Discrete-Time Sliding Mode Control for Robot Manipulators†

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Abstract In the real-field of control cases for robot manipulators, there always exists a modeling error, which results the model has the uncertainties in its parameters and/or structure. In many modern applications, digital computers are extensively used to implement control algorithms to control such systems. The discretization of the nonlinear dynamic equations of such systems results in a complicated discrete dynamic equations. Therefore, it will be difficult to design a discrete-time controller to give good tracking performances in the presence of certain uncertainties. In this paper, a discrete-time sliding mode control algorithm for nonlinear and time varying robot manipulators with uncertainties is presented. Sufficient conditions for guaranteeing the convergence of the discrete-time SMC system are derived. As example simulations, the proposed SMC algorithm is applied to a two-link robotic manipulator with unknown dynamics. The results of the simulation indicate that the developed control scheme is effective in manipulators and electro-mechanical system control.

Key Words: Sliding Mode Control(SMC), Discrete-time Controller, Robot Manipulator

1. INTRODUCTION

In recent years, methodology known as *sliding mode control* (SMC) has been researched actively, and the sliding mode control has effectively used in the tracking control of robot manipulators by many researchers[1,2,4-7,9,11-13]. The concept of sliding mode control has been studied in detail in Utkin [10], where it has been used to stabilize a class of non-linear systems. However, most of the related works [2,4,7,8] were based on the manipulator continuous-time linear parameterization model, which is a linear equation in terms of the unknown or un-precisely known parameter vector. In addition, most continuous-time control laws require a large amount of computation.

The discretization of the nonlinear dynamic equations of such systems results in a complicated discrete dynamic equations. Therefore, it will be difficult to design a discrete-time controller to give good tracking performances in the presence of certain uncertainties. Whereas, the digital control algorithm is usually very simple and can be easily implemented on-line using microcomputers, so it is predominantly more effective in actual application. But only a few literatures [1,3,5,6] have studied the manipulator discrete-time model and discrete-time control so far.

In this paper, based on the continuous-time SMC theory[8,10,14,15,16], when the sample time is small enough, the manipulator dynamic model is equivalent to a 2 order discrete-time equation whose

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coefficients are slowly time-varying and also possess some valuable characteristic relationships. We define the discrete time equation as the characteristic model of manipulators, and then a discrete-time SMC law is presented based on the manipulator characteristic model. Also, sufficient conditions for guaranteeing the convergence of the discrete-time SMC system are derived. With this control scheme, the problems resulting from the parametric and nonparametric uncertainty can be overcome effectively. As example simulations, the proposed SMC algorithm is applied to a two-link robotic manipulators with unknown dynamics. The results of the simulation indicate that the developed control scheme is effective in manipulators and electro-mechanical system control. This paper is organized as follows: In Section 2, the dynamics formulation and the manipulator characteristic model is introduced. In Section 3 and 4, a discrete-time SMC law is presented. A simulation example is given in Section 5. Finally, some conclusions are made in Section 6.

2. MANIPULATOR DYNAMIC MODEL AND CHARACTERISTIC MODEL

Consider a robotic manipulator with n degrees of freedom. The continuous Lagrangian dynamic model [2,4,7] is given by

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

Where $\tau \in \mathrm{R}^n$ is the vector of joint torques supplied by the actuators. $q,\dot{q},\ddot{q} \in \mathrm{R}^n$ are the position, velocity and acceleration of joint coordinates, with $q=[q_1,q_2,\cdots,q_n]^{\mathrm{T}}$. $M(q) \in \mathrm{R}^{n\times n}$ is a manipulator mass matrix. It is symmetric, positive definite and there exist two constant positive scalars $M_{\min}$ and $M_{\max}$ such that $M_{\min} \le \|M\| \le M_{\max}$. The vector $V(q,\dot{q})\dot{q} \in \mathrm{R}^n$ represents torques arising from centrifugal and Coriolis forces. The vector $G(q) \in \mathrm{R}^n$ represents torques due to gravity. If the designed $\tau(t,q,\dot{q})$ is continuous in t,q and $\dot{q}$ then the solution $(q,\dot{q})$ of (1) will be continuously differentiable.

