unitary matrix $\mathbf{Q}$ and an upper triangular matrix $\mathbf{R}$. Due to the structure of the upper triangular matrix $\mathbf{R}$, the detection is performed successively starting by the last component of the vector $\mathbf{x}$, i.e., x_N. In the conventional QR-decomposition, the diagonal elements of $\mathbf{R}$ are obtained in opposite order of signal detection. In the sorted QR-decomposition (SQRD), the columns of the channel matrix are reordered prior to each orthogonalization step, where $R_{1,1}$ is minimized over all the columns of the channel matrix, followed by $R_{2,2}$, and so on. Unfortunately, the SQRD does not sort signals in the order required by FSD.

In FSD, the efficient VBLAST algorithm is used to obtain the signal ordering prior to the QR-decomposition of the channel matrix. Despite that FSD achieves a quasi-ML performance using VBLAST ordering, the computational effort required by the ordering stage becomes exhaustive as the channel varies rapidly, where ordering is performed frequently.

The main idea of the proposed signal ordering algorithm is to obtain the $(p+1)$th minimum diagonal element of $\mathbf{R}$ prior to the orthogonalization step. Now, let $p = 1$, then $R_{1,1}$ is simply the norm of the channel matrix column whose order is the second in the norm sense. So, the column of $\mathbf{H}$ with the 2nd minimum norm is permuted with the first column. Thereafter, $R_{2,2}$ is computed in the same manner by considering only the remaining $(N-1)$ columns, etc. At the last iteration, the order of signal detection is same as that required by FSD algorithm.

Fig. 2 depicts an example of the proposed FSD-SQRD for $p = 1$, where $\mathbf{p}$ is the permutation vector. At the first iteration, the column with the 2nd minimum norm is permuted with the first column of $\mathbf{H}$ and the remaining $(N–1)$ vectors are orthogonalized. In the next iteration, the orthogonalized column with the 2nd minimum norm is permuted with the first remaining orthogonalized column. At the $(N-1)^{th}$ iteration, the remaining two columns correspond to the strongest and the weakest signals, i.e., the largest and the smallest received signal to noise ratio (SNR), respectively. Thus, the selected column corresponds to the signal with the largest SNR and the remaining one corresponds to the weakest signal. By doing so, the required order by FSD is obtained.

The introduced FSD signal sorting scheme is applied with the ZF and the MMSE performance-based criteria in the proposed FSD-ZF-SQRD and FSD-MMSE-SQRD algorithms, respectively. To apply the FSD-MMSE-SQRD, the channel matrix is extended to take into consideration the noise effect; that is,

$$\mathbf{H}_{ext} \in \mathbf{C}^{M+N \times N} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_N \end{bmatrix}, \tag{8}$$

Table 1 gives a detailed algorithmic description of the proposed FSD-MMSE-SQRD algorithm where the input n equals $(p + 1)$. $\mathbf{0}^N$ is the $N \times N$ matrix whose elements are all zeros, and $\min^k$ is the k-th minimum. The FSD-ZF-SQRD is obtained by simply replacing $\mathbf{H}_{ext}$ by $\mathbf{H}$, and line (7) in **Table 1** by the following line.

(7) Exchange columns i and k_i in $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{norm}$, and $\mathbf{p}$.

4. Analysis on the Computational Complexity

In this section, the computational complexities of the introduced signal ordering schemes are investigated. The complex floating points (flops) are given as a function of N and M, where the complex addition and multiplication operations are counted as one and three flops, respectively. Other operations are tracked back to the complex operations. For instance, real addition requires half a flop.

Table 2 gives the formulas for the computational complexities of the aforementioned signal sorting schemes. Hassibi's sorting scheme is a low complexity VBLAST algorithm used herein for comparison [6] and the assorted-ZF-QRD refers to the zero-forcing QRD without signal ordering.

