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On Fuzzy Almost r-minimal Continuous Functions between Fuzzy Minimal Spaces and Fuzzy Topological Spaces

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Abstract

The purpose of this paper is to introduce and investigate the concept of fuzzy almost r-minimal continuous function between fuzzy minimal spaces and fuzzy topological spaces. Particularly, we investigate characterizations for the fuzzy almost r-minimal continuity by using generalized fuzzy r-open sets.

Key words : r-minimal structure, fuzzy r-minimal continuous, fuzzy weakly r-minimal continuous, fuzzy almost r-minimal continuous

1. Introduction

The concept of fuzzy set was introduced by Zadeh [13]. Chang [2] defined fuzzy topological spaces using fuzzy sets. The concept of smooth topological space was introduced in [3, 10] by Chattopadhyay, Hazra, Samanta, and Ramadan, which is a generalization of fuzzy topological space. Yoo et al. [11] introduced the concept of fuzzy r-minimal space which is an extension of the smooth topological space. The author introduced the concepts of fuzzy r-minimal continuous function [8] and fuzzy weakly r-minimal continuous function [9] between fuzzy r-minimal spaces and fuzzy topological spaces. The purpose of this paper is to generalize the concept of fuzzy r-minimal continuous function. So, in this paper, we introduce the concept of fuzzy almost r-minimal continuous function between a fuzzy r-minimal space and a fuzzy topological space. In particular, we investigate characterizations for the fuzzy almost r-minimal continuity by using generalized fuzzy r-open sets - fuzzy r-semiopen sets, fuzzy r-preopen sets, fuzzy r-β-open sets, fuzzy r-regular open sets.

2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member A of I^X is called a fuzzy set of X. By $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$ we denote constant maps on X with value 0 and 1, respectively. For any $A \in I^X$, A^c denotes the complement $\tilde{\mathbf{1}} - A$. All other notations are standard notations of fuzzy set theory.

A *fuzzy point* x_α in X is a fuzzy set x_α defined as follows

$$x_\alpha(y) = \begin{cases} \alpha, & \text{if } y = x, \\ 0, & \text{if } y \neq x. \end{cases}$$

A fuzzy point x_α is said to belong to a fuzzy set A in X, denoted by $x_\alpha \in A$, if $\alpha \leq A(x)$ for $x \in X$. A fuzzy set A in X is the union of all fuzzy points which belong to A.

Let $f : X \to Y$ be a function and $A \in I^X$ and $B \in I^Y$. Then $f(A)$ is a fuzzy set in Y, defined by

$$f(A)(y) = \begin{cases} \sup_{z \in f^{-1}(y)} A(z), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases}$$

for $y \in Y$, and $f^{-1}(B)$ is a fuzzy set in X, defined by $f^{-1}(B)(x) = B(f(x)), x \in X$.

A *fuzzy topology* (or *smooth topology*) [3, 10] on X is a map $\mathcal{T} : I^X \to I$ which satisfies the following properties:
(1) $\mathcal{T}(\tilde{0}) = \mathcal{T}(\tilde{1}) = 1$.
(2) $\mathcal{T}(A_1 \cap A_2) \geq \mathcal{T}(A_1) \wedge \mathcal{T}(A_2)$ for $A_1, A_2 \in I^X$.
(3) $\mathcal{T}(\cup A_i) \geq \wedge \mathcal{T}(A_i)$ for $A_i \in I^X$.

The pair $(X, \mathcal{T})$ is called a *fuzzy topological space* [11]. $A \in I^X$ is said to be *fuzzy r-open* (resp., *fuzzy r-closed*) [6] if $\mathcal{T}(A) \geq r$ (resp., $\mathcal{T}(A^c) \geq r$).

The *r-closure* of A, denoted by $cl(A, r)$, is defined as $cl(A, r) = \cap\{B \in I^X : A \subseteq B \text{and } B \text{is fuzzy } r\text{-closed}\}$.

The *r-interior* of A, denoted by $int(A, r)$, is defined as $int(A, r) = \cup\{B \in I^X : B \subseteq A \text{and } B \text{is fuzzy } r\text{-open}\}$.