In joint space, the control problem for robot manipulators is to synthesize a control law for the torques such that the joint output, $q(t) \in \mathrm{R}^n$, traces the desired trajectory, $q_d(t) \in \mathrm{R}^n$, with a certain precision defined by

$$\tilde{\mathrm{q}} = [\tilde{q}\ \dot{\tilde{q}}]^{\mathrm{T}},\ \|\tilde{q}\| = \|q - q_d\| \le \gamma_1,\ \|\dot{\tilde{q}}\| = \|\dot{q} - \dot{q}_d\| \le \gamma_2$$
$$\gamma_1 > 0, \gamma_2 > 0 \tag{2}$$

It is assumed that $q_d(t), \dot{q}_d(t)$ and $\ddot{q}_d(t)$ are well defined and bounded for all operational time t.

When the sample time T_s is small enough, at instant $t = kT_s$, $\dot{q}$ and $\ddot{q}$ can be approximated by

$$\dot{q} \approx \frac{q(k)-q(k-1)}{T_s} \quad \text{and} \quad \ddot{q} \approx \frac{q(k+1)-2q(k)+q(k-1)}{T_s^2}$$

respectively. Using the above relationships the discrete-time representation of (1) becomes[4]

$$\frac{1}{T_s^2} M(q(k))\cdot q(k+1) + f_1(k)q(k) + f_2(k)q(k-1) + G(q(k)) = \tau(k) \tag{3}$$

Where

$$f_1(k) = -\frac{2}{T_s^2} M(q(k)) + \frac{1}{T_s} V(q(k),\dot{q}(k)) \tag{4}$$

$$f_2(k) = \frac{1}{T_s^2} M(q(k)) - \frac{1}{T_s} V(q(k), \dot{q}(k))$$

Pre-multiplying (3) by $T_s^2 M^{-1}(q(k))$ results in

$$q(k+1) = \Gamma(k)\cdot(-f_1(k)q(k) - f_2(k)q(k-1) - G(q(k)) + \tau(k)) \quad (5)$$

Where

$$\Gamma(k) = T_s^2 M^{-1}(q(k)) \quad (6)$$

Generally, parameters of robot manipulators are not known exactly, and are time varying.

Let $\hat{M}(q(k))$, $\hat{f}_1(k)$ and $\hat{f}_2(k)$ as the nominal value of $M(q(k))$, $f_1(k)$ and $f_2(k)$ respectively. Then, we can write

$$\tilde{M}(q(k)) = M(q(k)) - \hat{M}(q(k)) \quad (7)$$

$$\tilde{f}_1(k) = f_1(k) - \hat{f}_1(k)$$

Define

$$e(k) = q(k) - q_d(k) \quad \text{and} \quad \sigma(k) = e(k) + \Lambda e(k-1) \quad (8)$$

with Λ is chosen to satisfy $\sigma(k) = 0$.

Lemma 1:

(a) There exist positive scalar function $\bar{\eta}(k) \in \mathrm{R}$ such that

$$\Gamma(k)\cdot[\frac{1}{T_s^2}\tilde{M}(q(k))\cdot(q_d(k+1) + (1-\Lambda)e(k) + \Lambda e(k-1))] + \Gamma(k)\cdot[\tilde{f}_1(k)q(k) + \tilde{f}_2(k)q(k-1) + \tilde{G}(q(k)] \le \Gamma(k)\phi(k)\bar{\eta} \quad (9)$$

with

$$\phi(k) = 1 + \|\sigma(k)\| + \|\sigma(k)\|^2 \quad (10)$$

(b) There exist positive scalar function $\beta(k) \in \mathrm{R}$ such that

$$\Gamma(k)\cdot[\frac{1}{T_s^2}\tilde{M}(q(k))\cdot(q_d(k+1) + (1-\Lambda)e(k) + \Lambda e(k-1))] + \Gamma(k)\cdot[\tilde{f}_1(k)q(k) + \tilde{f}_2(k)q(k-1) + \tilde{G}(q(k)] - \Gamma(k)\phi(k)\bar{\eta} \le \beta(k) \quad (11)$$