Order of finding columns and rows of **Q** and **R**, respectively.
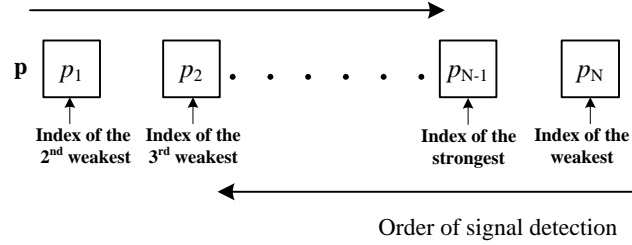

Fig. 2. Example of the proposed signal sorting algorithm.

Table 1. The proposed FSD-MMSE-SQRD algorithm.

Input: $\mathbf{R} = \mathbf{0}_N$, $\mathbf{Q} = \mathbf{H}_{ext}$, $\mathbf{p} = [1, 2, \cdots, N]$, n
1: **for** $i = 1$ to N **do**
2:    $\mathbf{norm}_i = \|\mathbf{q}_i\|^2$
3: **end for**
4: **for** $i = 1$ to N **do**
5:    $k = \min(n, N - i + 1)$
6:    $k_i = \underset{j=i,\cdots,N}{\arg\min^k} \mathbf{norm}_j$
7:    Exchange columns i and k_i in $\mathbf{R}$, $\mathbf{norm}$, and $\mathbf{p}$, and in the first $(M + i - 1)$ rows of $\mathbf{Q}$
8:    $R_{i,i} = |\mathbf{q}_i|$
9:    $\mathbf{q}_i = \mathbf{q}_i / R_{i,i}$
10:   **for** $j = i + 1$ to N **do**
11:      $R_{i,j} = \mathbf{q}_i^H \cdot \mathbf{q}_j$
12:      $\mathbf{q}_j = \mathbf{q}_j - R_{i,j} \cdot \mathbf{q}_i$
13:      $\mathbf{norm}_j = \mathbf{norm}_j - R_{i,j}^2$
14:   **end for**
15: **end for**
Output: $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{p}$

Table 2. Computational complexities of the sorting schemes.

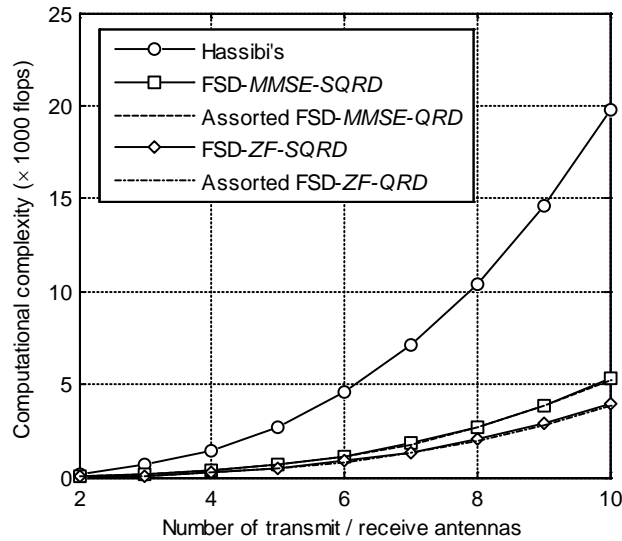| Ordering scheme	Complexity (flops)
Hassibi's	$6N^3 + 12N^2M + \dfrac{40}{3}N^2 + 4NM + \dfrac{39}{2}N - 7M - 30$
FSD-*MMSE-SQRD*	$\dfrac{4}{3}N^3 + 4N^2M + \dfrac{5}{4}N^2 - NM - \dfrac{13}{12}N$
FSD-*ZF-SQRD*	$4N^2M + \dfrac{3}{4}N^2 - NM - \dfrac{3}{4}N$
Assorted ZF-QRD	$4N^2M - \dfrac{1}{2}N^2 - NM + \dfrac{1}{2}N$

Fig. 3. Complexity in number of required floating point operations for Hassibi's scheme, the assorted QRD schemes, FSD-MMSE-SQRD, and FSD-ZF-SQRD.

