특별직교이방성 적층판의 고유진동수에 대한 형상비의 영향

The Influence of the Aspect Ratio on the Natural Frequency of the Specially Orthotropic Laminated Plates

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Abstract

Advanced composite structures are too difficult for such design engineers for construction and some simple but accurate enough methods are necessary. The simply supported laminated plates are analyzed by the specially orthotropic laminates theory. This method, however, may be too difficult for some practising engineers. In this paper, the result of analysis for such plate by means of the beam theory with unit width is reported. The plate aspect ratio considered is from 1 : 1 to 1 : 5. Most of the bridge and building slabs on girders have large aspect ratios. For such cases further simplification is possible by neglecting the effect of the longitudinal moment terms(M_x) on the relevant partial differential equations of equilibrium. In this paper. the influence of the aspect ratio on the natural frequency of the specially orthotropic laminated plates is studied and it is concluded that the method used is sufficiently accurate for engineering purposes. The result of this paper can be used for simply supported laminated plates analysis.

Keywords : Natural frequency, Aspect ratio, Specially orthotropic laminated plates

1. INTRODUCTION

The problem of deteriorated highway slabs is very serious all over the world. Before making any decision on repair work, reliable evaluation is necessary. One of the non-destructive methods is to evaluate the in-situ stiffness of the slab by obtaining the natural frequencies of the system. By comparing the in-situ stiffness with the one obtained at the design stage, the degree of damage can be estimated.

There are several means for slab system analysis such as

(1) Beam strip method,
(2) Composite beam theory between concrete slab and steel beams, and
(3) Gird analysis method for cross beams and girders.

Composite materials can be used economically and efficiently in broad civil engineering applications when standards and processes for analysis, design, fabrication, construction and quality control are established. The problem of deteriorating infrastructures is very serious in our country. The U.S. Civil Engineering Research Foundation (CERF) report, “High-Performance Construction Material and System: An Essential Program for America and its Infrastructure”, published, in collaboration with several organizations, U.S. Department of Transportation figures as follows:

(1) The road bridge condition in U.S.A at the year 2009, 149,654 of the Americans 603,259 bridges are structurally deficient or obsolete. (structurally deficient 71,177, functionally obsolete 78,477)
(2) 199,584 of these bridges are more than 50

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years old and unsuitable for current or projected traffic.

(3) Traffic delays alone will cost Americans \$115 billion per year in lost work time and fuel by the year 2009.

Steel girders become rusty. The reinforcing bars embedded in concrete beams or slabs are subject to corrosion caused by electro-chemical action. Underground fuel thanks are under similar condition. The U.S. Bureau of Standards (NIST) study showed that yearly loss caused by corrosion related damages mounted to 82 billion dollars, about 4.9% of GNP. About 32 billion dollars could be saved if existing technologies were used to prevent such losses.

These figures are in the United States of America, where various federal, state, and other agencies are doing their best in maintaining such structures in good condition. The issue of deteriorating and damaged infrastructures and lifelines has become a critically important subject in the United States as well as Japan and Europe. The problem in developing nations, where degree of construction quality control and maintenance are in question, must be much more profound [Kim 1995, Han 2010].

The advanced composite materials can be effectively used for repairing such structures. Because of the advantages of these materials, such repair job can fulfill two purposes :

(1) Repair of existing damage caused by corrosion, impact, earthquake, and others.

(2) Reinforcing the structure against anticipated future situation which will require increasing the load beyond the design parameters used for this structure.

Before making any decision on repair work, reliable non-destructive evaluation is necessary. One of the dependable methods is to evaluate the in-situ stiffness of the structure by means of obtaining the natural frequency. By comparing the in-situ stiffness with the one obtained at the design stage, the degree of damage can be estimated rather accurately.

The reinforced concrete slab can be assumed as a [0, 90, 0]r type specially orthotropic plate as a close approximation, assuming that the influence of B_{16}, B_{26}, D_{16} and D_{26} stiffness are negligible. Many of the bridge and building floor systems, including the girders and cross beams, also behave as similar specially orthotropic plates. Such plates are subject to the concentrated mass/masses in the form of traffic loads, or the test equipments such as the accelerator in addition to their own masses. Analysis of such problems is usually very difficult.

