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Condition based age replacement policy of used item

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Abstract. In most of literatures of age replacement policy, the authors consider the case that a new item starts operating at time zero and is to be replaced by new one at time T. It is, however, often to purchase used items because of the limited budget. In this paper, we consider age replacement policy of a used item whose age is t_0. The mathematical formulas of the expected cost rate per unit time are derived for both infinite-horizon case and finite-horizon case. For each case, we show that the optimal replacement age exists and is finite and investigate the effect of the age of the used item.

Key Words: *Age replacement policy, expected cost rate per unit time, infinite-horizon case, finite-horizon case*

1. INTRODUCTION

Since Barlow and Hunter (1960) proposed an age replacement policy in which an operating item is replaced at age T or at failure, whichever occurs first, the age replacement policy has been extensively studied by incorporating various types of repairs at failure and cost structures for repair. Beichelt (1976), Berg, Bienvenu and Cléroux (1986), Block, Borges and Savits (1988) and Sheu, Kuo and Nakagawa (1993) consider the age-replacement problem with age-dependent minimal repair and different cost structures. Cleroux, Dubuc and Tilquin (1979) and Bai and Yun (1986) consider age replacement policy based both on the system age and the minimal repair cost. Sheu and Griffith (1996), Sheu (1998), Sheu and Chien (2004) and Chien and Sheu (2006) consider age replacement policy of system subject to shocks. Sheu (1991), Sheu, Griffith and

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Nakagawa (1995), Jhang and Sheu (1999) and Sheu, Yeh, Lin and Juang (1999) consider age replacement policy with age dependent replacement and random repair cost.

Recently, condition based maintenance has attracted a great attention of researchers and engineers because it enables ones to make maintenance decisions based on the current information about the system. Scarf (2007) categorizes condition based maintenance models into three groups which are proportional hazard models, failure threshold models and two-phase failure models. Researchers consider the proportional hazard model (PHM) to develop the hazard rate function based on the monitored condition information and the age of the equipment. And they use the hazard rate function to determine optimal replacement time to either minimize the expected cost per unit time or maximize the long run average availability. (Makis and Jardine (1992), Wang, Scarf and Smith (2000), Jardine and Tsang (2006), Li et. al (2007)). Wang and Zhang (2005) point out that the proportional hazard model uses only the current observation, not all the previous history of health information. In failure threshold models, it is typically assumed that the unit fails if the condition of the unit is above the failure threshold. (Park (1988), Crister and Wang (1995)) Since the current condition may be unobservable, failure threshold models have been evolved by assuming that the condition can be calibrated by the indicator of condition (Crister, Wang and Sharp (1997), Wang (2002)). Typically, the specification of a failure threshold may not be feasible. Researchers consider two-phase failure models or delay-time model in which if the condition indicator is above the threshold then the component is assumed to be likely to fail. These models are used to optimize the monitoring interval (Christer and Wang (1992), Coolen and Dekker (1995), Christer (1999)).

In condition based maintenance or replacement policy, the conditional distribution of random residual life at the monitoring time given the monitored condition information plays an important role in determining the optimal maintenance time or optimal replacement time. There have been three different ways of modeling the conditional distribution of random residual life given the condition information and estimating in the conditional distribution. Proportional hazard model (PHM) has been considered by Kumar and Westberg (1997), Love and Guo (1991), Makis and Jardin (1991), Jardin et. al (1998), Banjevic et. al (2001). Accelerated life model (ALM) has been used by Kalbfleisch and Prentice (1980), Wang and Zhang (2005). And stochastic filtering and hidden Markov models (HMM) has been studied by Hontelez et. al (1996), Christer et. al (1997), Wang and Christer (2000), Bunks et. al (2000), Dong and He (2004), Lin and Markis (2003, 2004), Baruah and Chinnam (2005), Wang (2006), Lipi, Lim and Zuo (2010).

In this paper, we consider two-phase failure models and propose a condition based age replacement policy in which when the condition indicator of a system exceeds the warning threshold value, the system is replaced by new one either after planned time period T or at failure, whichever comes first. We formulate the expected cost per unit time for both the infinite-horizon case and the finite-horizon case. Our cost functions balance the cost ofpreventive replacement against the cost of failure given theprobability distributions of the time at which the condition indicator is beyond the warningthreshold and the time between the condition indicator crossing of the warning threshold and the consequent failure. We show the existence and uniqueness of the optimal replacement time which minimizes the expected cost rate per unit time.