Fig. 3 shows the computational complexities of the proposed algorithms with the conventional ones. Clearly, the proposed algorithms require much lower computational efforts compared to Hassibi's scheme. For instance, in 8×8 MIMO system, the proposed FSD-ZF-SQRD and FSD-MMSE-SQRD only require 19.5% and 26.3%, respectively, of the computational efforts required by Hassibi's scheme. The proposed signal sorting algorithms require only $1.25 \times (N^2 - N)$ additional flops compared to the assorted algorithms, i.e., without signal ordering.

The complexity of the conventional norm sorting FSD-norm is considered to be equivalent to that of the assorted QRD algorithm.

5. Threshold-based Complexity Reduction for the FSD

The first detected signal in the FSD, i.e., x_N, is the signal that suffers the largest noise amplification. Therefore, all the remaining (N-1) signals are more robust than the first ranked signal. Thus, if x_N experiences low noise amplification, the number of retained symbol replica candidates can be reduced without affecting the final performance. In contrast to the common adaptive symbol replica selection proposed in [19], FSD can have more reliable decision about the number of retained symbol replicas because its decision is based on the reliability of the signal suffering the largest noise amplification, whereas the algorithm in [19] does not take into consideration the weak signals.

Due to the structure of the matrix **R**, the signal x_N is interference-free, therefore, the robustness of the signal is directly proportional to $|R_{N,N}|$. Thus, based on the value of $|R_{N,N}|$, we decide the number of retained symbol replicas at the first full expansion stage. To accomplish that, we introduce three threshold vectors and the corresponding number of retained symbol replicas for the first full expansion level as follows:

$$P_1 = \begin{cases} |\Omega|, & \text{if } |R_{N,N}| \leq 0.75 \\ \frac{3}{4}|\Omega|, & \text{if } 0.75 < |R_{N,N}| \leq 1.0 \\ \frac{1}{2}|\Omega|, & \text{if } 1.0 < |R_{N,N}| \leq 1.5 \\ 1, & \text{if } |R_{N,N}| > 1.5 \end{cases} \tag{9}$$

$$P_2 = \begin{cases} |\Omega|, & \text{if } |R_{N,N}| \leq 0.5 \\ \frac{3}{4}|\Omega|, & \text{if } 0.5 < |R_{N,N}| \leq 0.75 \\ \frac{1}{2}|\Omega|, & \text{if } 0.75 < |R_{N,N}| \leq 1.0 \\ 1, & \text{if } |R_{N,N}| > 1.0 \end{cases} \tag{10}$$

and

$$P_3 = \begin{cases} |\Omega|, & \text{if } |R_{N,N}| \leq 0.4 \\ \frac{3}{4}|\Omega|, & \text{if } 0.4 < |R_{N,N}| \leq 0.6 \\ \frac{1}{2}|\Omega|, & \text{if } 0.6 < |R_{N,N}| \leq 0.8 \\ 1, & \text{if } |R_{N,N}| > 0.8. \end{cases} \tag{11}$$

Due to the fact that

$$P\{R_{N,N} \leq 0.4\} < P\{R_{N,N} \leq 0.5\} < P\{R_{N,N} \leq 0.75\}, \tag{12}$$

more complexity reduction can be achieved when the threshold vector P_3 is used as compared to the case of using the threshold vector P_2 or P_1. This is because the three aforementioned probabilities in (12) are the cases in which the proposed approach fails to achieve any complexity reduction using P_3, P_2, and P_1, respectively. Note that we obtained the threshold vectors using intensive simulations. In practice, the channel models are usually known, which indicates that an offline optimization of these threshold vectors is possible. Another alternative is to compute the probability density function (pdf) of $R_{N,N}$ and set the threshold vector as a trade-off between complexity reduction and performance degradation. The later option is suitable for systems that have preambles. These preambles can be used in the training phase to obtain optimized values of the threshold vector.

6. Simulation Results and Discussions

In this section, we investigate the bit error rate (BER) performance of the FSD in 4×4 MIMO spatial multiplexing system. Transmitted symbols are modulated using 4-QAM, 16-QAM, or 64-QAM mappers. The channel state information (CSI) is considered to be perfectly known at the receiver and unknown at the transmitter.