The most of the design engineers for construction has academic background of bachelors degree. Theories for advanced composite structures are too difficult for such engineers and some simple but accurate enough methods are necessary.

Most of the civil structures are large in sizes and the numbers of laminae are large, even though the thickness to length ratios are small enough to allow to neglect the transverse shear deformation effects in stress analysis. For such plates, the fiber orientations given above behave as specially orthotropic plates and simple formulas developed by the reference [Kim 1995, Han & Kim 2001, 2003] can be used.

Most of the bridge and building slabs on girders have large aspect ratios. For such cases further simplification is possible by neglecting the effect of the longitudinal moment terms (M_x) on the relevant partial differential equations of equilibrium [Han & Kim, 2001]. In this paper, the result of the study on the subject problem is presented. Even with such assumption, the specially orthotropic plate with boundary conditions other than Navier or Levy solution types, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical method for eigenvalue problems are also very much involved in seeking such a solution [Timosenko 1989, Ashton 1970, Whitney 1970, Pagano 1970].

The method of vibration analysis used is the one developed by the author. He developed and reported a simple but exact method of calculating the natural frequency of beam and tower structures with irregular cross sections and attached mass/masses. This method has been extended to two dimensional problems with several types of given conditions and has been reported at several international conferences.

2. METHOD OF ANALYSIS

2.1 Vibration Analysis

In this paper, the method of analysis given in detail, in the reference book [Kim 1995] is briefly repeated.

The magnitudes of the maximum deflection at a certain number of points are arbitrarily given as

$$w\ (i,j)(1) = W\ (i,j)(1) \tag{1}$$

where (i,j) denotes the point under consideration. This is absolutely arbitrary but educated guessing is good for accelerating convergence. The dynamic force corresponding to this (maximum) amplitude is

$$F(i,j)(1) = m(i,j)[\omega(i,j)(1)]^2 W(i,j)(1) \tag{2}$$

The "new" deflection caused by this force is a function of f and can be expressed as

$$w\ (i,j)(2) = f\ \{m(i,j)[\omega(i,j)(1)]^2 W(i,j)(1)\}$$
$$= \sum^{k,l} \triangle(i,j,k,l)\{m(i,j)[\omega(i,j)(1)]^2 W(k,l)(1)\} \tag{3}$$

where $\triangle$ is the deflection influence surface.

The relative (maximum) deflections at each point under consideration of a structural member under resonance condition, $w(i,j)(1)$ and $w(i,j)(2)$, have to remain unchanged and the following condition has to be held :

$$w\ (i,j)(1)/\ w\ (i,j)(2) = 1 \tag{4}$$

From this equation, $w(i,j)(1)$ at each point of (i,j) can be obtained. But they are not equal in most cases. Since the natural frequency of a structural member has to be equal at all points of the member, i.e. $w(i,j)$ should be equal for all (i,j), this step is repeated until sufficient equal magnitude of $w(i,j)$ is obtained at all (i,j) points.

However, in most cases, the difference between the maximum and the minimum values of $w(i,j)$ obtained by the first cycle of calculation is sufficiently negligible for engineering purposes. The accuracy can be improved by simply taking the average of the maximum and the minimum, or by taking the value of $w(i,j)$ where the deflection is the maximum. For the second cycle, $w(i,j)(2)$ is

$$w(i,j)(3) = f\{m(i,j)[\omega(i,j)(2)]^2 W(i,j)(2)\} \tag{5}$$

the absolute numerics of $w(i,j)(2)$ can be used for convenience.

2.2 Finite Difference Method

The method used in this paper requires the deflection influence surfaces. F.D.M is applied to the governing equation of the specially orthotropic plates,

$$D_1\frac{\partial^4 w}{\partial x^4} + 2D_3\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2\frac{\partial^4 w}{\partial y^4}$$
$$= q(x,y) - k\ w + Nx\frac{\partial^2 w}{\partial x^2}$$
$$+ Ny\frac{\partial^2 w}{\partial y^2} + 2Nxy\frac{\partial^2 w}{\partial x \partial y} \tag{6}$$

where, $D_1 = D_{11},\ D_2 = D_{22}, D_3 = (D_{12} + 2D_{66})$.