The remainder of this paper is organized as follows. Section 2 describes the condition based age replacement policy and develops the expected cost per unit time for both the infinite-horizon case and the finite-horizon case is formulated. In Section 3, the optimal replacement schedule is investigated. In Section 4, a numerical example is given to illustrate our results.

2. CONDITION BASED AGE REPLACEMENT POLICY

Let t_i be the i-th monitoring time and let Y_i be the condition information monitored at t_i, $i = 1, 2, \ldots$. Let X_i be the random residual life at monitoring time t_i.

Wang (2002) obtains the conditional distribution of X_i given that condition information up to t_i, denoted by $\Im_i = \{Y_1, Y_2, \ldots, Y_i\}$, is monitored. In his model, Y_i is assumed to be a random variable and to be dependent on X_i. The conditional probability density function of Y_i given X_i is given by $g(y_i \mid x_i)$. The conditional probability density function of X_i given $\Im_i = \{Y_1, Y_2, \ldots, Y_i\}$ is given by

$$p_i(x_i \mid \Im_i) = \frac{g(y_i \mid x_i) p_{i-1}(x_i + t_i - t_{i-1} \mid \Im_{i-1})}{\int_0^\infty g(y_i \mid x_i) p_{i-1}(x_i + t_i - t_{i-1} \mid \Im_{i-1}) dx_i} . \tag{2.1}$$

Let $t = \min_{i \geq 1}\{t_i | Y_i \geq Y_c\}$, where Y_c is the critical condition information (warning threshold point) after which the system begins deteriorating rapidly. In Figure 2.1, t is t_k and condition information up to age t is $\Im_t = \Im_k = \{Y_1, Y_2, \ldots, Y_k\}$. Then in order to minimize the maintenance cost, operating personnel may want to know the proper time at which the system should be replaced. Since unexpected failure typically causes extra cost for handling failure, it could take more cost than one for planned replacement to do replacement at failure. Hence it is worthy of considering the following age replacement policy based on condition information.

- **Step 1** At monitoring time t_i, $i = 1, 2, \ldots$, the condition information of the system is monitored and recorded.
- **Step 2** When the monitored indicator exceeds the warning threshold value, we consider the following age replacement policy at this point of time, denoted by $t = \min_{i \geq 1}\{t_i | Y_i \geq Y_c\}$.

『A system is replaced by new one either after planned time period *T* or at failure, whichever comes first. 』

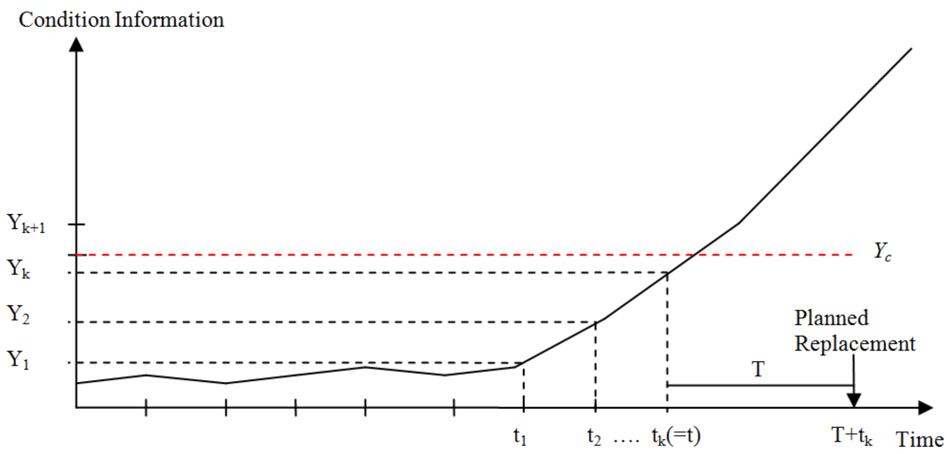

Figure 2.1. Condition based age replacement policy

3. EXPECTED COST AND OPTIMAL REPLACEMENT TIME

3.1. Infinite-horizon case

1) The long run expected cost per unit time

Let a cycle be defined as the duration from initial operation to replacement either planned or reactive. Let C_f and C_r be cost for failure and cost for planned replacement, respectively. And let X_t be the residual life of the system at age $t = \min_{i \geq 1}\{t_i | Y_i \geq Y_c\}$ and $\Im_t = \{Y_1, Y_2, ..., Y_k \mid k \text{ is such that } t_k = t\}$ be the history of condition information of the system up to age *t*.