Definition 2.1 ([11]). Let X be a nonempty set and $r \in (0, 1] = I_0$. A fuzzy family $\mathcal{M} : I^X \to I$ on X is said to have a *fuzzy r-minimal structure* if the family

$$\mathcal{M}_r = \{A \in I^X \mid \mathcal{M}(A) \geq r\}$$

contains $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$.

Then the $(X, \mathcal{M})$ is called a *fuzzy r-minimal space* (simply r-FMS) if $\mathcal{M}$ has a fuzzy r-minimal structure. Every member of $\mathcal{M}_r$ is called a *fuzzy r-minimal open* set. A fuzzy set A is called a *fuzzy r-minimal closed* set if the

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complement of A (simply, A^c) is a fuzzy r-minimal open set.

Let $(X, \mathcal{M})$ be an r-FMS and $r \in I_0$. The fuzzy r-minimal closure and the fuzzy r-minimal interior of A [11], denoted by $mC(A, r)$ and $mI(A, r)$, respectively, are defined as

$mC(A, r) = \cap\{B \in I^X : B^c \in \mathcal{M}_r \text{and } A \subseteq B\}$,
$mI(A, r) = \cup\{B \in I^X : B \in \mathcal{M}_r \text{and } B \subseteq A\}$.

Theorem 2.2 ([11]). Let $(X, \mathcal{M})$ be an r-FMS and A, B in I^X.

(1) $mI(A, r) \subseteq A$ and if A is a fuzzy r-minimal open set, then $mI(A, r) = A$.

(2) $A \subseteq mC(A, r)$ and if A is a fuzzy r-minimal closed set, then $mC(A, r) = A$.

(3) If $A \subseteq B$, then $mI(A, r) \subseteq mI(B, r)$ and $mC(A, r) \subseteq mC(B, r)$.

(4) $mI(A, r) \cap mI(B, r) \supseteq mI(A \cap B, r)$ and $mC(A, r) \cup mC(B, r) \subseteq mC(A \cup B, r)$.

(5) $mI(mI(A, r), r) = mI(A, r)$ and $mC(mC(A, r), r) = mC(A, r)$.

(6) $\tilde{\mathbf{1}} - mC(A, r) = mI(\tilde{\mathbf{1}} - A, r)$ and $\tilde{\mathbf{1}} - mI(A, r) = mC(\tilde{\mathbf{1}} - A, r)$.

3. Fuzzy Almost r-minimal Continuous Functions

Definition 3.1. Let $(X, \mathcal{M}_X)$ be an r-FMS and (Y, σ) a fuzzy topological space. Then $f : X \to Y$ is said to be *fuzzy almost r-minimal continuous* if for a fuzzy point x_α and for each fuzzy r-open set V with $f(x_\alpha) \in V$, there exists a fuzzy r-minimal open set U such that $x_\alpha \in U$ and $f(U) \subseteq int(cl(V, r), r)$.

We recall that: Let $(X, \mathcal{M}_X)$ be an r-FMS and (Y, σ) a fuzzy topological space. Then $f : X \to Y$ is said to be

(1) *fuzzy r-minimal continuous* [8] if for every fuzzy r-open set A in Y, $f^{-1}(A)$ is fuzzy r-minimal open in X;

(2) *fuzzy weakly r-minimal continuous* [9] if for a fuzzy point x_α and for each fuzzy r-open set V with $f(x_\alpha) \in V$, there exists a fuzzy r-minimal open set U such that $x_\alpha \in U$ and $f(U) \subseteq cl(V, r)$.

From the above definitions, easily we have the following implications:

fuzzy r-minimal continuity $\Rightarrow$ fuzzy almost r-minimal continuity $\Rightarrow$ fuzzy weakly r-minimal continuity

Example 3.2. Let $X = I$ and let A, B, C and D be fuzzy sets as the following:

$$A(x) = \frac{1}{2}x, \;\; x \in I;$$

$$B(x) = -\frac{1}{2}(x - 1), \;\; x \in I;$$

$$C(x) = \begin{cases} \frac{1}{2}(x + 1), & \text{if } 0 \le x \le \frac{1}{2}, \\ -\frac{1}{2}(x - 2), & \text{if } \frac{1}{2} < x \le 1; \end{cases}$$

and

$$D(x) = \begin{cases} -\frac{1}{2}(2x - 1), & \text{if } 0 \le x \le \frac{1}{2}, \\ \frac{1}{2}(2x - 1), & \text{if } \frac{1}{2} < x \le 1. \end{cases}$$

