Opportunistic Relay Selection for Joint Decode-and-Forward Based Two-Way Relaying with Network Coding

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Abstract

This paper investigates the capacity rate problems for a joint decode-and-forward (JDF) based two-way relaying with network coding. We first characterize the achievable rate region for a conventional three-node network scenario along with the calculation of the corresponding maximal sum-rate. Then, for the goal of maximizing the system sum-rate, opportunistic relay selection is examined for multi-relay networks. As a result, a novel strategy for the implementation of relay selection is proposed, which depends on the instantaneous channel state and allows a single best relay to help the two-way information exchange. The JDF scheme and the scheme using relay selection are analyzed in terms of outage probability, after which the corresponding exact expressions are developed over Rayleigh fading channels. For the purpose of comparison, outage probabilities of the amplify-and-forward (AF) scheme and those of the scheme using relay selection are also derived. Finally, simulation experiments are done and performance comparisons are conducted. The results verify that the proposed strategy is an appropriate method for the implementation of relay selection and can achieve significant performance gains in terms of outage probability regardless of the symmetry or asymmetry of the channels. Compared with the AF scheme and the scheme using relay selection, the conventional JDF scheme and that using relay selection perform well at low signal-to-noise ratios (SNRs).

Keywords: Two-way relaying, joint decode-and-forward, network coding, relay selection

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1. Introduction

Two-way relaying has recently received increasing attention from the research community because of its efficiency in using spectral resources. Originally, two widely used physical layer operations, namely, amplify-and-forward (AF) and decode-and-forward (DF) [1], have been utilized by two-way relay transmission. The authors of [2] pointed out that there is a close relationship between two-way relaying and network coding which has been proposed by Ahlswede et al. in their seminal paper [3]. Initially, network coding on wired networks has received more attention. Inspired by the broadcast nature of the wireless medium, Zhang et al. [4] creatively applied the idea of network coding into wireless networks and proposed a new concept called physical-layer network coding (PNC). Since the use of network coding can help achieve more gain in terms of spectral efficiency, there have been extensive interest in employing the concept of network coding in two-way relaying scenarios. Popovski et al. [5] summarized the schemes of two-way relay transmission with network coding and conducted the performance analyses in terms of maximal sum-rate. Moreover, a new two-phase DF scheme called joint decode-and-forward (JDF) has been introduced. Similar to the conventional DF scheme proposed in [1], JDF scheme requires additional operations at relay nodes employing such approaches as MAC-XOR network coding [6] instead of superposition coding [2].

In this paper, our investigations focus on the half-duplex JDF based two-way relaying. We first characterize the achievable rate region of a conventional three-node scenario along with the calculation of the corresponding maximal sum-rate. Then, according to the calculated maximal sum-rate, a novel relay selection strategy is proposed, which maximizes the value of sum-rate accordingly. Subsequently, the JDF scheme and the scheme using relay selection are analyzed in terms of outage probability, after which the corresponding exact expressions are developed over Rayleigh fading channels. It is worth mentioning that the derived expressions are applicable to both symmetric and asymmetric channels, and to all rate, SNR values and link gains. Moreover, for the purpose of comparison, outage probabilities of the AF scheme and those of the scheme using relay selection are also derived. The results indicate that the proposed strategy is an appropriate method for the implementation of relay selection and can achieve significant performance gains in terms of outage probability regardless of the symmetry or asymmetry of the channels. Furthermore, performance comparisons on outage probability between the JDF and AF schemes are conducted, along with the comparisons of the two schemes that use relay selection. The results show that compared with the AF scheme and the scheme using relay selection, the conventional JDF scheme and that using relay selection perform well at low SNRs.

