Nonlinear Speed Control of PM Synchronous Motor with Extended Kalman Filter Observer

Nga Thi-Thuy Vu^{*} · Jin-Woo Jung^{**}

Abstract

This paper proposes a nonlinear speed controller for a permanent magnet synchronous motor (PMSM). In this paper, the load torque is estimated by an extended Kalman filter (EKF) observer because the proposed controller needs its knowledge. To confirm the effectiveness of the proposed control scheme, simulations and experiments are performed under motor parameter variations with a prototype PMSM drive system.

Key Words : Extended Kalman filter(EKF), Nonlinear control, Permanent magnet synchronous motor(PMSM), Speed control

1. Introduction

Permanent magnet synchronous motors (PMSM) are widely used in industrial applications because of their high power density, high efficiency, and rugged construction. However, PMSMs are nonlinear multivariable systems. Moreover, the model parameters can be changed by temperature and current, and the load torque is also widely unknown. Thus, it is difficult to achieve

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Date of submit : 2010. 11. 30
First assessment : 2010. 12. 3
Completion of assessment : 2011. 1. 11 high-performance speed or position control with linear control methods such as a PI control or an LQ regulator. Therefore, nonlinear control techniques can be a promising alternative to precisely control the PMSM.

Recently, many researchers presented various nonlinear control methods [1–9] to deal with the above problems by directly considering the nonlinear PMSM dynamics. In [3], a nonlinear backstepping control scheme has been proposed to control the speed of the PMSM. The most appealing point of that method is to use the virtual control variable to make the high-order system simple, and thus the final control outputs can be derived step-by-step through appropriate Lyapunov functions. However, this control scheme requires the exact model knowledge of the control system. The fuzzy techniques [4–6] and a sliding-mode control [7–9] have also been reported for speed control of the PMSM. Although these control methods can obtain good performance, they are quite complex to be implemented. In [10-11] the authors have presented the feedback linearization techniques for designing the controller of nonlinear systems such as robot manipulators, induction motors, and PMSMs. In with comparison the others, the feedback linearization techniques are very useful methodologies for AC motor control. The controllers, which have been presented in [10-11], however, require full knowledge of the system parameters and load conditions with sufficient accuracy.

This paper proposes a nonlinear speed controller for PMSM. The controller consists of a nonlinear compensating term and a stabilizing term. The extended Kalman filter (EKF) observer is used for estimating the load torque which the proposed controller needs. To evaluate the performance of the proposed regulator, simulation and experimental results are presented in the presence of motor parameter variations with a prototype PMSM drive system.

2. Nonlinear Controller Design

2.1 PMSM model

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Based on the d-q reference theory, the stator voltage equations of a three-phase surface-mounted PMSM can be expressed as (1), and Fig. 1 illustrates the equivalent circuit of the PMSM.

$$V_{qs} = R_s i_{qs} + L_s \frac{di_{qs}}{dt} + \omega L_s i_{ds} + \lambda_m \omega$$
$$V_{ds} = R_s i_{ds} + L_s \frac{di_{ds}}{dt} - \omega L_s i_{qs}$$
(1)

where R_s is the stator resistance, L_s is the stator

inductance, ω is the electrical rotor angular speed, λ_m is the magnetic flux, i_{qs} is the *q*-axis current, i_{ds} is the *d*-axis current, V_{qs} is the *q*-axis voltage, and V_{ds} is the *d*-axis voltage, respectively.



Fig. 1. PMSM equivalent circuit

Also, the electromagnetic torque (T_c) can be given by

$$T_e = \frac{3}{2} \frac{p}{2} \lambda_m i_{qs} = \frac{2J}{p} \frac{d\omega}{dt} + \frac{2}{p} B\omega + T_L$$
(2)

where p is the number of poles, J is the rotor inertia, B is the viscous friction coefficient, and T_L is the load torque plus parameter imprecision, respectively.

From (1) and (2) the PMSM can be represented by the following differential equations:

$$\begin{aligned} \mathbf{\mathscr{E}} &= k_1 i_{qs} - k_2 \omega - k_3 T_L \\ \mathbf{\mathscr{E}}_{qs}^{\mathbf{k}} &= -k_4 i_{qs} - k_5 \omega + k_6 V_{qs} - \omega i_{ds} \\ \mathbf{\mathscr{E}}_{ds}^{\mathbf{k}} &= -k_4 i_{ds} + k_6 V_{ds} + \omega i_{qs} \end{aligned}$$
(3)

where $k_i > 0$, $i = 1 \cdots 6$ are the parameter values given by the nominal values of p, R_s , L_s , J, B, and λ_{m} respectively.