Fig. 4 shows the BER performance of the FSD for different signal ordering schemes in a 4×4 MIMO system using 4-QAM modulation. Without signal ordering, the performance of FSD is highly degraded and the diversity order, i.e., the slope of the BER curve, is lower than that of SD. When applying FSD-*norm* signal ordering, the BER performance is improved by approximately 2.7dB at BER of 10^{-4} whereas no improvement in the diversity order is achieved. On the other hand, when FSD-*VBLAST* ordering is applied, FSD algorithm achieves the same diversity order of SD with a 0.2dB performance lagging at BER $= 10^{-4}$. When the proposed FSD-*ZF-SQRD* signal ordering is employed, FSD attains the diversity order of SD with a degradation of less than 0.6dB compared to SD at BER of 10^{-4}. At the same target BER, FSD lags the optimum performance by only 0.4dB when the proposed FSD-*MMSE-SQRD* sorting algorithm is employed. This small degradation in the BER performance is tolerable considering the low computational complexity of the proposed algorithms compared with the conventional FSD-*VBLAST* signal ordering. Due to space limitation, we consider the proposed FSD-*ZF-SQRD* for the following comparisons.

Fig. 5 depicts the performance of the FSD employing the proposed FSD-ZF-SQRD algorithm using different QAM schemes. At BER of 10^{-4}, a degradation of 0.6dB, 0.3dB, and 0.25dB are remarked in the BER performance of the FSD when employing FSD-ZF-SQRD in a 4×4 system using 4-QAM, 16-QAM, and 64-QAM, respectively. That is, as the modulation order increases, the degradation due to the use of the proposed FSD-SQRD algorithms vanishes compared with the performance of FSD employing the complex FSD-VBLAST sorting scheme.
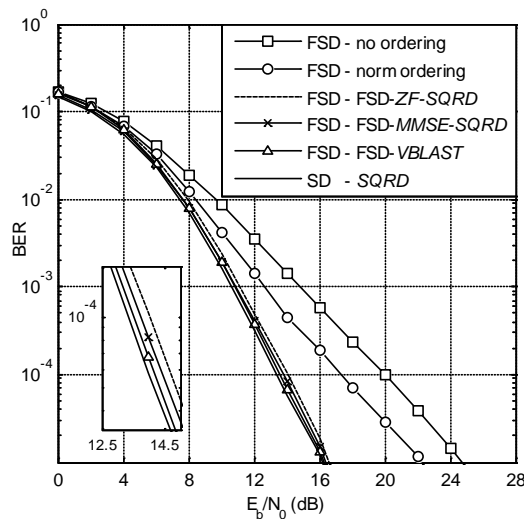

Fig. 4. BER performance of FSD for different signal sorting schemes using 4-QAM.
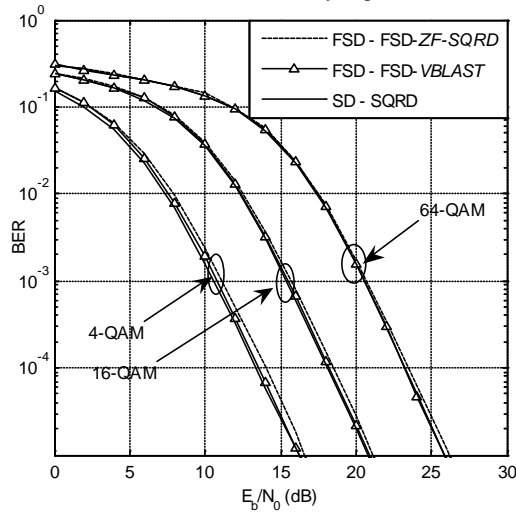
Fig. 5. BER performance of FSD for different signal sorting and modulation schemes.
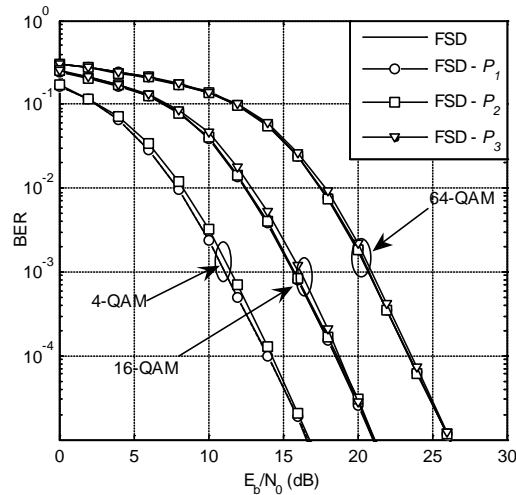