2. Related Work

Recently, two-way relay transmission has been investigated from many pespectives. Li et al. [7] derived the exact outage probabilities for both AF and DF schemes and further proposed an adaptive protocol for bidirectional communications. However, the expressions provided are applicable for the DF scheme only when the data rate is greater than $0.5\text{bit}/\text{s}\cdot\text{Hz}$. On the other hand, previous works [8][9][10][11][12] have also derived the outage probabilities for the AF scheme. Moreover, opportunistic relay selection has been considered in [9][10][11][12] to optimize overall performance. In contrast to the AF scheme, limited studies have conducted on two-way DF relaying with network coding. In [13][14][15][16], the capacity rate problems of the DF scheme have been investigated. However, these studies focused on the DF scheme using either time-sharing or superposition coding, and network coding protocols have been excluded. Katti et al. [17] proposed a joint network coding and superposition coding (JNSC) strategy for two-way relay transmission and found that JNSC can achieve gains in overall throughput. However, outage probability has been excluded in their work. Reference [2] proposed a relay selection policy for the DF scheme using superposition coding, although this work excluded network coding protocols. Reference [18] recently studied the DF scheme using relay selection from a diversity point of view, but no exact outage probabilities have been derived.

3. System Model

We consider a perfectly synchronized three-node wireless communication network, where each terminal has a single antenna and operates in a half-duplex manner. In this network scenario (**Fig. 1**), terminal A(B) wants to send s_A (s_B) to B(A) with target rate r_A (r_B). Suppose s_A (s_B) has unit average power. We use P_A , P_B and P_R to respectively denote the average transmit powers consumed by terminals A, B and R. In addition, we let h_{AR} (h_{BR}) denote the path gain from A(B) to B(A). Without loss in generality, we assume that h_{AR} and h_{BR} are independent complex Gaussian random variables, which are denoted by $h_{AR} \sim \mathcal{CN}(0, g_1)$ and $h_{BR} \sim \mathcal{CN}(0, g_2)$, respectively. Then, it can be obtained that $|h_{AR}|^2$ and $|h_{BR}|^2$ are independent exponential random variables with parameters $1/g_1$ and $1/g_2$, respectively. We assume that all the channels are reciprocal, i.e., $h_{AR} = h_{RA}$ and $h_{BR} = h_{RB}$, where h_{RA} (h_{RB}) represents the path gain from R to A(B), and quasi-static, namely, the path gains are regarded as constant within two phases or time slots. In particular, it is assumed that the two-way relaying network is able to obtain perfect channel state information (CSI). Furthermore, i.i.d. additive white Gaussian noises (AWGN) at A, B and R are denoted by $w_A \sim \mathcal{CN}(0, \sigma^2)$, $w_B \sim \mathcal{CN}(0, \sigma^2)$ and $w_R \sim \mathcal{CN}(0, \sigma^2)$, respectively.

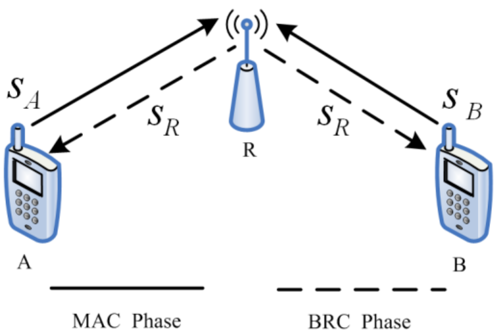


Fig. 1. Two-Way relaying system model

As shown in **Fig. 1**, the information exchange between A and B consists of two phases. In the multiple access (MAC) phase, both A and B transmit their messages to the relay node R, simultaneously. Then the relay node receives an additive Gaussian noise corrupted superposition of the transmitted signals given by

$$y_R = \sqrt{P_A} h_{AR} s_A + \sqrt{P_B} h_{BR} s_B + w_R \,. \tag{1}$$

In the broadcast (BRC) phase, after jointly decoding both s_A and s_B, the relay node applies a canonical network coding operation to obtain a encoded signal s_R and forwards that to A and B. Accordingly, A(B) receives $\sqrt{P_R} h_{RA} s_R + w_A$ ($\sqrt{P_R} h_{RB} s_R + w_B$). Since both A and B have known their own transmitted messages, A(B) can obtain s_B (s_A) correctly through perfect decoding.

4. Achievable Rate Region and Relay Selection Strategy

4.1 Achievable rate region

In this subsection, the achievable rate region of a JDF based two-way relaying scenario is characterized. This subsection also presents the caculation of the corresponding maximal sum-rate. For simplicity, P_A, P_B and P_R are assumed as equal, i.e., $P_A = P_B = P_R = P$. Actually, such an assumption is widely adopted in a large number of studiess [9][11][12][18]. In addition, $\gamma = P/\sigma^2$ is used to denote the SNR.