Proof: It is well known that M is positive definite and a bounded matrix function of $q(k)$. $V(q(k), \dot{q}(k))$ is a function at most quadratic in $\dot{q}(k)$, thus $f_1(k)$, $f_2(k)$ and $G(k)$ are bounded. Noting that $q_d(k)$ and $\dot{q}_d(k)$ are bounded, we have that $q(k)$ and $\dot{q}(k)$ are bounded on $\|e(k)\|$ and $\|\dot{e}(k)\|$ respectively. From (8), we see that $\|e(k)\|$ is bounded on $\|\sigma(k)\|$. Therefore, it is clear that there exist bounded nonlinear functions $r_1(k), r_2(k) \in R$ and $\bar{\theta}(k) \in R^n$ such that

$$\Gamma(k)\cdot[\frac{1}{T_s^2}\tilde{M}(q(k))\cdot(q_d(k+1) + (1-\Lambda)e(k) + \Lambda e(k-1))] + \Gamma(k)\cdot[\tilde{f}_1(k)q(k) + \tilde{f}_2(k)q(k-1) + \tilde{G}(q(k)] = \bar{\theta} + r_1\|\sigma\| + r_2\|\sigma\|^2 \quad (12)$$

Now, let $\bar{\eta}(k)$ be the maximum value of functions $r_1(k), r_2(k)$ and $\bar{\theta}(k)$. Then, clearly (9) holds. This implies that there exist bounded functions such that (b) holds.

3. DISCRETE-TIME COMPUTED TORQUE CONTROL

In (3), if $M(q(k))$, $f_1(k)$ and $f_2(k)$ are completely known, then $f_1(k) = \hat{f}_1(k)$, $f_2(k) = \hat{f}_2(k)$ and $G(q(k)) = \hat{G}(q(k))$ The desired torque can be computed at every instant by using the computed torque method (also called inverse dynamics) as follows

$$\tau(k) = u_{eq}(k) \tag{13}$$

with $u_{eq}(k)$ as

$$u_{eq}(k) = \frac{1}{T_s^2}\hat{M}(q(k))\cdot(q_d(k+1)+(1-\Lambda)e(k)+\Lambda e(k-1))+ \hat{f}_1(k)q(k)+\hat{f}_2(k)q(k-1)+\hat{G}(q(k)) \tag{14}$$

Then, from (3) and (12) we get

$$\frac{1}{T_s^2}M(q(k))\cdot q(k+1)+f_1(k)q(k)+f_2(k)q(k-1)+G(q(k))$$

$$=\frac{1}{T_s^2}\hat{M}(q(k))\cdot(q_d(k+1)+(1-\Lambda)e(k)+\Lambda e(k-1))+ \hat{f}_1(k)q(k)+\hat{f}_2(k)q(k-1)+\hat{G}(q(k))$$

$$\Rightarrow \frac{1}{T_s^2}\hat{M}(q(k))\cdot[q(k+1)-q_d(k+1)+(1-\Lambda)e(k)+\Lambda e(k-1)]=0$$

$$\Rightarrow \frac{1}{T_s^2}\hat{M}(q(k))\cdot(e(k+1)+(1-\Lambda)e(k)+\Lambda e(k-1))=0 \tag{15}$$

Therefore, it can be seen that $\Lambda = diag(\Lambda_1, \Lambda_2, \cdots, \Lambda_n)$ and Λ_i can be chosen to satisfy (8).

4. DISCRETE-TIME SLIDING MODE CONTROL

In this section, we propose a discrete-time sliding mode control law to compute the control input torque for the uncertain system (3) as follows

$$\tau(k) = u_{eq}(k)+u_{sm}(k) \tag{16}$$

where $u_{eq}(k)$ is nominal torque vector shown as (14) and

$$u_{sm}(k)=\begin{cases} -\phi(k)\eta_{sm}\frac{\sigma(k)}{\|\sigma(k)\|}, \text{ if } \|\sigma(k)\|>\varepsilon \\ \qquad ;\eta_{sm}>0, \quad \varepsilon>0 \\ -\phi(k)\eta_{sm}\frac{\sigma(k)}{\varepsilon}, \quad \text{otherwise} \end{cases} \tag{17}$$

with $\phi(k)$ defined by (10). In (14), we see that $u_{sm}(k)$ is the sliding mode torque vector with $\sigma(k)$ as the discrete-time sliding (hyper) surface and gain η_{sm} is defined by Lemma 1 as $\eta_{sm} \geq \bar{\eta}(k)$. Then, we have the following result.