Let us consider two fuzzy topologies $\mathcal{M}_1$ and $\mathcal{M}_2$ defined as the following:

$$\mathcal{M}_1(\mu) = \begin{cases} 1, & \text{if } \mu = \tilde{\mathbf{0}}_{\mathbf{X}}, \tilde{\mathbf{1}}_{\mathbf{X}}, \\ \frac{1}{2}, & \text{if } \mu = C, \\ 0, & \text{otherwise}; \end{cases}$$

$$\mathcal{M}_2(\mu) = \begin{cases} 1, & \text{if } \mu = \tilde{\mathbf{0}}_{\mathbf{X}}, \tilde{\mathbf{1}}_{\mathbf{X}}, \\ \frac{1}{2}, & \text{if } \mu = D, \\ 0, & \text{otherwise}. \end{cases}$$

And let us consider a fuzzy r-minimal structure $\mathcal{N}$ defined as the following:

$$\mathcal{N}(\mu) = \begin{cases} 1, & \text{if } \mu = \tilde{\mathbf{0}}_{\mathbf{X}}, \tilde{\mathbf{1}}_{\mathbf{X}}, \\ \frac{2}{3}, & \text{if } \mu = A, B, \\ 0, & \text{otherwise}. \end{cases}$$

Then:

(1) The identity function $f : (X, \mathcal{N}) \to (X, \mathcal{M}_1)$ is fuzzy almost $\frac{1}{2}$-minimal continuous but not fuzzy $\frac{1}{2}$-minimal continuous.

(2) The identity function $g : (X, \mathcal{N}) \to (X, \mathcal{M}_2)$ is fuzzy weakly $\frac{1}{2}$-minimal continuous but not fuzzy almost $\frac{1}{2}$-minimal continuous.

Let $(X, \mathcal{M})$ be a FTS and $A \in I^X$. Then a fuzzy set A is said to be *fuzzy r-regular open* (resp., *fuzzy r-regular closed* [7] if $A = int(cl(A, r), r)$ (resp., $A = cl(int(A, r), r)$).

Theorem 3.3. Let $f : X \to Y$ be a function between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y, σ). Then the following statements are equivalent:

(1) f is fuzzy almost r-minimal continuous.

(2) $f^{-1}(B) \subseteq mI(f^{-1}(int(cl(B, r), r)), r)$ for each fuzzy r-open set B of Y.

(3) $mC(f^{-1}(cl(int(F, r), r)), r) \subseteq f^{-1}(F)$ for each fuzzy r-closed set F in Y.

(4) $f^{-1}(F) = mC(f^{-1}(F), r)$ for an fuzzy r-regular closed set F in Y.

(5) $f^{-1}(V) = mI(f^{-1}(V), r)$ for an fuzzy r-regular open set V in Y.

Proof. (1) ⇒ (2) Let B be a fuzzy r-open set in Y. Then for $x_\alpha \in f^{-1}(V)$, there exists a fuzzy r-minimal open set U containing x_α such that $f(U) \subseteq int(cl(B,r),r)$. Since $x_\alpha \in U \subseteq f^{-1}(mI(mC(B,r),r))$, $x_\alpha \in mI(f^{-1}(int(cl(B,r),r)),r)$. Thus we have (2).

(2) ⇒ (1) For a fuzzy point x_α, let V be a fuzzy r-open set containing $f(x_\alpha)$. By (2), we have $x_\alpha \in mI(f^{-1}(int(cl(V,r),r)),r)$. So there exists a fuzzy r-minimal open set U such that $x_\alpha \in U \subseteq f^{-1}(int(cl(V,r),r))$. This fact implies

$$f(U) \subseteq f(f^{-1}(int(cl(V,r),r))) \subseteq int(cl(V,r),r).$$

Hence f is fuzzy almost r-minimal continuous.