Let V_t be the cost incurred during a cycle and Z_t be the length of a cycle. Then we have $V_t = C_f \cdot I(X_t < T) + C_r \cdot I(X_t \geq T)$ and $Z_t = t + X_t \cdot I(X_t < T) + T \cdot I(X_t \geq T)$.

Hence the expected cost incurred during a cycle and the expected length of a cycle can be obtained as follows.

$$E[V_t \mid \Im_t] = C_f \cdot P(X_t < T \mid \Im_t) + C_r \cdot P(X_t \geq T \mid \Im_t)$$

and

$$E(Z_t \mid \Im_t) = t + E[X_t \cdot I(X_t < T) \mid \Re_t) + E[T \cdot I(X_t \geq T) \mid \Im_t]$$
$$= t + \int_0^T x f_t(x \mid \Im_t) dx + T \cdot P(X_t \geq T \mid \Im_t)$$

where $f_t(\cdot \mid \Im_t)$ is the conditional probability density function of X_t given that the condition information $\Im_t$ is monitored. If *t* is such that $t = t_k$, $k = 1, 2, ..., n$, then it can be obtained from Wang (2002) that

$$f_t(x \mid \Im_t) = p_k(x_k \mid \Im_k) = \frac{g(y_k \mid x_k) p_{k-1}(x_k + t_k - t_{k-1} \mid \Im_{k-1})}{\int_0^\infty g(y_k \mid x_k) p_{k-1}(x_k + t_k - t_{k-1} \mid \Im_{k-1}) dx_k}.$$

Long run expected cost $C_t(T)$ at age t can be defined as follows.

$$C(T \mid \Im_t) = \lim_{s\to\infty} \frac{K(s)}{s} = \frac{E(\text{cost incurred if during a cycle})}{E(\text{length of a cycle})} = \frac{E[V_t \mid \Im_t]}{E[Z_t \mid \Im_t]},$$

where $K(s)$ is the cumulative expected cost due to the series of cycles in an internal $(0, s)$. $K(s)/s$ is the expected cost per unit time. The second equality holds due to Renewal Reward Theorem. (See Ross(1992) for details.)

Therefore we have the long run expected cost $C_t(T)$ at age t as follows.

$$C_t(T \mid \Im_t) = \frac{E[V_t \mid \Im_t]}{E[Z_t \mid \Im_t]} = \frac{C_f \cdot P(X_t < T \mid \Im_t) + C_r \cdot P(X_t \geq T \mid \Im_t)}{t + \int_o^T f_t(x \mid \Im_t) dx + T \cdot P(X_t \geq T \mid \Im_t)}. \quad (3.1)$$

2) The optimal replacement age *T which minimizes long run expected cost.**

Taking a partial integration of $\int_0^T x f_t(x \mid \Im_t) dx$ and replacing $P(X_t \geq T \mid \Im_t)$ by $1 - P(X_t < T \mid \Im_t)$, we have simplified long run expected cost as follows.

$$C_t(T \mid \Im_t) = \frac{(C_f - C_r) P(X_t \leq T \mid \Im_t) + C_r}{t + T - \int_o^T P(X_t \leq x \mid \Im_t) dx} = \frac{(C_f - C_r) F_t(T \mid \Im_t) + C_r}{t + \int_o^T R_t(x \mid \Im_t) dx},$$

where $F_t(x \mid \Im_t) = P(X_t \leq x \mid \Im_t)$ and $R_t(x \mid \Im_t) = P(X_t \geq x \mid \Im_t)$.