Theorem 1 Consider the system (3) with the control law (13). The closed-loop system is globally stable in the sense that the tracking error $\sigma(k)$ is globally bounded by

$$\|\sigma(k)\| \leq \varepsilon \tag{18}$$

Proof: Choose a Lyapunov function:

$$V(k) = \frac{1}{2}\sigma(k)^{\mathrm{T}}\sigma(k) \tag{19}$$

From (8) and (5) we get

$$\begin{aligned}\sigma(k+1) &= q(k+1)-q_d(k+1)+\Lambda e(k) \\ &= \Gamma(k)\cdot[-f_1(k)q(k)-f_2(k)q(k-1)- \\ &\quad G(q(k))+\tau(k)]-q_d(k+1)+\Lambda e(k)\end{aligned} \tag{20}$$

By substituting (12) ,(13) and (14) into (20) yields

$$\begin{aligned}\sigma(k+1) &= -\Gamma(k)\cdot[f_1(k)q(k)+f_2(k)q(k-1)+G(q(k))] \\ &+\Gamma(k)\cdot[\frac{1}{T_s^2}\hat{M}(q(k))\cdot(q_d(k+1)+(1-\Lambda)e(k)+\Lambda e(k-1))] \\ &+\Gamma(k)\cdot(\hat{f}_1(k)q(k)+\hat{f}_2(k)q(k-1)+\hat{G}(q(k))) \\ &+\Gamma(k)\cdot u_{sm}(k)-q_d(k+1)+\Lambda e(k)\end{aligned}$$

$$
\begin{aligned}
&= \Lambda e(k) + (1-\Lambda)e(k) + \Lambda e(k-1) \\
&\quad + \Gamma(k)\cdot[\frac{1}{T_s^2}\tilde{M}(q(k))\cdot(q_d(k+1)+(1-\Lambda)e(k)+\Lambda e(k-1))] \\
&\quad + \Gamma(k)\cdot[\tilde{f}_1(k)q(k)+\tilde{f}_2(k)q(k\text{-}1)+\tilde{G}(q(k)]+\Gamma(k)\cdot u_{sm}(k) \\
&= e(k) + \Lambda e(k-1) \\
&\quad + \Gamma(k)\cdot[\frac{1}{T_s^2}\tilde{M}(q(k))\cdot(q_d(k+1)+(1-\Lambda)e(k)+\Lambda e(k-1))] \\
&\quad + \Gamma(k)\cdot[\tilde{f}_1(k)q(k)+\tilde{f}_2(k)q(k\text{-}1)+\tilde{G}(q(k)]+\Gamma(k)\cdot u_{sm}(k) \\
&= \sigma(k) + \Gamma(k)\cdot[\frac{1}{T_s^2}\tilde{M}(q(k))\cdot(q_d(k+1)+(1-\Lambda)e(k)+\Lambda e(k-1))] \\
&\quad + \Gamma(k)\cdot[\tilde{f}_1(k)q(k)+\tilde{f}_2(k)q(k\text{-}1)+\tilde{G}(q(k)]+\Gamma(k)\cdot u_{sm}(k)
\end{aligned} \tag{21}
$$

Now we let

$$\Delta\sigma(k+1) = \sigma(k+1) - \sigma(k) \tag{22}$$

Then from (21) and (22) we get

$$
\begin{aligned}
&\Delta\sigma(k+1) = \\
&\quad \Gamma(k)\cdot[\frac{1}{T_s^2}\tilde{M}(q(k))\cdot(q_d(k+1)+(1-\Lambda)e(k)+\Lambda e(k-1))] \\
&\quad + \Gamma(k)\cdot[\tilde{f}_1(k)q(k)+\tilde{f}_2(k)q(k\text{-}1)+\tilde{G}(q(k)]+\Gamma(k)\cdot u_{sm}(k) \\
&= \Gamma(k)\cdot h_M(k) + \Gamma(k)\cdot h_f(k) + \Gamma(k)\cdot u_{sm}(k)
\end{aligned} \tag{23}
$$

where

$$h_M(k) = \frac{1}{T_s^2}\tilde{M}(q(k))\cdot(q_d(k+1)+(1-\Lambda)e(k)+\Lambda e(k-1))$$

$$h_f(k) = \tilde{f}_1(k)q(k) + \tilde{f}_2(k)q(k\text{-}1) + \tilde{G}(q(k)$$

Squaring both side of (22) gives

$$
\begin{aligned}
&\sigma(k+1)^2 = \sigma(k)^2 + 2\sigma(k)\Delta\sigma(k+1) + \Delta\sigma(k+1)^2 \\
&\rightarrow \sigma(k+1)^2 - \sigma(k)^2 = 2\sigma(k)\Delta\sigma(k+1) + \Delta\sigma(k+1)^2 \\
&\rightarrow 2\sigma(k)\Delta\sigma(k+1) + \Delta\sigma(k+1)^2 \\
&\qquad = 2\sigma(k)\cdot\Gamma(k)[h_M(k)+h_f(k)+u_{sm}(k)] + \\
&\qquad\quad \Gamma(k)^2[h_M(k)+h_f(k)+u_{sm}(k)]^2
\end{aligned} \tag{24}
$$

