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The Effect of Neglecting the Longitudinal Moment Terms in Analyzing Composite Laminate Plates

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ABSTRACT

Some laminate orientations have decreasing values of D_{16}, B_{16}, D_{26} and B_{26} stiffnesses as the ply number increases. For such plates, the fiber orientations given above behave as specially orthotropic plates and simple formulas developed by the senior author. Most of the bridge and building slabs on girders have large aspect ratios. For such cases further simplification is possible by neglecting the effect of the longitudinal moment terms(Mx) on the relevant partial differential equations of equilibrium. In this paper, the result of the study on the subject problem is presented.

요 지

몇 가지 섬유 배향각을 가진 적층판은 층수가 증가하면 D_{16}, B_{16}, D_{26} 및 B_{26} 강성이 감소하게 되어 특별직교이방성판처럼 거동함을 밝히고, 간단한 공식들을 개발하여 발표한 바 있다. 대부분의 교량이나 건물의 상판은 형상비가 큰 경우가 많은데, 이런 구조물의 평형방정식에 대한 종방향 모멘트항(Mx)의 영향은 매우 작아서, 더욱 간단한 해석이 가능하다. 본 논문에는 이러한 문제들에 대한 연구결과를 제시하였다.

Key Words: effect of neglecting the longitudinal moment terms(종방향모멘트 무시효과), aspect ratio (형상비), finite difference method(유한차분법), natural frequencies(고유진동수)

1. INTRODUCTION

The future of material industry will depend on if and when the conventional construction materials are replaced by advanced composite materials. If composite materials are used for construction, the quantity is huge : in tons, not in kilos or pounds. Composite materials can be used economically and efficiently in broad civil engineering applications when standards and processes for analysis, design, fabrication, construction and quality control are established.

The problem of deteriorated highway slabs is very serious all over the world. The U.S. Civil Engineering Research Foundation (CERF) report, "High - Performance Construction Material and System : An Essential Program for America and its Infrastructure", published, in collaboration with several organizations, U.S. Department of Transportation figures as follows :

(1) The road bridge condition in U.S.A at the year 2009, 149,654 of the Americans 603,259 bridges are structurally deficient or obsolete. (structurally deficient 71,177, functionally obsolete 78,477)

(2) 199,584 of these bridges are more than 50 years old and unsuitable for current or projected traffic.

(3) Traffic delays alone will cost Americans $115 billion per year in lost work time and fuel by the year 2009.

Steel girders become rusty. The reinforcing bars embedded in concrete beams or slabs are subject to corrosion caused by electro - chemical action. Underground fuel thanks are under similar condition. The U.S. Bureau of Standards (NIST) study showed that yearly loss caused by corrosion related damages mounted to 82 billion dollars, about 4.9% of GNP. About 32 billion dollars could be saved if existing technologies were used to prevent such losses [Han, 2010].

These figures are in the United States of America, where various federal, state, and other agencies are doing their best in maintaining such structures in good condition. The issue of deteriorating and damaged infrastructures and lifelines has become a critically important subject in the United States as well as Japan and Europe. The problem in developing nations, where degree of construction quality control and maintenance are in question, must be much more profound [Kim 1995,

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Han, 2010].

One of the dependable methods is to evaluate the in-situ stiffness of the structure by means of obtaining the natural frequency. By comparing the in-situ stiffness with the one obtained at the design stage, the degree of damage can be estimated rather accurately.

The reinforced concrete slab can be assumed as a [0,90,0]r type specially orthotropic plate as a close approximation, assuming that the influence of B_{16}, B_{26}, D_{16} and D_{26} stiffnesses are negligible. Many of the bridge and building floor systems, including the girders and cross beams, also behave as similar specially orthotropic plates. Such plates are subject to the concentrated mass/masses in the form of traffic loads, or the test equipments such as the accelerator in addition to their own masses. Analysis of such problems is usually very difficult.

The most of the design engineers for construction has academic background of bachelors degree. Theories for advanced composite structures are too difficult for such engineers and some simple but accurate enough methods are necessary.

The author has reported that some laminate orientations such as $[\alpha,\beta]_{r,}$ $[\alpha,\beta,\gamma]_{r,}$ $[\alpha,\beta,\beta,\alpha,\alpha,\beta]_{r,}$ and $[\alpha,\beta,\beta,\gamma,\alpha,\alpha,\beta]_{r,}$ with $\alpha=-\beta$, and $\gamma=0^{o}$ or 90^{o}, and with increasing r, have decreasing values of B_{16}, B_{26}, D_{16} and D_{26} stiffnesses. Most of the civil and architectural structures are large in sizes and the numbers of laminae are large, even though the thickness to length ratios are small enough to allow to neglect the transverse shear deformation effects in stress analysis. For such plates, the fiber orientations given above behave as specially orthotropic plates and simple formulas developed by the reference [Kim 1995] can be used. Most of the bridge and building slabs on girders have large aspect ratios. For such cases further simplification is possible by neglecting the effect of the longitudinal moment terms (M_x) on the relevant partial differential equations of equilibrium [Kim, 1967].