The number of the pivotal points required in the case of the order of error $\triangle^2$, where $\triangle$ is the mesh size, is five for the central differences, This makes the procedure at the boundaries complicated. In order to solve such problem, the three simultaneous partial differential equations of equilibrium with three dependent variables, w, M_x and M_y, are used instead of Eq.(6) with $N_x = N_y = N_{xy} = 0$ [Kim 1974, Han & Kim 2001].

$$\frac{\partial^2 Mx}{\partial x^2} - 4D_{66}\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^2 My}{\partial y^2} = -q(x,y) + kw(x,y) \quad (7)$$

$$M_x = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} \quad (8)$$

$$M_y = -D_{12}\frac{\partial^2 w}{\partial x^2} - D_{22}\frac{\partial^2 w}{\partial y^2} \quad (9)$$

If F.D.M is applied to these equations, the resulting matrix equation is very large in sizes, but the tridiagonal matrix calculation scheme used by Kim [Kim 1974] is very efficient to solve such equations. Since one of the few efficient analytical solutions of the specially orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, F.D.M is used to solve this problem. The result is satisfactory as expected.

By neglecting the Mx terms, the sizes of the matrices needed to solve the resulting linear equations are reduced to two thirds of the “non-modified” equations (4).

In order to confirm the accuracy of the F.D.M., [A/B/A]r type laminate with aspect ratio of a/b = 1m/1m = 1 is considered.

For simplicity, it is assumed that $A = 0°, B = 90°$, and r=1. Since one of the few efficient analytical solutions of the specially orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, F.D.M. is used to solve this problem and the result is compared with the Navier solution.

Calculation is carried out with different mesh sizes and the maximum errors at the center of the plate are as follows. 10 × 10 case is 0.140 % and 20 × 20 case is 0.035 %. The error is less than 1%. This is smaller than the predicted theoretical errors ; If F.D.M is applied to these equations, the resulting matrix equation is very large in sizes, but the tridiagonal matrix calculation scheme used by author is very efficient to solve such equations [Kim 1974].

Since one of the few efficient analytical solutions of the specially orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, F.D.M is used to solve this problem. The result is satisfactory as expected.

3. NUMERICAL EXAMINATION

3.1 Structure under consideration

The specially orthotropic laminated plate exist in field considered as shown in Fig. 1. In design and analysis, an engineer may try to use symmetric sections if possible. It is much easier for analysis since all bending-extension coupling stiffness terms vanish. The laminate does not have an inherent tendency to twist when extended or contracted by any reason including thermal effect.

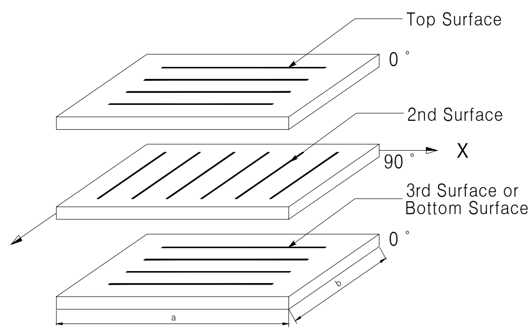

Fig. 1 Configuration of [a/b/a]r Laminated Plate

The material properties of glass fiber are :

$E_1 =$ 67.36 GPa, $E_2 =$ 8.12 GPa,

$G_{12} =$ 3.0217 GPa

$\nu_{12} =$ 0.272, $\nu_{21} =$ 0.0328, r = 1

Ply thickness = 0.005 m
Orientation : [90°, 0°, 90°]r

The stiffnesses are :
D_{11} = 2,929, D_{22} = 18,492, D_{12} = 627
a = nb, n = an integer 1~5
and D_{66} = 849, b = 3 m
Loading : q = 286.65 N/m^2

Boundary condition of plate is as shown in Fig. 2 and Fig. 3.