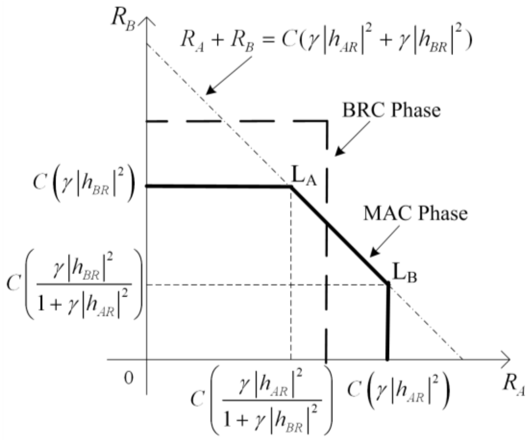


Fig. 2. Achievable rate region

(1) MAC Phase

The capacity region of the multiple access channel, which is well known, is reststed because the aforementioned multiple access phase is identical to the classical discrete memoryless Gaussian MAC channel [16][19]. As shown in **Fig. 2**, the achievable rate region of MAC phase given by

$$Rg_{MAC} = \left\{ [r_A, r_B] \in \mathbb{R}_+^2 : r_A \le R_{\overline{AR}}, r_B \le R_{\overline{BR}}, r_A + r_B \le R_{\Sigma} \right\} \tag{2}$$

is a pentagon with the following rate constraints

$$R_{\overline{AR}} = \tfrac{1}{2}\log_2\left(1+\gamma|h_{AR}|^2\right), R_{\overline{BR}} = \tfrac{1}{2}\log_2\left(1+\gamma|h_{BR}|^2\right), R_{\Sigma} = \tfrac{1}{2}\log_2\left(1+\gamma|h_{AR}|^2 + \gamma|h_{BR}|^2\right), \tag{3}$$

where $[r_A, r_B]$ refers to the rate pair of A and B, and $R_{\overline{AR}}$, $R_{\overline{BR}}$ and R_{Σ} are the rate constraints of links A $\rightarrow$ R, B $\rightarrow$ R and the sum-rate, respectively.

(2) BRC Phase

For the BRC phase, the achievable rate region is only determined by the rate constraints of links R $\rightarrow$ A and R $\rightarrow$ B. Hence in this phase, the achievable rate region can be denoted as follows:

$$Rg_{BRC} = \left\{ [r_A, r_B] \in \mathbb{R}_+^2 : r_A \le R_{\overline{RB}}, r_B \le R_{\overline{RA}} \right\}, \tag{4}$$

where $R_{\overline{RA}}$ and $R_{\overline{RB}}$ are the rate constraints of R $\rightarrow$ A and R $\rightarrow$ B, respectively. As all the links are reciprocal, $R_{\overline{RA}}$ and $R_{\overline{RB}}$ are equal to $R_{\overline{AR}}$ and $R_{\overline{BR}}$, respectively.

Consequently, the ultimate achievable rate region of JDF based two-way relaying is scaled by Eqs. (2) and (4), simultaneously, because a complete information exchange for two-way relay transmission is composed of the MAC and BRC phases. Therefore, the ultimate achievable rate region can be given as $Rg_{JDF} = Rg_{MAC} \cap Rg_{BRC}$. The maximal sum-rate refers to the maximal sum of a practical rate pair $[r_A, r_B]$, which is constrained in Rg_{JDF}. As **Fig. 2** depicts, the sum-rate can be maximized if the rate pair $[r_A, r_B]$ lies on the line $\overline{L_A L_B} : r_A + r_B = R_\Sigma$, whereas $r_A + r_B < R_\Sigma$ in all other points of the achievable rate region. For the symmetric traffics, i.e., $r_A = r_B$, the sum-rate is maximized if there is an intersection point between $\overline{L_A L_B}$ and $\overline{r_A = r_B}$.