In this paper, the result of the study on the subject problem is presented. Even with such assumption, the specially orthotropic plate with boundary conditions other than Navier or Levy solution types, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical method for eigenvalue problems are also very much involved in seeking such a solution [Han & Kim, 2001, 2003, 2009, Han & Suk, 2010, Kim, 1995].

The method of vibration analysis used is the one developed. He developed and reported, in 1974, a simple but exact method of calculating the natural frequency of beam and tower structures with irregular cross sections and attached mass/masses [Kim 1974].

Since 1989, this method has been extended to two dimensional problems with several types of given conditions and has been reported at several international conferences. This method uses the deflection influence surfaces. The finite difference method is used for this purpose, in this paper.

2. METHOD OF ANALYSIS

2.1 Vibration Analysis

In this paper, the method of analysis given in detail, in the reference book [Kim 1995] is briefly repeated.

The magnitudes of the maximum deflection at a certain number of points are arbitrarily given as

$$w\ (i,j)(1) = W\ (i,j)(1) \tag{1}$$

where (i,j) denotes the point under consideration. This is absolutely arbitrary but educated guessing is good for accelerating convergence. The dynamic force corresponding to this(maximum) amplitude is

$$F(i,j)(1) = m(i,j)[\omega(i,j)(1)]^2 W\ (i,j)(1) \tag{2}$$

The "new" deflection caused by this force is a function of f and can be expressed as

$$w(i,j)(2) = f\ \{m(k,l)[\omega(i,j)(1)]^2 W\ (k,l)(1)\}$$
$$= \sum^{k,l} \triangle(i,j,k,l)\{m(k,l)[\omega(i,j)(1)]^2 W(k,l)(1)\} \tag{3}$$

where $\triangle$ is the deflection influence surface. The relative (maximum) deflections at each point under consideration of a structural member under resonance condition, $w(i,j)(1)$ and $w(i,j)(2)$, have to remain unchanged and the following condition has to be held :

$$w\ (i,j)(1)\ /\ \ w\ (i,j)(2) \tag{4}$$

From this equation, $w(i,j)(1)$ at each point of (i,j) can be obtained. But they are not equal in most cases. Since the natural frequency of a structural member has to be equal at all points of the member, i.e. $w(i,j)$ should be equal for all (i,j), this step is repeated until sufficient equal magnitude of $w(i,j)$ is obtained at all (i,j) points. However, in most cases, the difference between the maximum and the minimum values of $w(i,j)$ obtained by the first cycle of calculation is sufficiently negligible for engineering purposes. The accuracy can be improved by simply taking the average of the maximum and the minimum, or by taking the value of $w(i,j)$ where the deflection is the maximum. For the second cycle, $W(i,j)(2)$ in

$$w(i,j)(3) = f\{m(i,j)[\omega(i,j)(2)]^2 W(i,j)(2)\} \quad (5)$$

the absolute numerics of $W(i,j)(2)$ can be used for convenience.

2.2 Finite Difference Method

The method used in this paper requires the deflection influence surfaces. F.D.M is applied to the governing equation of the specially orthotropic plates,

$$\begin{aligned} &D_1\frac{\partial^4 w}{\partial x^4} + 2D_3\frac{\partial^4 w}{\partial x^2\partial y^2} + D_2\frac{\partial^4 w}{\partial y^4} \\ &= q(x,y) - k\ w + Nx\frac{\partial^2 w}{\partial x^2} \\ &+ Ny\frac{\partial^2 w}{\partial y^2} + 2Nxy\frac{\partial^2 w}{\partial x\partial y} \end{aligned} \quad (6)$$

where

$D_1 = D_{11},\ D_2 = D_{22},\ D_3 = (D_{12} + 2D_{66})$.

The number of the pivotal points required in the case of the order of error $\triangle^2$, where $\triangle$ is the mesh size, is five for the central differences, This makes the procedure at the boundaries complicated. In order to solve such problem, the three simultaneous partial differential equations of equilibrium with three dependent variables, w, M_x and M_y are used instead of Eq.(6) with $N_x = N_y = N_{xy} = 0$ [Kim 1965].

$$\begin{aligned} &\frac{\partial^2 Mx}{\partial x^2} - 4D_{66}\frac{\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^2 My}{\partial y^2} \\ &= -q(x,y) + kw(x,y) \end{aligned} \quad (7)$$

$$M_x = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} \quad (8)$$

$$M_y = -D_{12}\frac{\partial^2 w}{\partial x^2} - D_{22}\frac{\partial^2 w}{\partial y^2} \quad (9)$$

If F.D.M is applied to these equations, the resulting matrix equation is very large in sizes, but the tridiagonal matrix calculation scheme used by Kim [Kim 1967] is very efficient to solve such equations. Since one of the few efficient analytical solutions of the specially orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, F.D.M is used to solve this problem. The result is satisfactory as expected.