(2) ⇒ (3) Let F be any fuzzy r-closed set of Y. Then from (2), it follows

$$\begin{aligned} f^{-1}(\tilde{\mathbf{1}}_Y - F) &\subseteq mI(f^{-1}(int(cl(\tilde{\mathbf{1}}_Y - F,r),r)),r) \\ &= mI(f^{-1}(\tilde{\mathbf{1}}_Y - cl(int(F,r),r)),r) \\ &= mI(\tilde{\mathbf{1}}_X - f^{-1}(cl(int(F,r),r)),r) \\ &= \tilde{\mathbf{1}}_X - mC(f^{-1}(cl(int(F,r),r)),r). \end{aligned}$$

This implies $mC(f^{-1}(cl(int(F,r),r)),r) \subseteq f^{-1}(F)$.

(3) ⇒ (4) For any fuzzy r-regular closed set F of Y, since F is $F = cl(int(F,r),r)$ and fuzzy r-closed, we have $mC(f^{-1}(F),r) = mC(f^{-1}(cl(int(F,r),r)),r) \subseteq f^{-1}(F)$. So $f^{-1}(F) = mC(f^{-1}(F),r)$.

(4) ⇒ (5) Obvious.

(5) ⇒ (1) Let V be a fuzzy r-open set containing $f(x_\alpha)$. Since $int(cl(V,r),r)$ is fuzzy r-regular open, from (5),

$x_\alpha \in f^{-1}(V) \subseteq f^{-1}(int(cl(V,r),r)) = mI(f^{-1}(int(cl(V,r),r)),r)$.

So there is a fuzzy r-minimal open set U such that $x_\alpha \in U \subseteq f^{-1}(int(cl(V,r),r))$. This implies $f(U) \subseteq int(cl(V,r),r)$, and so f is fuzzy almost r-minimal continuous. □

Let X be a nonempty set and $\mathcal{M} : I^X \to I$ a fuzzy family on X. The fuzzy family $\mathcal{M}$ is said to have the property $(\mathcal{U})$ [11] if for $A_i \in \mathcal{M}$ $(i \in J)$,

$$\mathcal{M}(\cup A_i) \geq \wedge \mathcal{M}(A_i).$$

Theorem 3.4 ([11]). Let $(X,\mathcal{M})$ be an r-FMS with the property $(\mathcal{U})$. Then

(1) For $A \in I^X$, $mI(A,r) = A$ if and only if A is fuzzy r-minimal open.

(2) For $F \in I^X$, $mC(F,r) = F$ if and only if F is fuzzy r-minimal closed.

Corollary 3.5. Let $f : X \to Y$ be a function between an r-FMS $(X,\mathcal{M}_X)$ and a fuzzy topological space (Y,σ). If $\mathcal{M}_X$ has the property $(\mathcal{U})$, then the following statements are equivalent:

(1) f is fuzzy almost r-minimal continuous.

(2) $f^{-1}(B) \subseteq mI(f^{-1}(int(cl(B,r),r)),r)$ for each fuzzy r-open set B of Y.

(3) $mC(f^{-1}(cl(int(F,r),r)),r) \subseteq f^{-1}(F)$ for each fuzzy r-closed set F in Y.

(4) $f^{-1}(B)$ is fuzzy r-minimal open for each fuzzy r-regular open set B of Y.

(5) $f^{-1}(B)$ is fuzzy r-minimal closed for each fuzzy r-regular closed set B of Y.

Definition 3.6. Let (X,τ) be a FTS and $A \in I^X$. Then a fuzzy set A is said to be

(1) *fuzzy r-semiopen* [6] if $A \subseteq cl(int(A,r),r)$;

(2) *fuzzy r-preopen* [5] if $A \subseteq int(cl(A,r),r)$;

(3) *fuzzy r-β-open* [1] if $A \subseteq cl(int(cl(A,r),r),r)$.

A fuzzy set A is called a *fuzzy r-semiclosed* (resp., *fuzzy r-preclosed*, *fuzzy r-β-closed*) set if the complement of A is a fuzzy r-semiopen (resp., fuzzy r-preopen, fuzzy r-β-open) set.