Theorem 1: For the JDF based two-way relaying scheme with symmetric target rates (i.e., $r_A = r_B$), the maximal sum-rate is given by:

$$R_{sum} = \begin{cases} R_\Sigma, \text{if } \max(\gamma|h_{AR}|^2, \gamma|h_{BR}|^2) \le \min\left[\gamma|h_{AR}|^2(\gamma|h_{AR}|^2+1), \gamma|h_{BR}|^2(\gamma|h_{BR}|^2+1)\right]; \\ 2\min\left(R_{\overline{RA}}, R_{\overline{RB}}\right), \text{if } \max(\gamma|h_{AR}|^2, \gamma|h_{BR}|^2) > \min\left[\gamma|h_{AR}|^2(\gamma|h_{AR}|^2+1), \gamma|h_{BR}|^2(\gamma|h_{BR}|^2+1)\right]. \end{cases} \tag{5}$$

Proof: The case where $|h_{AR}|^2 \ge |h_{BR}|^2$, i.e., $R_{\overline{AR}} \ge R_{\overline{BR}}$ is first considered. To facilitate the proof, the following two cases are examined.

(a) For $\gamma|h_{BR}|^2 \ge \frac{\gamma|h_{AR}|^2}{1+\gamma|h_{BR}|^2}$, i.e., $R_{\overline{BR}} \ge R_\Sigma - R_{\overline{BR}}$, the corresponding achievable rate region can be given as follows:

$$Rg_{JDF} = \left\{ [r_A, r_B] \in \mathbb{R}_+^2 : r_A \le R_{\overline{AR}}, r_B \le R_{\overline{BR}}, r_A + r_B \le R_\Sigma \right\}. \tag{6}$$

Given that $R_{\overline{BR}} \ge \frac{R_\Sigma}{2}$, line $\overline{r_A = r_B}$ always has an intersection point with the segment $\overline{L_A L_B}$. Therefore, the maximal sum-rate R_{sum} can be given by $R_{sum} = r_A + r_B = R_\Sigma$.

(b) For $\gamma|h_{BR}|^2 < \frac{\gamma|h_{AR}|^2}{1+\gamma|h_{BR}|^2}$, i.e., $R_{\overline{BR}} < R_\Sigma - R_{\overline{BR}}$, the corresponding achievable rate region can be expressed as follows:

$$Rg_{JDF} = \left\{ [r_A, r_B] \in \mathbb{R}_+^2 : r_A \le R_{\overline{AR}}, r_B \le R_{\overline{BR}} \right\}. \tag{7}$$

Then the maximal sum-rate can be obtained directly, which is given by $R_{sum} = r_A + r_B = 2R_{\overline{AR}} = 2R_{\overline{BR}}$. For the case where $|h_{AR}|^2 < |h_{BR}|^2$, the maximal sum-rate can be obtained using a similar method. However, this operation is omitted in the current work due to space limitations. By combining cases of $|h_{AR}|^2 \ge |h_{BR}|^2$ and $|h_{AR}|^2 < |h_{BR}|^2$, Eq. (5) can be obtained.

4.2 Relay selection strategy

We now consider a JDF based two-way relaying network with *N* relay nodes. Our aim, here, is to maximize the system sum-rate by opportunistically choosing the “best” relay node among *N* candidates. We first keep on discussing a number of remarkable phenomena for the aforementioned single relay network. After that, we extend the obtained results into multi-relay systems, from which a novel strategy for the implementation of relay selection is proposed. Similar to reference [20], only a single "best" relay is selected to retransmit the encoded signals with overall relay power equal to P. Our assumption, here, is that the system operates in the symmetric traffic mode, and the two source terminals exchange information at their maximal rates at all times. This indicates that $r_A = r_B = {}^{R_{sum}}\!/_2$, where R_{sum} is given in Eq. (5). For notational simplicity, $x = \gamma |h_{AR}|^2$, $y = \gamma |h_{BR}|^2$ and ${}^{R_{sum}}\!/_2 = r_0$ are assumed. Moreover, to facilitate the analysis, two complementary events are defined as follows:

$$\mathbf{E_1} = \{x + y > \min[x(x+2), y(y+2)]\}, \quad \mathbf{E_2} = \overline{\mathbf{E_1}}. \tag{8}$$

Then, according to Eq. (5), the maximal rate r_0 can be expressed as follows:

$$r_0 = \begin{cases} \frac{1}{2}\min[\log_2(1+x), \log_2(1+y)], \text{if } \mathbf{E_1}; \\ \frac{1}{4}\log_2(1+x+y), \text{if } \mathbf{E}_2. \end{cases} \tag{9}$$