Fig. 6. BER performance of FSD using the proposed FSD-ZF-SQRD for different threshold values.

Table 3. Achieved reduction in the complexity of FSD by the proposed threshold-based approach in a 4×4 MIMO system.

Threshold	Reduced Computational Complexity (%)		
	4-QAM	16-QAM	64-QAM
P_1	12.04	12.08	12.09
P_2	28.03	30.52	31.10
P_3	Not applied	37.14	46.33

Fig. 6 shows the BER performance of the FSD when the number of retained symbol replicas at the first detection level is controlled by the threshold vectors P_1, P_2, or P_3. In the case of using P_1, the complexity reduction does not have any effect on the achieved performance of FSD for the different applied modulation schemes. By contrast, when the threshold vector P_2 is used, the performance of FSD using 4-QAM is degraded at medium Eb/No values, whereas

the performance is still intact when 16-QAM or 64-QAM is used.

Therefore, the vector P_3, which is tighter than P_1 and P_2, is not used with 4-QAM signalling scheme. In the case of using P_3, we remark a slight degradation in the BER performance when 16-QAM or 64-QAM is used. For instance, at BER of 10^{-3}, the maximum degradation is about 0.5dB and 0.3dB for 16-QAM and 64-QAM, respectively. This degradation in the performance is tolerable due to the remarkable reduction in the computational complexity.

Table 3 gives the achieved percentage of reduction in the computational complexity of FSD for different modulation schemes and threshold vectors. When the threshold vector P_1 is used, about 12% of complexity reduction is achieved for all modulation schemes. On the other side, when P_2 is used, the reduction in the complexity of FSD increases as the modulation order increases attaining 31.1% reduction in the case of 64-QAM. In the case of P_3, 37.14% and 46.33% of the computational efforts required by the conventional FSD are avoided when 16-QAM and 64-QAM are used, respectively.

We conclude that for high modulation orders, the set of candidates for each symbol becomes large. Therefore, when we apply a high complexity reduction, e.g., using P_3, there will still be enough retained candidates to obtain a quasi-ML solution. However, in the case of low modulation orders, e.g., QPSK, when the threshold vector P_3 is applied, there will be a few retained candidates at each detection level. These retained candidates might not be sufficient to obtain a quasi-ML performance.

7. Conclusions

In this paper, we proposed two techniques to reduce the computational complexity of FSD in the ordering and tree search stages, respectively. In the ordering stage, we proposed QRD-based FSD signal ordering scheme (FSD-*SQRD*) that leads to quasi-ML performance while requiring only a fraction of the computational complexity of the conventional FSD-VBLAST ordering algorithm. The proposed ordering algorithm is then extended to the MMSE case, where better performance is obtained. In the tree-search stage, we introduced a complexity reduction approach for FSD algorithm based on the reliability of the signal with the lowest received SNR value. That is, if the lowest SNR of the received signals is higher than a pre-defined threshold, the number of retained symbol replicas is reduced leading to the reduction in the computational complexity of the FSD algorithm with negligible degradation in the BER performance. The proposed improvements for FSD remarkably reduce the computational efforts required to achieve quasi-ML performance. As a consequence, FSD can be considered as a prominent detection scheme for next generation communication systems.

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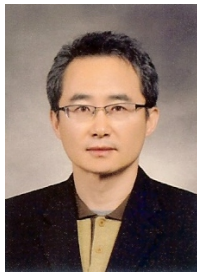
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