Theorem 3.7. Let $f : X \to Y$ be a function between an r-FMS $(X,\mathcal{M}_X)$ and a fuzzy topological space (Y,σ). Then the following statements are equivalent:

(1) f is fuzzy almost r-minimal continuous.

(2) $mC(f^{-1}(G),r) \subseteq f^{-1}(cl(G,r))$ for each fuzzy r-β-open set G in Y.

(3) $mC(f^{-1}(G),r) \subseteq f^{-1}(cl(G,r))$ for each fuzzy r-semiopen set G in Y.

Proof. (1) ⇒ (2) Let G be a fuzzy r-β-open set. Then since $cl(G,r)$ is fuzzy r-regular closed, from Theorem 3.3 (4), it follows

$$\begin{aligned} mC(f^{-1}(G),r)) &\subseteq mC(f^{-1}(cl(G,r)),r) \\ &= f^{-1}(cl(G,r)). \end{aligned}$$

(2) ⇒ (3) Since every fuzzy r-semiopen set is fuzzy r-β-open, it is obvious.

(3) ⇒ (1) Let F be a fuzzy r-regular closed set. Then F is fuzzy r-semiopen, and so from (3), we have

$$mC(f^{-1}(F),r) \subseteq f^{-1}(cl(F,r)) = f^{-1}(F).$$

Hence, from Theorem 3.3, f is fuzzy almost r-minimal continuous. □

Theorem 3.8. Let $f : X \to Y$ be a function between an r-FMS $(X,\mathcal{M}_X)$ and a fuzzy topological space (Y,σ). Then f is fuzzy almost r-minimal continuous if and only if $mC(f^{-1}(cl(int(cl(G,r),r),r)),r) \subseteq f^{-1}(cl(G,r))$ for each fuzzy r-preopen set G in Y.

Proof. Suppose f is fuzzy almost r-minimal continuous and let G be a fuzzy r-preopen set in Y. Then since $cl(G,r) = cl(int(cl(G,r),r),r)$ and $cl(G,r)$ is fuzzy r-regular closed, from Theorem 3.3,

$$\begin{aligned} f^{-1}(cl(G,r)) &= mC(f^{-1}(cl(G,r)),r) \\ &= mC(f^{-1}(cl(int(cl(G,r),r),r)),r). \end{aligned}$$

Thus it implies $mC(f^{-1}(cl(int(cl(G,r),r),r)),r) \subseteq f^{-1}(cl(G,r))$.

For the converse, let A be a fuzzy r-regular closed set in Y. Then since $int(A,r)$ is fuzzy r-preopen, from hypothesis and $A = cl(int(A,r),r)$, it follows

$$\begin{aligned} f^{-1}(A) &= f^{-1}(cl(int(A,r),r)) \\ &\supseteq mC(f^{-1}(cl(int(cl(int(A,r),r),r),r)),r) \\ &= mC(f^{-1}(cl(int(A,r),r)),r) \\ &= mC(f^{-1}(A),r). \end{aligned}$$

This implies $f^{-1}(A) = mCl(f^{-1}(A),r)$, and hence by Theorem 3.3, f is fuzzy almost r-minimal continuous. □

Theorem 3.9. Let $f : X \to Y$ be a function between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y,σ). Then f is fuzzy almost r-minimal continuous if and only if $f^{-1}(G) \subseteq mI(f^{-1}(int(cl(G,r),r)),r)$ for each fuzzy r-preopen set G in Y.

Proof. Suppose f is fuzzy almost r-minimal continuous and let G be a fuzzy r-preopen set in Y. Since $int(cl(G,r),r)$ is fuzzy r-regular open, from Theorem 3.3, it follows $f^{-1}(G) \subseteq f^{-1}(int(cl(G,r),r)) = mI(f^{-1}(int(cl(G,r),r)),r)$.