Taking a derivative of $C_t(T \mid \Im_t)$ with respect to T yields

$$\frac{d}{dT} C_t(T \mid \Im_t) = \frac{d}{dT} \frac{(C_f - C_r) F_t(T \mid \Im_t) + C_r}{t + \int_o^T R_t(x \mid \Im_t) dx}$$

$$= \frac{1}{[t + \int_o^T R_t(x \mid \Im_t) dx]^2} \times$$

$$\left\{ (C_f - C_r) f_t(T \mid \Im_t) [t + \int_o^T R_t(x \mid \Im_t) dx] - [(C_f - C_r) F_t(T \mid \Im_t) + C_r] R_t(T \mid \Im_t) \right\}$$

$$= \frac{R_t(T \mid \Im_t)}{\left[t + \int_0^T R_t(x \mid \Im_t) dx\right]^2}$$

$$\times \left((C_f - C_r) \left\{ h_t(T \mid \Im_t) \left[\mathrm{t} + \int_0^T R_t(x \mid \Im_t) dx \right] + R_t(T \mid \Im_t) \right\} - C_f \right), \quad (3.2)$$

where $h_t(u \mid \Im_t) = \dfrac{f_t(u \mid \Im_t)}{R_t(u \mid \Im_t)}$.

Our goal is to find an optimal age T^* for replacement such that $C_t(T \mid \Im_t)$ is minimized. It is well known that the necessary and sufficient condition for T^* to be optimal is $\dfrac{d}{dT} C_t(T \mid \Im_t)\Big|_{T=T^*} = 0$. That is, the optimal age T^* for replacement is the solution of the following equation if it exits.

$$h_t(T \mid \Im_t)[t + \int_o^T R_t(x \mid \Im_t)dx] + R_t(T \mid \Im_t) = \frac{C_f}{C_f - C_r}. \tag{3.3}$$

The following theorem formally state the existence and the uniqueness of the optimal replacement age in the proposed age replace policy.

Theorem 1. Suppose that $h_t(x \mid \Im_t)$ is strictly increasing in $x \geq 0$ and goes to infinity as x goes to infinity. Then there exists the optimal age T^* for replacement which minimizes $C_t(T \mid \Im_t)$ in (3.1) provided that $t \cdot h_t(0 \mid \Im_t) < \dfrac{C_r}{C_f - C_r}$.

Proof. Let $\xi(T)$ be the left-sided term in the equation (3.3). Then it is sufficient to show that there exists T^* such that $\xi(T^*) = \dfrac{C_f}{C_f - C_r}$. When $T = 0$, it is obvious from the assumption that $\xi(T = 0) = t \cdot h_t(0 \mid \Im_t) + R_t(0 \mid \Im_t) = t \cdot h_t(0 \mid \Im_t) + 1 < \dfrac{C_f}{C_f - C_r}$.

Taking a derivative of $\xi(T)$ with respect to T yields

$$\frac{d\xi(T)}{dT} = \frac{d}{dT} h_t(T \mid \Im_t)[t + \textstyle\int_o^T R_t(x \mid \Im_t)dx] + h_t(T \mid \Im_t)R_t(T \mid \Im_t) - f_t(T \mid \Im_t)$$
$$= \frac{d}{dT} h_t(T \mid \Im_t)[t + \textstyle\int_o^T R_t(x \mid \Im_t)dx]$$

Since $h_t(x \mid \Im_t)$ is strictly increasing in $x \geq 0$, it is clear that $\dfrac{d\xi(T)}{dT} > 0$ and then $\xi(T)$ is strictly increasing. Finally, we have

$$\lim_{T\to\infty} \xi(T) = \lim_{T\to\infty}\Big(h_t(T \mid \Im_t)[t + \textstyle\int_o^T R_t(x \mid \Im_t)dx] + R_t(T \mid \Im_t)\Big)$$
$$= \lim_{T\to\infty}[t \cdot h_t(T \mid \Im_t)] + [\lim_{T\to\infty} h_t(T \mid \Im_t)][\lim_{T\to\infty} \textstyle\int_o^T R_t(x \mid \Im_t)dx] + \lim_{T\to\infty} R_t(T \mid \Im_t)$$
$$= \lim_{T\to\infty}[t \cdot h_t(T \mid \Im_t)] + [\lim_{T\to\infty} h_t(T \mid \Im_t)]E[X_t \mid \Im_t] = \infty$$

Hence there exists optimal age T* for replacement which minimizes the long-run expected cost $C_t(T \mid \Im_t)$ in (3.1) and it is unique. ■