2.2 Speed Regulator Design and Stability Analysis

In this paper, the following assumptions, which are widely used in most papers, will be made to design a nonlinear speed regulator:

A1: θ , i_{qs} , i_{ds} are measurable.

A2: The load torque T_L is unknown and it changes very slowly, i.e., $P_L^{\&}$ can be set as 0.

A3: The desired rotor angular speed, ω_d , is twice differentiable, and ω_d , \mathcal{A}_d , \mathcal{A}_d are bounded.

Next, let us define the rotor position error (Θ_e), rotor speed error (ω_e) and rotor angular acceleration error (n_e) as

$$\begin{aligned} \theta_e &= \int_0^t \omega_e d\tau = \int_0^t (\omega - \omega_d) d\tau \\ \omega_e &= \omega - \omega_d \\ \eta &= \mathscr{A} = k_1 i_{qs} - k_2 \omega - k_3 T_L \\ \eta_e &= \mathscr{A} = \mathscr{A} - \mathscr{A} = \eta - \mathscr{A} \end{aligned}$$
(4)

Let the control inputs V_{qs} and V_{ds} be defined as

$$V_{qs} = \frac{1}{k_1 k_6} (u_{ffq} + u_{fbq}), \quad V_{ds} = \frac{1}{k_6} (u_{ffd} + u_{fbd})$$
$$u_{ffq} = \bigotimes_{d}^{\infty} + k_2 \eta + k_1 k_4 i_{qs} + k_1 k_5 \omega + k_1 \omega i_{ds}$$
$$u_{ffd} = k_4 i_{ds} - \omega i_{qs}$$
(5)

where u_{lkl} and u_{lkl} are the linearizing control terms used to compensate for the nonlinear characteristics of PMSM, and u_{lkl} and u_{lkl} are the feedback control terms used to stabilize the error dynamics.

Then, the model (3) can be transformed into the following:

$$\begin{aligned}
\theta_e^{\mathbf{x}} &= \omega_e \\
\theta_e^{\mathbf{x}} &= \eta_e \\
\theta_e^{\mathbf{x}} &= u_{fbq} \\
\theta_{ds}^{\mathbf{x}} &= u_{fbd}
\end{aligned}$$
(6)

The model (6) can be rewritten in the state–space form below.

$$\mathbf{k} = A\mathbf{x} + B\mathbf{u} \tag{7}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} \theta_e \\ \omega_e \\ \eta_e \\ i_{ds} \end{bmatrix},$$
$$u = \begin{bmatrix} u_{fbq} \\ u_{fbd} \end{bmatrix}.$$

By defining the feedback control terms (u_{flx_l} and u_{flx_l}) as $u_{fl} = [u_{flx_l}, u_{flx_l}]^T = Kx$, the closed-loop control system can be expressed by:

$$\mathbf{x} = (A + BK)\mathbf{x} \tag{8}$$

where $K \in \mathbb{R}^{2\times 4}$ denotes the controller gain matrix.

Since the pair (A, B) can be stabilized, the standard results [12] imply the existence of a stabilizing gain, K, such that for some positive-definite matrix P_c

$$P_{c}(A + BK) + (A + BK)^{T} P_{c} < 0$$
(9)

Let us define the Lyapunov function as $V_c = x^T P_c x$ From the closed-loop system (8), its time derivative is written as

$$V_{c}^{\&} = \frac{d}{dt} x^{T} P_{c} x = 2x^{T} P_{c} [A + BK] x \le 0$$
(10)

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The above equation (10) implies that the origin x = 0 is exponentially stable. Therefore, the following Theorem 1 can be obtained.

Theorem 1: Consider the closed-loop control system of (3), (5), and $u_{tb} = [u_{tbx}, u_{tbx}]^{T} = Kx$ Assume that the gain matrix K stabilizes (A + BK). Then, x exponentially converges to zero.

2.3 EKF-Based Load Torque Observer

The proposed nonlinear speed regulator needs the knowledge of the rotor angular acceleration (η) . As shown in (4), it can be easily obtained if the load torque is given. Accordingly, the control performance can be severely degraded if the term T_L is unknown. In this paper, an observer based on Extended Kalman filter (EKF) [13] is used to estimate the load torque because it is well-known that the EKF-based observer is insensitive to motor parameters and load torque variations, and can also suppress the measurement errors caused from the sensors [14].