For the converse, let U be fuzzy r-regular open. Then U is also fuzzy r-preopen. Since $U = int(cl(U,r),r)$, by hypothesis, $f^{-1}(U) \subseteq mI(f^{-1}(int(cl(U,r),r)),r) = mI(f^{-1}(U),r)$. This implies $f^{-1}(U) = mI(f^{-1}(U),r)$ and so f is fuzzy almost r-minimal continuous. □

Definition 3.10 ([12]). Let $(X, \mathcal{M}_X)$ be an r-FMS and $\mathcal{C} = \{A_i \in I^X : i \in J\}$. $\mathcal{C}$ is called a *fuzzy r-minimal cover* if $\cup\{A_i : i \in J\} = \tilde{1}_X$. It is a *fuzzy r-minimal open cover* if each A_i is a fuzzy r-minimal open set. A subcover of a fuzzy r-minimal cover $\mathcal{A}$ is a subfamily of it which also is a fuzzy r-minimal cover. X is said to be *fuzzy r-minimal compact* (resp., *almost fuzzy r-minimal compact*, *nearly fuzzy r-minimal compact*) if for every fuzzy r-minimal open cover $\mathcal{C} = \{A_i \in I^X : i \in J\}$ of X, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $\tilde{1}_X = \cup_{i\in J_0} A_i$ (resp., $\tilde{1}_X = \cup_{i\in J_0} mC(A_i,r)$, $\tilde{1}_X = \cup_{i\in J_0} mI(mC(A_i,r),r)$).

Definition 3.11 ([4]). Let (X,τ) be a fuzzy topological space. X is said to be *r-fuzzy compact* (resp., *r-fuzzy almost compact*, *r-fuzzy nearly compact*) if for every fuzzy r-open cover $\mathcal{C} = \{A_i \in I^X : \tau(A_i) \geq r, i \in J\}$ of A, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $\tilde{1}_X = \cup_{i\in J_0} A_i$ (resp., $\tilde{1}_X = \cup_{i\in J_0} cl(A_i,r)$, $\tilde{1}_X = \cup_{i\in J_0} int(cl(A_i,r),r)$).

Theorem 3.12. Let $f : X \to Y$ be a fuzzy almost r-minimal continuous surjection between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y,σ). If X is fuzzy r-minimal compact, then Y is r-fuzzy nearly compact.

Proof. Let $\mathcal{C} = \{B_i \in I^Y : i \in J\}$ be a fuzzy r-open cover of Y. Then for each $x(i)_\alpha \in f^{-1}(B_i)$ for $B_i \in \mathcal{C}$, since f is fuzzy almost r-minimal continuous, there exists a fuzzy r-minimal open set $U(x(i)_\alpha)$ such that $x(i)_\alpha \in U(x(i)_\alpha) \subseteq f^{-1}(int(cl(B_i,r),r))$. So the collection $\{U(x(i)_\alpha) : x(i)_\alpha \in X\}$ is a fuzzy r-minimal open cover in X. Since X is fuzzy r-minimal compact, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $\tilde{1}_X = \cup_{j\in J_0} U(x(j)_\alpha) \subseteq \cup_{j\in J_0} f^{-1}(int(cl(B_j,r),r))$. Hence $\tilde{1}_Y = \cup_{j\in J_0} int(cl(B_j,r),r)$. □

Theorem 3.13. Let $f : X \to Y$ be a fuzzy almost r-minimal continuous surjection between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y,σ). If X is fuzzy r-minimal compact and if $\mathcal{M}_X$ has the property $(\mathcal{U})$, then Y is r-fuzzy nearly compact.

Proof. Let $\mathcal{C} = \{B_i \in I^Y : i \in J\}$ be a fuzzy r-open cover of Y. Then by the property $(\mathcal{U})$, the fuzzy family $\mathcal{C}' = \{mI(f^{-1}(int(cl(B_i,r),r)),r) : B_i \in \mathcal{C}$for $i \in J\}$ is a fuzzy r-minimal open cover of X. Since X is fuzzy r-minimal compact, there exists a finite subset J_0 of J such that $\tilde{1}_X = \cup_{j\in J_0} mI(f^{-1}(int(cl(B_j,r),r)),r) = \cup_{j\in J_0} f^{-1}(int(cl(B_j,r),r))$.

This implies $\tilde{1}_Y = \cup_{j\in J_0} int(cl(B_j,r),r)$ and so Y is r-fuzzy nearly compact. □

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