Remark. If the condition, $t \cdot h_t(0|\Im_t) < \frac{C_r}{C_f - C_r}$, in Theorem 1 does not hold, there is no guarantee that there exist the optimal age for replacement. We may consider two possible cases which are (i) $t \cdot h_t(0|\Im_t) = \frac{C_r}{C_f - C_r}$ and (ii) $t \cdot h_t(0|\Im_t) > \frac{C_r}{C_f - C_r}$. If $t \cdot h_t(0|\Im_t) = \frac{C_r}{C_f - C_r}$, then the long-run expected cost $C_t(T \mid \Im_t)$ is minimized at $T=0$. Hence, the optimal replacement policy is to replace the system at the current monitoring time. If $t \cdot h_t(0|\Im_t) > \frac{C_r}{C_f - C_r}$, then $C_t(T \mid \Im_t)$ is minimized at $T<0$. Hence is recommended to replace the system at the current monitoring time.

3.2. One-replacement-cycle case

When the planning time span is finite, we can not apply Renewal Reward Theorem. Sheu et. al. (1999) consider total cost per unit time between two successive replacement when the planning time span is finite. We also utilize total cost per unit time between two successive replacements for the case that the planning time span is finite.

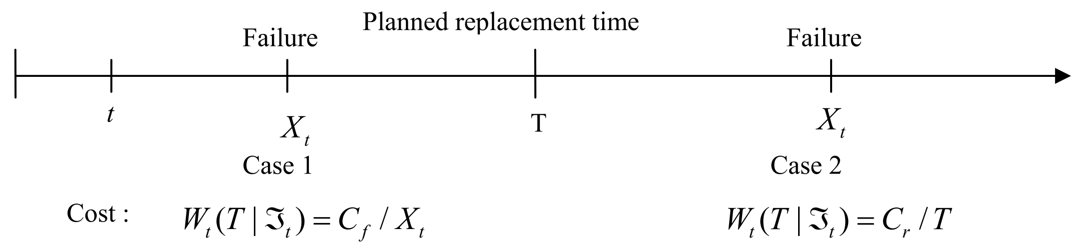

Figure 3.1. Two possible failures and related cost per unit time

1) The expected cost per unit time between two successive replacements

Let C_f and C_r be cost for failure and cost for planned replacement, respectively. We can consider two possible cases of failure as shown in Figure 3.1 and can obtain total cost per unit time between two successive replacements at age t as follows.

$$W_t(T \mid \Im_t) = \left(\frac{C_f}{X_t}\right) I_{(0,T]}(X_t) + \left(\frac{C_r}{T}\right) I_{(T,\infty)}(X_t).$$

Then the expected value of $W(T)$ becomes

$$E[W_t(T \mid \Im_t)] = E[I_{(0,T)}(X_t) \cdot \left(\frac{C_f}{X_t}\right) \mid \Im_t] + E[I_{(T,\infty)}(X_t) \cdot \left(\frac{C_r}{T}\right) \mid \Im_t]$$

$$= C_f \cdot \int_0^T \frac{1}{x} f_t(x \mid \Im_t)dx + C_r \cdot \frac{P(X_t > T \mid \Im_t)}{T} \tag{3.4}$$

2) The optimal replacement time *Twhich minimizes $E[W_t(T) \mid \Im_t]$**

Taking a derivative of $E[W_t(T) \mid \Im_t]$ with respective to T yields

$$\frac{d}{dT} E[W_t(T|\Im_t)] = (C_f - C_r) \cdot \frac{1}{T} f_t(T|\Im_t) - C_r \cdot \frac{1}{T^2} R_t(T|\Im_t)$$
$$= \frac{1}{T^2} R_t(T|\Im_t)\{(C_f - C_r) \cdot T h_t(T|\Im_t) - C_r\}. \tag{3.5}$$

Our goal is to find an optimal age T^* for replacement such that $E[W_t(T) \mid \Im_t]$ is minimized. It is well known that the necessary and sufficient condition for T^* to be optimal is $\left.\frac{d}{dT} E[W_t(T \mid \Im_t)]\right|_{T=T^*} = 0$. That is, the optimal age T^* for replacement is the solution of the following equation if it exits.

$$T \cdot h_t(T|\Im_t) = \frac{C_r}{C_f - C_r} \tag{3.6}$$

The remaining is to show that there exists T^* satisfying $\left.\frac{d}{dT} E[W_t(T) \mid \Im_t]\right|_{T=T^*} = 0$. That is officially stated in the following theorem.