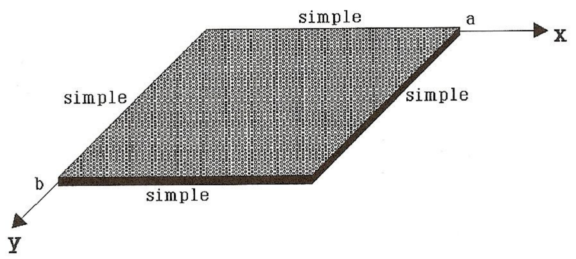

Fig. 2 Boundary Condition of Plate (SS)

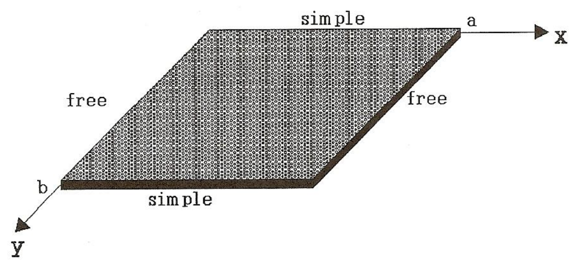

Fig. 3 Boundary Condition of Plate (SF)

3.2 Numerical Results

In order to study the influence of M_x on the equilibrium equations, three cases are considered :
Case A : w , M_x and M_y are considered.
Case B : w and M_y are considered, M_x is neglected.
Case C : beam with unit width.

F.D.M. is used to obtain w, M_x, M_y and obtain the natural frequency. The result is as shown in Fig. 4 and Fig. 5.

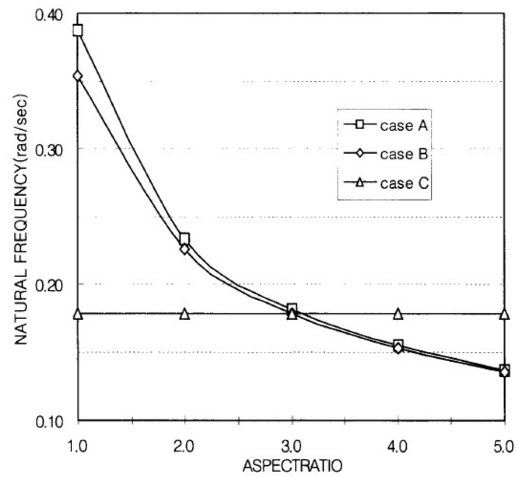

Fig. 4 Natural Frequency of Case A, Case B, Case C (SS)

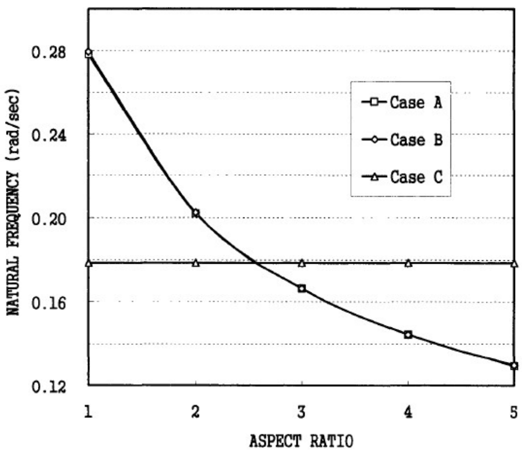

Fig. 5 Natural Frequency of Case A, Case B, Case C (SF)

Table 1 Ratios of the natural frequencies (SS case)

a/b	1	2	3	4	5
ω_A/ω_B	1.0960	1.0303	1.0189	1.0137	1.0107

The result of numerical examination is quite promising. Plates with all edges simple supported (SS), the ratios of the natural frequencies at the center of the uniformly loaded plate are as shown in Table 1.

Plates with one side simple and the other side free supported (SF), the ratios of the natural frequencies at the center of the uniformly loaded plate are as shown in Table 2.

It is concluded that, for all boundary conditions, neglecting M_x terms is acceptable if the aspect

Table 2 Ratios of the natural frequencies (SF case)

a/b	1	2	3	4	5
ω_A/ω_B	0.9957	0.9977	0.9985	0.9988	0.9991

ratio (a/b) is equal to or larger than 2.

4. CONCLUSION

Many engineers use the load distribution factor and beam theory to design panel systems. The purpose of this paper is to demonstrate, to the practicing engineers, how to apply the specially orthotropic plate theory to the slab systems made of plate girders and cross beams.