When , $\|\sigma(k)\| > \varepsilon$ from (17) and Lemma 1, can be

$$\sigma(k)\cdot u_{sm}(k) = -\phi(k)\eta_{sm}\|\sigma(k)\| \tag{25}$$

$$
\begin{aligned}
&2\sigma(k)\Delta\sigma(k+1) + \Delta\sigma(k+1)^2 \\
&\quad = 2\sigma(k)\cdot\Gamma(k)[h_M(k)+h_f(k)+u_{sm}(k)] + \\
&\quad \Gamma(k)^2[h_M(k)+h_f(k)+u_{sm}(k)]^2 \\
&\quad \le -2\sigma(k)\beta(k) + \beta(k)^2 < 0
\end{aligned} \tag{26}
$$

Thus , $V(k+1) < V(k)$ for all $0 < \beta(k) < 2$. This implies that $\sigma(k)$ is bounded by $\|\sigma(k)\| \le \varepsilon$.

5. ALLEVIATION OF THE COMPUTATIONAL BURDEN

In the control law of (16), the computed torque term $u_{eq}(k)$ contains many trigonometric functions. In this section, we exclude these trigonometric functions. Instead of the computed torque vector $u_{eq}(k)$ in (16), we replace $u_{eq}(k)$ with another sliding mode control torque vector as:

$$
\hat{u}_{eq}(k) = \begin{cases} -\phi(k)\eta_{eq}\dfrac{\sigma(k)}{\|\sigma(k)\|}, \text{ if } \|\sigma(k)\| > \varepsilon \\ \qquad ;\eta_{eq} > 0,\ \ \varepsilon > 0 \\ -\phi(k)\eta_{eq}\dfrac{\sigma(k)}{\varepsilon}, \ \ \text{otherwise} \end{cases} \tag{27}
$$

The discrete sliding mode control law can now be designed

$$
\begin{aligned}
\hat{u}_{sm}(k) &= \hat{u}_{eq}(k) + u_{sm}(k) \\
&= \begin{cases} -\phi(k)(\eta_{eq}+\eta_{sm})\dfrac{\sigma(k)}{\|\sigma(k)\|}, \text{ if } \|\sigma(k)\| > \varepsilon \\ \qquad ;\eta_{eq} > 0,\ \eta_{sm} > 0,\ \ \varepsilon > 0 \\ -\phi(k)(\eta_{eq}+\eta_{sm})\dfrac{\sigma(k)}{\varepsilon}, \ \ \text{otherwise} \end{cases}
\end{aligned} \tag{28}
$$

The stability proof (28) can be obtained as the same way in Theorem 1.

6. SIMULATION RESULTS

A simple two-link robot manipulator shown in Figure 1 has been simulated, controlled by the discrete sliding mode control of (28), which is developed in this paper. The manipulator was modeled as a set of nonlinear coupled differential equations as described in [7]

$$\tau_1 = m_2 l_2^2(\ddot{q}_1+\ddot{q}_2)+m_2 l_1 l_2 c_2(2\ddot{q}_1+\ddot{q}_2)+(m_1+m_2)l_1^2\ddot{q}_1 - m_2 l_1 l_2 s_2 \dot{q}_2^2 - 2m_2 l_1 l_2 s_2 \dot{q}_1\dot{q}_2 + m_2 l_2 q s_{12} + (m_1+m_2) l_1 g s_1$$

$$\tau_2 = m_2 l_2^2(\ddot{q}_1+\ddot{q}_2)+m_2 l_1 l_2 c_2 \ddot{q}_1 + m_2 l_1 l_2 s_2 \dot{q}_1^2 + m_2 l_1 g s_{12} \quad (29)$$

where $c_1 := \cos(q_1)$, $s_{12} := \sin(q_1+q_2)$, etc.

The desired trajectories are chosen as

$$q_{d1} = 1+0.2\sin(\pi t), \quad q_{d2} = 1-0.2\cos(\pi t)$$

for $t \in [0,12]$.