From (2), the load torque observer can be represented as the following continuous-time state space equation:

 $\begin{aligned} \mathbf{x}_{\sigma} &= A_{\sigma} \mathbf{x}_{\sigma} + B_{\sigma} u_{\sigma} \\ y_{\sigma} &= C_{\sigma} \mathbf{x}_{c} \\ \hat{T}_{L} &= C_{T} \mathbf{x}_{\sigma} \end{aligned} \tag{11}$

where
$$T_L$$
 is an estimate of T_L , and
 $x_o = \begin{bmatrix} \hat{\theta} & \hat{\omega} & \hat{T}_L \end{bmatrix}^T$, $x_c = \begin{bmatrix} \theta & \omega & T_L \end{bmatrix}^T$, $u_o = T_e$,
 $A_o = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{p}{2}k_2 & -k_3 \\ 0 & 0 & 0 \end{bmatrix}$, $B_o = \begin{bmatrix} 0 \\ k_3 \\ 0 \end{bmatrix}$,

 $C_T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

By introducing an Euler approximation method, the approximate discrete-time model of the PMSM can be expressed as

$$\begin{aligned} x_o(k+1) &= A_{od} x_o(k) + B_{od} u_o(k) \\ y_o(k) &= C_o x_c(k) \\ \hat{T}_L(k) &= C_T x_o(k) \end{aligned} \tag{12}$$

where

$$A_{od} = \begin{bmatrix} T & T & 0 \\ 0 & T(1 - \frac{p}{2}k_2) & -Tk_3 \\ 0 & 0 & T \end{bmatrix}, \quad B_{od} = \begin{bmatrix} 0 \\ Tk_3 \\ 0 \end{bmatrix},$$

T is the sampling time, and $x_0(k)$, $u_o(k)$, $y_o(k)$ are the values of x_0 , u_0 , y_o at the sampling instant *k*.

Fig. 2 depicts an algorithm of the EKF-based load torque observer [13] used in this paper. It is noted that P is the error covariant matrix, L is the Kalman gain, Q and R are the noise covariance matrices, \bar{x}_o is the prediction of x_o , \bar{P} is the prediction of P, and $x_o(0)$ and P(0) are the initial values of x_o and P, respectively.



Fig. 2. Algorithm of EKF-based load torque observer

As shown in Fig. 2, the EKF algorithm to estimate the load torque consists of two steps. The first step performs the prediction of two quantities $(\bar{x}_o(k), \bar{P}(k))$ based on the previous estimates. That is, the predicted value $\bar{x}_o(k)$ is calculated from the previous state $x_o(k-1)$ and previous input $u_o(k-1)$, and the predicted value $\bar{P}(k)$ is derived from the previous error covariant matrix P(k-1) and noise covariance matrix Q. In the second step, the Kalman gain L(k) is computed by using $\bar{P}(k)$ and R, and then the state $x_o(k)$ is estimated with L(k) and $\bar{x}_o(k)$. Also, P(k) is updated for the next calculation using $\bar{P}(k)$ and L(k). Finally, the estimated load torque (\hat{T}_L) can be obtained from x_o and C_T :

3. Simulations and Experiments

To confirm the effectiveness of the proposed nonlinear speed controller, simulations and experiments are performed. Table 1 shows the nominal parameters of a prototype PMSM considered in this paper. According to the parameters shown in Table 1, the dynamic equations can be rewritten as follows:

$$\mathcal{B} = 3539.6i_{qs} - 0.2484\omega - 4968.8T_L$$

$$\mathcal{B}_{qs} = -170.1i_{qs} - 13.6\omega + 171.8V_{qs} - \omega i_{ds}$$

$$\mathcal{B}_{ds} = -170.1i_{ds} + 171.8V_{ds} + \omega i_{qs}$$
(13)

Table 1. Specifications of PMSM

Number of poles (p)	12
Stator resistance (R_s)	0.99[Ω]
Stator inductance (L_s)	5.82[mH]
Magnetic flux (λ_m)	0.0792[V·sec/rad]
Equivalent inertia (J)	$0.0012[kg.m^2]$
Viscous friction coefficient (B)	0.0003[N·m·sec/rad]