Most of the bridge and building slabs have plate aspect ratios larger than 2. For such cases, design analysis becomes much simpler if influence of the longitudinal moment (M_x) terms on the relevant differential equations of equilibrium can be neglected. The result of the study on this subject is presented in this paper.

It is concluded that, for all boundary conditions, neglecting M_x terms is acceptable if the aspect ratio (a/b) is equal to or larger than 2. This conclusion gives good guide line for design of bridge and building slabs on main girders.

REFERENCES

1. Ashton, J. E., "*Anisotropic Plate Analysis- Boundary Condition*", Jour. of Composite Materials, April, 1970, pp.162-171,
2. Goldberg, John E. and Kim, D. H., "*The Effect of Neglecting the Radial Moment Term in Analyzing a Sectorial Plate by Means of Finite Differences*", Proc, of the Seventh International Symposium on Space Technology and Sciences, Tokyo, Japan, 1967.
3. Han, B. K. and Kim D. H., "*Analysis of Steel Bridges by Means of Specially Orthotropic Plate Theory*", Jour. of KSSC, vol 13, No. 1, 2001, pp.61-69.
4. Han, B. K, and Kim, D. H., "*Analysis of Prstressed Concrete Slab Bridge by Beam Theory*", Jour. of KISM, vol. 7, No. 2, 2003, pp.115-124.
5. Han, B. K, and Kim, D. H., "*Simple Method of Analysis for Reinforced Concrete Slab Bridges*", Jour. of KISM, vol. 7, No. 4, 2003, pp.110-118.
6. Han, B. K. and Suk, J. W., "*The Influence of the Aspect Ratio on the Natural Frequency of the Composite Laminated Plates*", Jour. of the Korean Society for Advanced Composite Structures, vol. 1, No. 2, 2010, pp.14-18.
7. Kim. D. H., "*A Method of Vibration Analysis of Irreqularly Shaped Structural Members*", Proceedings, International Symposium on Engineering Problems in Creating Coastal Industrial Sites, Seoul, Korea, October, 1974.
8. Kim, D. H., "*Simple Method of Anaysis for Preliminary Design of Certain Composite Laminated Primary Structures for Civil Construction II*", Journal of Materials Processing Technology, 55, Elsevier, London, 1993.
9. Kim, D. H., "*Composite Structure for Civil and Architectural Engineering*", E & FN Spon, 1st edition, London, 1995,
10. Pagano, N. J., "*Exact Solution for Rectangular Bidirectional Composites and Sandwich Plates*", Jour. of Composites Materials, vol. 4, No. 1, 1970, pp.20-34.
11. Stephen P. Timoshenko, and S. Woinowsky- krieger, "*Theory of Plates and Shells, Second Edition*", Mcgraw Hill Book Co., 1989.
12. Whitney, J. M. and Leissa, A. W., "*Analysis of a Simply Supported Laminated Anisotropic Rectangular Plate*", Jour. of AIAA, vol. 8, No. 1, 1970, pp.28-33, 1970.

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요 지

건설기술자들에게는 첨단 복합재료구조에 대한 이론이 너무 어려워서 간단하면서도 쉽게 적용할 수 있는 정확한 방법을 필요로 하고 있다. 단순지지된 적층판을 특별직교이방성 적층판 이론에 의하여 해석하였다. 본 연구에서는 형상비를 1 : 1 ~ 1 : 5까지 변화시켜가며 해석을 수행하였다. 대부분의 교량이나 건물의 상판은 형상비가 큰 경우가 많은데, 이런 구조물의 평형방정식에 대한 종방향 모멘트항(Mx)의 영향은 매우 작아서, 더욱 간단한 해석이 가능하다. 본 논문에서는 특별직교이방성 적층판의 고유진동수에 대한 형상비의 영향을 연구하였으며 이 방법을 사용하면 충분히 정확한 값을 산출할 수 있다. 본 논문의 연구의 결과는 단순지지된 특별직교이방성 적층판의 해석에 이용할 수 있다.

핵심 용어 : 고유진동수, 형상비, 특별직교이방성 적층판