From Eq. (9), r_0 is conditioned on the instantaneous channel state. For the case of $\mathbf{E}_1$, r_0 is determined only by the link with the minimal path gain, whereas for the case of $\mathbf{E}_2$, r_0 is related to the two-way links simultaneously. Our focus here is to design a strategy, which maximizes the sum-rate by choosing the best relay among N candidates. Thus, for a two-way relaying network with N relay nodes, the best relay should be chosen in two stages. The first stage is selecting which event between $\mathbf{E}_1$ or $\mathbf{E}_2$ occurs for each relay, from which two source terminals comprising the sub-system are determined. Following the above judgment, the maximal sum-rates are calculated accordingly. Consequently, comparisons of the calculated maximal sum-rates are conducted, and the node providing the largest maximal sum-rate is chosen as the "best" relay. Thus, the best relay can be selected using the following strategy:

$$k = \arg\max\left\{\left[\min\left(|h_{AR_m}|^2, |h_{BR_m}|^2\right), m \in Set1\right]; |h_{AR_n}|^2 + |h_{BR_n}|^2, n \in Set2\right\}, \tag{10}$$

where

$$Set\ 1 = \{i : x_i + y_i > \min[x_i(x_i+2), y_i(y_i+2)], i \in \{1, 2, \cdots, N\}\}, \tag{11a}$$

and

$$Set\ 2 = \{i : i \in \{1, 2, \cdots, N\} \cap i \notin Set\,1\}. \tag{11b}$$

Note that, here, we generally extend the notation for the already introduced variables by an additional index for the i-th relay node. In the following section, the proposed relay selection strategy is also called JDF scheme with diversity.

5. Outage Analysis

In this section, we derive the exact outage probability expressions for the JDF scheme and that with diversity. As a first step, we present the definitions of outage rate and outage probability by letting r and p denote the two parameters, respectively.

Definition 1: Outage rate r is a data rate that can be supported with a probability $1-p$, i.e., $p = \mathrm{P}(R < r)$, where R denotes the maximal data rate that a user can achieve, p represents the outage probability and $1-p$ stands for the complementary value of outage probability.

5.1 JDF Scheme with Single Relay

We first make some descriptions about the following used variables. Recall that variables x and y are defined as $x=\gamma|h_{AR}|^2$ and $y=\gamma|h_{BR}|^2$, respectively. According to the assumption in section 3, we can deduce easily that x and y are independent exponentially distributed random variables with parameters $\lambda_1 = 1/\gamma g_1$ and $\lambda_2 = 1/\gamma g_2$, respectively. For notational simplicity, $z=2^{2r}-1$, $a_1=\lambda_1$, $b_1=-\frac{1}{2}-\frac{\lambda_2}{2\lambda_1}$, $a_2=\lambda_2$ and $b_2=-\frac{1}{2}-\frac{\lambda_1}{2\lambda_2}$ are assumed.

According to the outage definition, for terminals A, B and the whole system, outage probabilities can be depicted as $p_{JDF-A}=\mathrm{P}(r_A<r)$, $p_{JDF-B}=\mathrm{P}(r_B<r)$ and $p_{JDF}=\mathrm{P}(r_A<r, r_B<r)$, respectively. Given that $r_A=r_B=r_0$, p_{JDF-A}, p_{JDF-B} and p_{JDF} are equal to each other. For simplicity, p_{JDF} is used to represent the three outage probabilities above. As depicted in Eq. (9), r_0 is conditioned on events $\mathbf{E}_1$ and $\mathbf{E}_2$, exclusively. Therefore, we conduct the outage analysis by considering $\mathbf{E}_1$ and $\mathbf{E}_2$, separately. For the case of $\mathbf{E}_1$, the outage probability can be shown as