Parameters used in the simulation were

$$l_1 = l_2 = 1\,\text{m},\ m_1 = m_2 = 1\,\text{kg}$$

While the manipulator was being operated, m_1 and m_2 were changed from $1\,\text{kg}$ to $1.5\,\text{kg}$ and from $1\,\text{kg}$ to $1.25\,\text{kg}$ respectively at $t = 6\,\text{sec}$. In the simulation, the sample time Ts = $0.1ms$.

The plant initial states were set as

$$q_{d1}(k)=1,\ q_{d1}(k)=0.8,\ q_{d1}(k-1)=0.2\pi,\ q_{d2}(k-1)=0$$

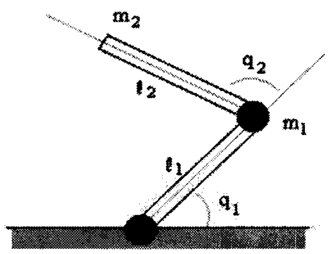
We applied the control algorithm of (28) to the system (29) with the controller parameters were chosen to be

$$\eta_{eq}+\eta_{sm}=55,\ \varepsilon=0.03$$

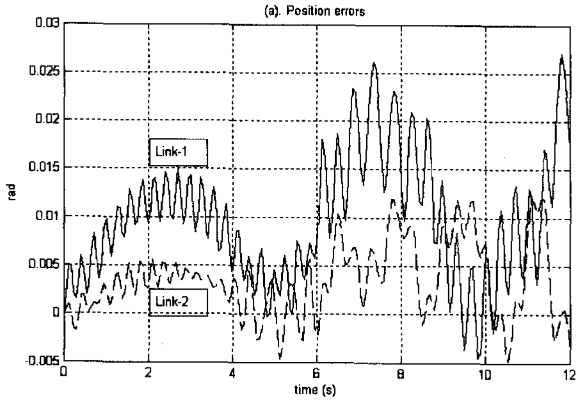
The simulation results are shown in Figure 2: (a) position errors; (b) control torques for each link of the manipulator; (c) link trajectories. It can be seen in Figure 2(a) and (c) that the tracking errors were increasing due to the mass change of at t=6sec. However, the system is stable with the desired bounded tracking errors for the given parameter changes.

7. CONCLUSIONS

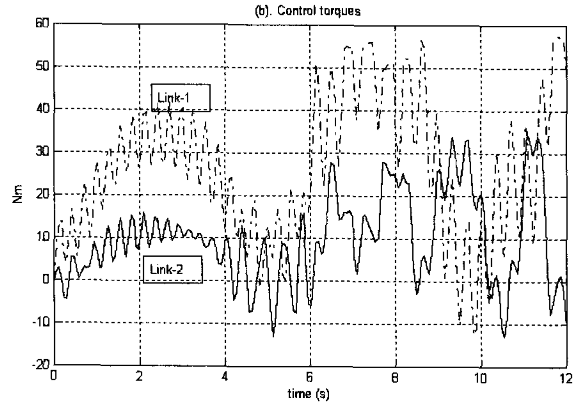
In this paper, discrete sliding mode control algorithms have been proposed for robust trajectory following control of robot manipulators. The proposed algorithms eliminate the requirement for measurement of joint acceleration, or for calculation of a regressor. Therefore the computation load required is roughly the same as that of a PID controller. The implementation of the control scheme is very simple. Only the measurements of the position tracking errors and the velocity tracking errors are required. Computer simulation results shown good properties of the proposed algorithms under large manipulator parameter uncertainties.



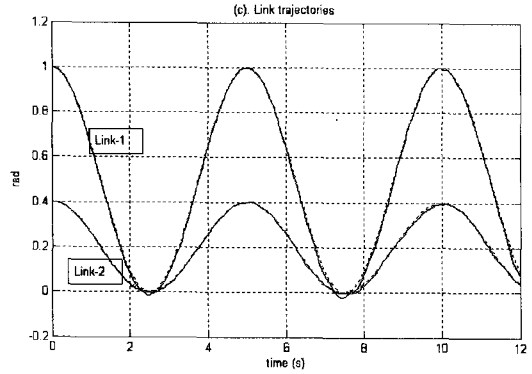
<Figure 1> A simple two-rigid-link robot manipulator



(a). Position errors



(b). Control torques



(c). Link trajectories

<Figure 2> Simulation Results

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