Theorem 2. Suppose that $h_t(x \mid \Im_t)$ is strictly increasing in $x \geq 0$. Then there exists the optimal replacement age T^* which minimizes $E[W_t(T)|\Im_t]$ in (3.4).

Proof. Let $\delta(T)$ be the left-sided term in the equation (3.6). When $T = 0$, it is obvious that $\delta(T = 0) = 0 \cdot h_t(0|\Im_t) < \frac{C_f}{C_f - C_r}$. Taking a derivative of $\delta(T)$ with respect to T yields

$$\frac{d\delta(T)}{dT} = h_t(T|\Im_t) + T \cdot \frac{d}{dT} h_t(T|\Im_t).$$

Since $h_t(x \mid \Im_t)$ is strictly increasing in $x \geq 0$, it is clear that $\frac{d\delta(T)}{dT} > 0$ and then $\delta(T)$ is strictly increasing. Finally, it is clear that $lim_{T\to\infty} \delta(T) = lim_{T\to\infty} T \cdot h_t(T|\Im_t) = \infty$

Hence there exists optimal age T^* for replacement which minimizes the long-run expected cost $E[W_t(T)|\Im_t]$ in (3.4) and it is unique. ■

4. EXAMPLE

Wang (2002) considers the data of overall vibration level of six bearings, which is from a fatigue experiment. It can be seen from Figure 4.1 that all bearings tend to fail after the vibration level is beyond 10 while the life time of bearings varies from around 100 hours to over 1000 hours. Hence it is reasonable to make a decision for planned replacement when the vibration level is beyond 10 which is warning threshold point.

With the estimated model parameters and given condition information history outlined in Wang (2002), the long run expected cost can be calculated with equation (3.1) assuming that the cost for preventive replacements (C_r) is \$1000 and the cost for failure replacements (C_f) is \$2000. For the monitoring time point the 256th hour($=\min_{i\geq 1}\{t_i|Y_i \geq 10\}$), the long run average cost is plotted as a function of preventive replacement time in Figure 4.2 for bearing 4, from which we can see that the optimum replacement time is at the 279th hour to achieve minimum long run average cost of 3.63.