$$p_{JDF}^{E_1} = \mathrm{P}(r_0 \le r, \mathbf{E_1}) = p_1 + p_2, \tag{12}$$

where

$$\begin{aligned} p_1 &= 1-\exp\left[-\lambda_1\left(z+\tfrac{1}{2}\right)^2+\tfrac{\lambda_1}{4}\right]-2\lambda_1\exp\left[\tfrac{\lambda_1}{4}+\tfrac{\lambda_2}{2}+\tfrac{\lambda_2^2}{4\lambda_1}\right]\times\varphi(b_1,a_1) \\ &\quad +\left[1-\exp(-\lambda_2 z)\right]\exp\left[\tfrac{\lambda_1}{4}-\lambda_1\left(z+\tfrac{1}{2}\right)^2\right] \end{aligned}, \tag{13}$$

$$\begin{aligned} p_2 &= 1-\exp\left[-\lambda_2\left(z+\tfrac{1}{2}\right)^2+\tfrac{\lambda_2}{4}\right]-2\lambda_2\exp\left[\tfrac{\lambda_2}{4}+\tfrac{\lambda_1}{2}+\tfrac{\lambda_1^2}{4\lambda_2}\right]\times\varphi(b_2,a_2) \\ &\quad +\left[1-\exp(-\lambda_1 z)\right]\exp\left[\tfrac{\lambda_2}{4}-\lambda_1\left(z+\tfrac{1}{2}\right)^2\right] \end{aligned}. \tag{14}$$

Note that in Eqs. (13) and (14), function $\varphi(u,v)$ are defined as follows:

$$\varphi(u,v) = \frac{\Gamma\left[1,v(-u)^2\right]-\Gamma\left[1,v(z-u)^2\right]}{2v} + (\tfrac{1}{2}+u)\frac{\Gamma\left[\frac{1}{2},v(-u)^2\right]-\Gamma\left[\frac{1}{2},v(z-u)^2\right]}{2v^{\frac{1}{2}}}, \tag{15}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function.

For the case of $\mathbf{E}_2$, the outage probability can be expressed as follows:

$$p_{JDF}^{E_2} = \mathrm{P}(r_0 \le r, \mathbf{E}_2) = \begin{cases} q_{11}+q_{12}, \lambda_1=\lambda_2, \\ q_{21}+q_{22}, \lambda_1 \ne \lambda_2. \end{cases} \tag{16}$$

where

$$q_{11} = 2\lambda_1\exp\left(\tfrac{\lambda_1}{4}+\tfrac{\lambda_2}{2}+\tfrac{\lambda_2^2}{4\lambda_1}\right)\times\varphi(b_1,a_1)-\lambda_1\tfrac{z^2}{2}\exp\left[-\lambda_2(z^2+2z)\right]-\tfrac{\lambda_1}{\lambda_1+\lambda_2}\left\{1-\exp\left[-\tfrac{(\lambda_1+\lambda_2)(z^2+2z)}{2}\right]\right\}, \tag{17}$$

$$q_{12} = 2\lambda_2\exp\left(\tfrac{\lambda_2}{4}+\tfrac{\lambda_1}{2}+\tfrac{\lambda_1^2}{4\lambda_2}\right)\times\varphi(b_2,a_2)-\lambda_2\tfrac{z^2}{2}\exp\left[-\lambda_1(z^2+2z)\right]-\tfrac{\lambda_2}{\lambda_1+\lambda_2}\left\{1-\exp\left[-\tfrac{(\lambda_1+\lambda_2)(z^2+2z)}{2}\right]\right\}, \tag{18}$$

$$\begin{aligned} q_{21} &= 2\lambda_1\exp\left(\tfrac{\lambda_1}{4}+\tfrac{\lambda_2}{2}+\tfrac{\lambda_2^2}{4\lambda_1}\right)\times\varphi(b_1,a_1)-\tfrac{\lambda_1}{\lambda_1+\lambda_2}\left\{1-\exp\left[-\tfrac{(\lambda_1+\lambda_2)(z^2+2z)}{2}\right]\right\} \\ &\quad -\tfrac{\lambda_1}{\lambda_2-\lambda_1}\exp\left[-\lambda_1 z-\lambda_1\tfrac{z^2}{2}-\lambda_2 z\right]\left[\exp\left(-\lambda_1\tfrac{z^2}{2}\right)-\exp\left(-\lambda_2\tfrac{z^2}{2}\right)\right], \end{aligned}, \tag{19}$$