To design the EKF-based load torque observer,

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the following matrices are chosen:

$$A_{od} = \begin{bmatrix} T & T & 0 \\ 0 & -0.49T & -4968.8T \\ 0 & 0 & T \end{bmatrix}, B_{od} = \begin{bmatrix} 0 \\ 4968.8T \\ 0 \end{bmatrix},$$
$$x_o(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, P(0) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & -0.01 \\ 0 & -0.01 & 2 \end{bmatrix},$$
$$Q = 5 \times 10^{-6} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R = 0.001.$$

From the EKF algorithm shown in Fig. 2, the Kalman gain L is below calculated with respect to time.

$$L(k) = \overline{P}(k)C_o^T \left[C_o\overline{P}(k)C_o^T + R\right]^{-1}$$

Next, the observer-based feedback control terms can be expressed as

$$u_{fb} = \begin{bmatrix} u_{fbq} & u_{fbd} \end{bmatrix}^T = K\hat{x}$$
(14)

where $\hat{x} = [\theta_e, \omega_e, \hat{\eta}_e, i_{ds}]^T$, $\hat{\eta} = \delta = k_1 i_{qs} - k_2 \omega - k_3 \hat{T}_L$, $\hat{\eta}_e = \delta = \hat{\eta} - \delta d$.

Finally, the following controller gain is used:

$$K = 10^8 \times \begin{bmatrix} -5.2006 & -0.0194 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Fig. 3 shows a block diagram of the observer-based speed controller. Based on Fig. 3, simulations and experiments are executed to evaluate the performance of the proposed control

algorithm. Fig. 4 shows an overall block diagram of the proposed nonlinear control system. In this paper, a space vector PWM (SVPWM) technique is used due to well-known benefits, and PWM frequency and sampling frequency (1/T) are selected as 5 [kHz] considering switching loss and current ripple.



Fig. 3. Block diagram of the observer-based speed controller



Fig. 4. Overall block diagram of the proposed nonlinear control system

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Fig. 5 and 6 show the simulation results of two conditions (nominal parameters and 200[%] variations of some system parameters) by using Matlab/Simulink. In this case, the motor desired

speed (a_d) increases from 157.08[rad/sec] to 314.16 [rad/sec] and then decreases to 157.08[rad/sec]. Fig. 5 shows the simulation results ($a_{d_i} a_i, a_{e_i}, T_L, \hat{T}_L, i_{q_s}, i_{d_s}, V_{arb}, i_a$) under nominal parameters. On the other hand, Fig. 6 shows the simulation results under 200[%] variations of some parameters (R_s, L_s, J) to verify the robustness of the proposed control scheme. In Fig. 6, the proposed nonlinear speed controller shows good control performance under even model parameter variations.



Fig. 5. Simulation results under nominal parameters



Fig. 6. Simulation results under 200[%] variations of some system parameters (R_{s} , L_{s} , J)

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Fig. 7. Experimental setup

Fig. 7 illustrates the experimental test setup to implement the proposed control algorithm. As shown in Fig. 7, it includes a PMSM, a brake, a host PC, a three-phase PWM inverter with a TMS320F28335 DSP. Fig. 8 shows the experimental results under the same conditions as Fig. 5, whereas Fig. 9 shows the experimental results under the same condition as Fig. 6. Fig. 8 (a) and 9 (a) show the desired speed (ω_d), measured speed (ω), and speed error (ω_e). Fig. 8 (b) and 9 (b) show the load torque (T_L) and estimated load torque (\hat{T}_L). Fig. 8



Fig. 8. Experimental results under nominal parameters

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(c) and 9 (c) show the measured q-axis current (i_{qs}) and d-axis current (i_{ds}) . Fig. 8 (d) and 9 (d) show the phase a voltage (V_{an}) and phase a current (i_{a}) .

From simulation and experimental results, it is shown that actual motor speed reaches the desired speed within about 10[msec], and the steady-state speed error is almost zero. Therefore, it is clearly realized that the proposed control scheme can accurately and quickly follow the reference trajectory of a PMSM in the presence of motor

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parameter variations.

4. Conclusion

This paper presented a nonlinear control scheme for PMSM drive systems. In this paper, an extended Kalman filter (EKF) observer was employed to estimate the load torque. Through the simulation and experimental results, it was verified that the proposed control method can precisely and rapidly



Fig. 9. Experimental results under 200[%] variations of some parameters (Rs, Ls, J)

track the desired speed even under the variations of motor parameters and unknown load torque conditions.

This research was supported by Basic Science Research Program National through the Research Foundation Korea (NRF) funded of bv the Ministry of Education, Science and Technology (No. 2010-0009577).

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