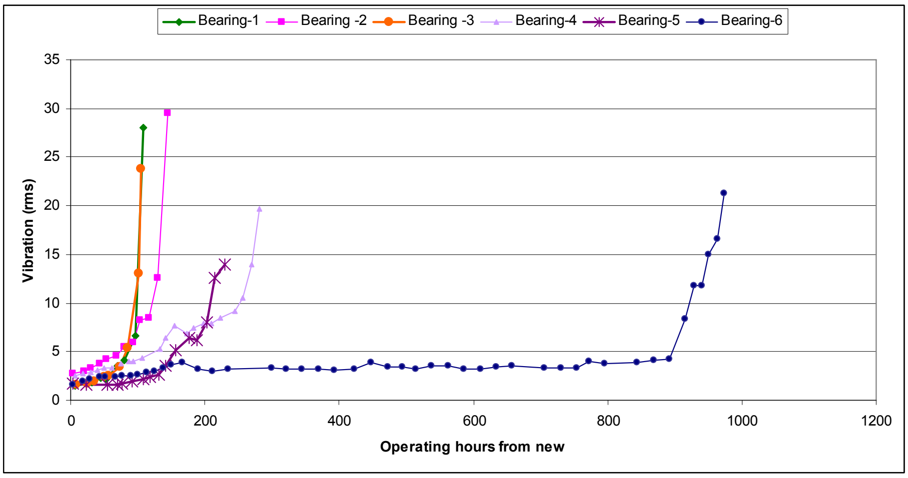

Figure 4.1.Vibration data of six bearings

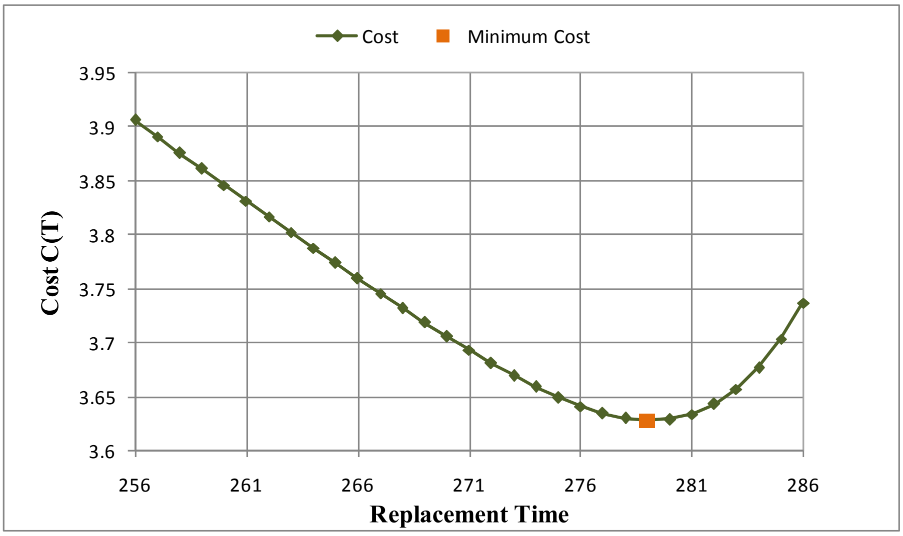

Figure 4.2. Optimum preventive replacement time minimizing the long run average cost

REFERNCES

Bai, D.S. and Yun, W.Y. (1986). An age replacement policy with minimal repair cost limit, *IEEE Transactions on Reliability*, **35**, 452–454.

Banjevic D., Jardine A.K.S., Makis V., Ennis M. (2001). A control-limit policy and software for condition based maintenance optimization, *INFOR,*$\,$**39**, 32-50.

Barlow, R.E. and Hunter, L.C. (1960). Optimum preventive maintenance policies, *Operations Research*, **8**, 90–100.

Baruah, P. and Chinnam R.B. (2005). HMM for diagnostics and prognostics in maching processes, *I. J. Prod. Res.,* **43**, 1275-1293.

Beichelt, F. (1976). A general preventive maintenance policy, *Mathem. Operations Forschung und Statistik,*$\,$**7**, 927–932.

Berg, M., Bienvenu, M. and Cléroux, R. (1986). Age replacement policy with age-dependent minimal repair, *INFOR*,**24**, 26–32.

Block, H.W., Borges, W.S. and Savits, T.H. (1988). A general age replacement model with minimal repair, *Naval Research Logistics*, **35**, 365–372.

Bunks C., McCarthy D., and Al-Ani T. (2000). Condition based maintenance of machine using hidden Markov models, *Mech. Sys. & Sig. Pro.,* **14**, 597-612.

Chien, Y.H. and Sheu, S.H. (2006). Extended optimal age-replacement policy with minimal repair of a system subject to shocks, *European Journal of Operational Research,* **174**, 169–181.

Cleroux, R., Dubuc, S. and Tilquin, C. (1979). The age replacement problem with minimal repair and random repair costs, *Operations Research*, **27**, 1158-1167.

Christer, A. H. (1999). Developments in delay time analysis for modelling plant maintenance. *Journal of the Operational Research Society,* **50***,* 1120-1137.

Christer, A.H and Wang, W. (1995). A simple condition monitoring model for a direct monitoring process, *E. J. Opl. Res.,* **82**, 258-269.

Christer, A.H. and Wang, W. (1992). A model of condition monitoring inspection of production plant, *I. J. Prod. Res.,* **30**, 2199-2211.

Christer, A. H., Wang, W. and Sharp, J. (1997).A state space condition monitoringmodel for furnace erosion prediction and replacement, *European Journal of Operational Research,* **101***,* 1-14.

Coolen, F. P. A. and Dekker, R. (1995). Analysis of a 2-phase model for optimization of condition monitoring intervals, *IEEE Transactions on Reliability,* **55**, 505-511.

Dong, M. and He, D. (2004). Hidden semi-Markov models for machinery health diagnosis and prognosis, *Trans. North Amer. Manu. Res. Ins. of SME,* **32**, 199-206.

Hontelez, J.A.M., Burger, H.H. and Wijnmalen, D.J.D. (1996). Optimum condition based maintenance policies for deteriorating systems with partial information, *Rel. Eng. & Sys. Safety,* **51**, 267-274.

Jardine, A.K.S., Makis, V., Banjevic, D., Braticevic, D., and Ennis M. (1998). A decision optimization model for condition based maintenance, *J. Qua. Main. Eng*., **4**, 115-121.