By neglecting the M_x terms, the sizes of the matrices needed to solve the resulting linear equations are reduced to two thirds of the "non-modified" equations (4).

3. NUMERICAL EXAMINATION

3.1 Structure under Consideration

The plate considered is as shown in Fig.1.

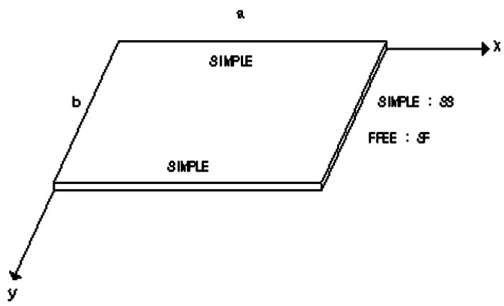

Fig. 1 Plate under consideration.

The material properties are :

$E_1 = 67.36$ GPa,

$E_2 = 8.12$ GPa,

$G_{12} = 3.0217$ GPa

$\nu_{12} = 0.272$, $\nu_{21} = 0.0328$,

r = 1

Ply thickness = 0.005 m

Orientation : [90°, 0°, 90°]r

The stiffnesses are :

$D_{11} = 2929$,

$D_{22} = 18492$,

$D_{12} = 627$

a = nb,

n = an integer 1 ~ 5

and $D_{66} = 849$,

b = 3 m

Loading : q = 286.65 N/m²

3.2 Numerical Results

In order to study the influence of M_x on the equilibrium equations, two cases are considered :

Case A : w , M_x and M_y are considered.

Case B : w and M_y are considered, i.e.,

M_x is neglected.

F.D.M. is used to obtain w, M_x, M_y and the natural frequency.

The result is as shown in Tables 1 to 5.

Table 1 Deflection at the center of the plate (m)

Aspect Ratio (a : b)	Case	SS	SF
1:1	A	0.1434E-01	0.1619E-01
	B	0.1525E-01	0.1648E-01
	A/B	0.9403	0.9824
1:2	A	0.1698E-01	0.1643E-01
	B	0.1643E-01	0.1648E-01
	A/B	1.0335	0.9970
1:3	A	0.1654E-01	0.1648E-01
	B	0.1648E-01	0.1648E-01
	A/B	1.0036	1.0000

Table 2 Moment My at center of the plate (N-m)

Aspect Ratio (a : b)	Case	SS	SF
1:1	A	0.2873E+03	0.3163E+03
	B	0.3016E+03	0.3225E+03
	A/B	0.9526	0.9808
1:2	A	0.3329E+03	0.3216E+03
	B	0.3217E+03	0.3225E+03
	A/B	1.0348	0.9972
1:3	A	0.3235E+03	0.3226E+03
	B	0.3225E+03	0.3225E+03
	A/B	1.0031	1.0003

Table 3 Moment Mx at the center of the plate (N-m)

Aspect Ratio (a : b)	Case	SS	SF
1:1	A	0.4676E+02	0.8996E+01
	B	0.2804E+02	0.1088E+02
	A/B	1.6676	0.8268
1:2	A	0.1268E+02	0.1125E+02
	B	0.1156E+02	0.1092E+02
	A/B	1.0969	1.0302
1:3	A	0.1038E+02	0.1099E+02
	B	0.1095E+02	0.1093E+02
	A/B	0.9480	1.0054

Table 4 Natural frequency (SS)

Aspect Ratio (a / b)	Natural Frequency (rad/sec)		Case A/ Case B
	Case A	Case B	
1	0.3879841	0.3540070	1.0960
2	0.2328229	0.2259199	1.0303
3	0.1823675	0.1789838	1.0189

Table 5 Natural frequency (SF)

Aspect Ratio (a / b)	Natural Frequency (rad/sec)		Case A/ Case B
	Case A	Case B	
1	0.2783243	0.2795208	0.9957
2	0.2018968	0.20235530	0.9977
3	0.1663077	0.1665625	0.9985

4. CONCLUSION

Most of the bridge and building slabs have plate aspect ratios larger than 2. For such cases, design analysis becomes much simpler if influence of the longitudinal moment (M_x) terms on the relevant differential equations of equilibrium can be neglected.

The result of the study on this subject is presented in this paper. The result of numerical examination is quite promising.

Plates with all edges simple supported (SS), the ratios of the natural frequencies and the deflections at the center of the uniformly loaded plate are :

For SS case :

Aspect Ratios	1	2	3
δ_A/δ_B	0.9403	1.0335	1.0036
ω_A/ω_B	1.0960	1.0303	1.0189

For SF case :

Aspect Ratios	1	2	3
δ_A/δ_B	0.9824	0.9979	1.0000
ω_A/ω_B	0.9957	0.9977	0.9985

It is concluded that, for all boundary conditions, neglecting M_x terms is acceptable if the aspect ratio (a/b) is equal to or larger than 2. This conclusion gives good guide line for design of bridge.

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