Jardine, A. K. S. and Tsang, A. H. C. (2006). Maintenance, Replacement, and Reliability: Theory and Applications, CRC Press, Taylor and Frances.

Jhang, J.P. and Sheu, S.H. (1999). Opportunity-based age replacement policy with minimal repair, *Reliability Engineering and System Safety*, **64**, 339–344.

Kalbfleisch, J.D. and Prentice, R.L. (1980).The Statistical Analysis of Failure Time Data,Wiley, New York.

Li, C., Xuhua, C.,Yongsheng, B. and Zhonghua C. (2007). Age replacement model based condition information, The Eighth International Conference on Electronic Measurement and Instruments ICEMI, **2**, 468-471.

Lin, D. and Makis, V. (2003). Recursive filters for a partially observable system subject to random failures, *Adv. Appl. Prob*., **35**, 207-227.

Lin, D. and Makis, V. (2004). Filters and parameter estimation for a partially observable system subject to random failures with continuous-range observations, *Adv. Appl. Prob*., **36**, 1212-1230.

Lipi, T.F., Lim, J.H. and Zuo, M.J. (2010). Age Replacement Policy Based On Condition Information, *The Proceedings of the 4th Asian Pacific International Symposium - Advanced Reliability Modeling IV*, 472-479.

Love, C.E. andGuo, R. (1991). Using proportional hazard modeling in plant maintenance, *Quality and Reliability Engineering International,* **7**, 7-17.

Makis, V. andJardine, A.K.S. (1992). Optimal replacement in the proportional hazards model, *INFOR*, **30**, pp 172-183.

Kumar, D., and Westberg, U. (1997). Maintenance scheduling under age replacement policy using proportional hazard modeling and total-time-on-test plotting, *Euro. J. Opl. Res.,* **99**, 507-515.

Park, K.S. (1988). Optimal wear-limit replacement with wear-dependent failure, *IEEETransactions on Reliability,* **37**, 293-294

Ross, S.M. (1992). Applied probability models with optimization applications, Courier Dover Publications, New York.

Scarf, P. A. (2007). A framework for condition monitoring and condition based maintenance, *Quality Technology & Quantitative Management*, **4**, 301-312.

Sheu, S.H. (1991). A general age replacement model with minimal repair and general random repair cost, *Microelectronics & Reliability*, **31**, 1009-1017.

Sheu, S.H. (1998). A generalized age and block replacement of a system subject to shocks, *European Journal of Operational Research*, **108**, 345–362.

Sheu, S.H. and Chien, Y.H. (2004). Optimal age-replacement policy of a system subject to hocks with random lead-time, *European Journal of Operational Research*, **159**, 132–144.

Sheu, S.H., Kuo, C.M. and Nakagawa, T. (1993). Extended optimal age replacement policy with minimal repair, *PAIRO Recherch eOperationnelle*, **27**, 337–351.

Sheu, S.H. and Griffith, W.S. (1996). Optimal number of minimal repairs before replacement of a system subject to shocks, *Naval Research Logistics*, **43**, 319–333.

Sheu, S.H., Griffith, W.S. and Nakagawa, T. (1995). Extended optimal replacement model with random minimal repair costs, *European J. Operational Research*, **85**, 636–649.

Sheu, S.H.,Yeh, R.H., Lin, Y.B. and Juang, M.G. (1999). A Bayesian perspective on age replacement with minimal repair, *Reliability Engineering & System Safety*, **65**, 55-64.

Wang, W. (2002). A model to predict Residual life of rolling element bearings given monitored condition information to date, *IMA Journal of management mathematics*, 13, 3-16.

Wang, W. (2007). A prognosis model for wear prediction based on oil based monitoring, *Journal of the Operational Research Society,* **58**, 887-893.

Wang ,W. and Christer, A.H. (2000). Towards a general condition based maintenance model for a stochastic dynamic system, *J. Opl. Res. Soc.* 51, 145-155.

Wang, W., Scarf, P.A. and Smith, M. A.J. (2000). On the application of a model ofcondition based maintenance, *Journal of the Operational Research Society,* **51***,* 1218-1227.

Wang, W. and Zhang, W. (2005). A model to predict the residual life of aircraft engines based upon oil analysis data, *Naval Research Logistics*, ***52***(3), 276-284.