$$\begin{aligned} q_{22} &= 2\lambda_2\exp\left(\tfrac{\lambda_2}{4}+\tfrac{\lambda_1}{2}+\tfrac{\lambda_1^2}{4\lambda_2}\right)\times\varphi(b_2,a_2)-\tfrac{\lambda_2}{\lambda_1+\lambda_2}\left\{1-\exp\left[-\tfrac{(\lambda_1+\lambda_2)(z^2+2z)}{2}\right]\right\} \\ &\quad -\tfrac{\lambda_2}{\lambda_1-\lambda_2}\exp\left[-\lambda_2 z-\lambda_2\tfrac{z^2}{2}-\lambda_1 z\right]\left[\exp\left(-\lambda_2\tfrac{z^2}{2}\right)-\exp\left(-\lambda_1\tfrac{z^2}{2}\right)\right]. \end{aligned}, \tag{20}$$

and function $\varphi(u,v)$ is given in Eq. (15).

From Eqs. (12) and (16), the average outage probability can be obtained directly, which is given by the following

$$p_{JDF} = p_{JDF}^{E_1} + p_{JDF}^{E_2} . \tag{21}$$

Proof: The proof can be found in appendix A.
Of note is that Eq. (21) also represents the outage probabilities of p_{JDF-A} and p_{JDF-B}.

5.2 JDF Scheme Using Relay Selection

In this subsection, we derive the outage probabilities for the JDF scheme using relay selection. To facilitate the analysis, we first consider a network that contains only two relay nodes (i.e., $N=2$). Subsequently, we extend N to all natural numbers and derive the outage probabilities accordingly. As discussed in the previous section, two complementary events should be considered when conducting an outage analysis for a single relay network. Thus, for a two-relay system proposed in the current work, the number of combinations of the two defined events increases to four. Accordingly, for a two-relay system, the outage probability can be given by the following

$$p_{JDF-D}^{(2)} = \begin{cases} \left(p_{JDF}^{E_1}\right)^2, \text{ if } [Cs_1, Cs_2] = [\mathbf{E_1}, \mathbf{E}_1]; \\ p_{JDF}^{E_1} p_{JDF}^{E_2}, \text{ if } [Cs_1, Cs_2] \in \{[\mathbf{E_1}, \mathbf{E}_2], [\mathbf{E}_2, \mathbf{E}_1]\}; \\ \left(p_{JDF}^{E_2}\right)^2, \text{ if } [Cs_1, Cs_2] = [\mathbf{E}_2, \mathbf{E}_2]. \end{cases} . \tag{22}$$

In Eq. (22) above, $[Cs_1, Cs_2]$ is the possible combination pair of events $\mathbf{E}_1$ and $\mathbf{E}_2$.
From Eq. (22), the average outage probability of a two-relay JDF system can be calculated directly, which is shown as follows:

$$p_{JDF-D}^{(2)} = \left(p_{JDF}^{E_1}\right)^2 + 2 p_{JDF}^{E_1} p_{JDF}^{E_2} + \left(p_{JDF}^{E_2}\right)^2 = \left(p_{JDF}\right)^2 . \tag{23}$$

In the following, we extend the number of relay nodes to arbitrary natural numbers. We use mathematical induction [21] to prove that for arbitrary natural numbers N, the average outage probability can be given by the following

$$p_{JDF-D}^{(N)} = \left(p_{JDF}\right)^N . \tag{24}$$

Proof: As depicted in Eq. (23), the statement of Eq. (24) is true for $N=2$. The statement is further assumed true for $N = M \, (M \geq 3)$, i.e., $p_{JDF-D}^{(M)} = \left(p_{JDF}\right)^M$. Then, for $N = M+1$, the average outage probability can be given as follows:

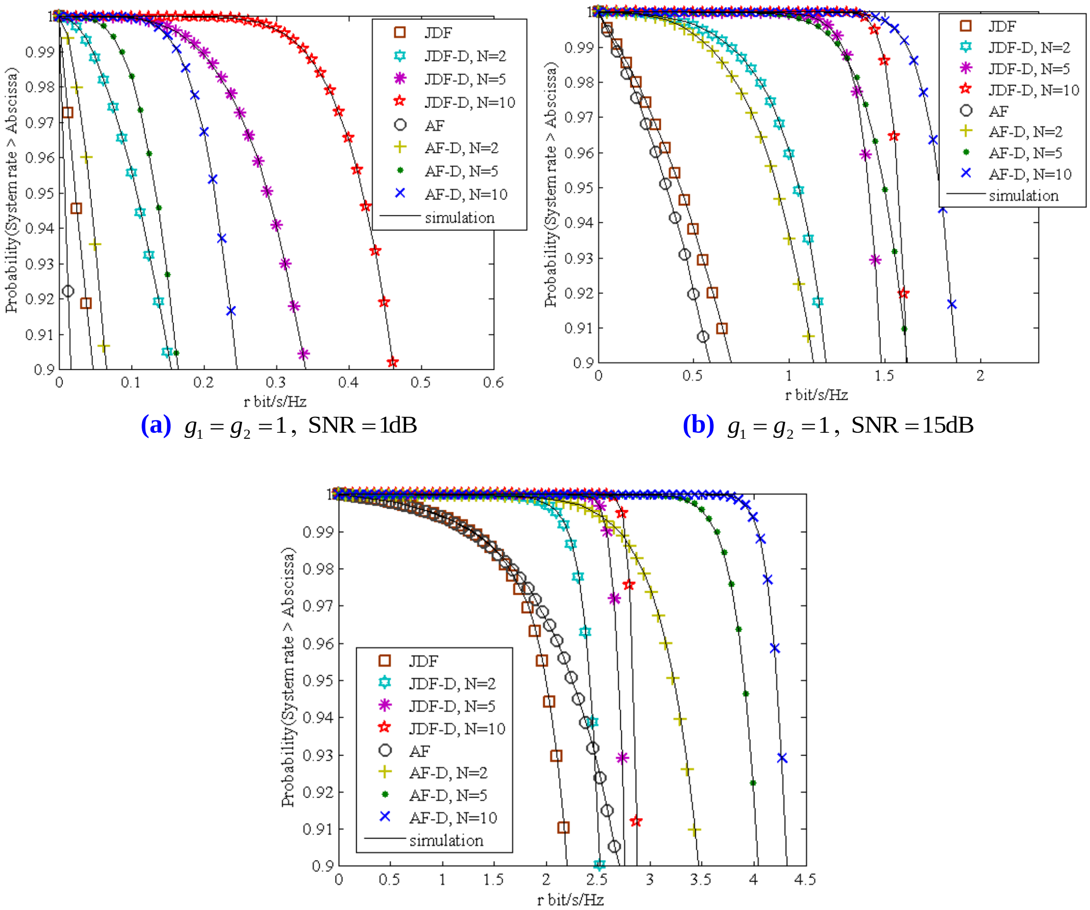
$$p_{JDF-D}^{(M+1)} = p_{JDF}^{E_1} p_{JDF-D}^{(M)} + p_{JDF}^{E_2} p_{JDF-D}^{(M)} = p_{JDF} \cdot p_{JDF-D}^{(M)} = \left(p_{JDF}\right)^{M+1} . \tag{25}$$

The statement above shows that $N = M+1$ is true. Therefore, Eq. (24) is proven true for all natural numbers N.

6. Numerical Results

In this section, we provide some numerical results to compare the outage probabilities of the JDF and AF schemes as well as to comapre the two schemes that use relay selection. Of note is that the theoretic values of outage probability for the AF scheme and that using relay selection are numerically calculated using exact expressions (See Appendix B). To measure the overall performance clearly, the complementary value of the outage probability is applied in the following figures.

In **Fig. 3**, we plot the complementary values of outage probability as functions of outage rate r. As depicted in the three sub-figures, a symmetric channel (i.e., $g_1 = g_2 = 1$) is considered. Three SNR cases (i.e., low, moderate and high SNRs) are also included. The first observation is that the theoretic values are extremely close to the simulation results, thus validating the accuracy of the derived expressions. Second, the JDF scheme performs better than the AF scheme for low and moderate SNRs, whereas the AF scheme outperforms the JDF scheme at high SNRs. Third, in contrast to the conventional JDF scheme, that with diversity can achieve significant performance gains irrespective of the values of outage rate and SNR, which is attributed to the system's ability to explore the diversity gain. In addition, the JDF scheme with diversity outperforms the AF scheme using relay selection when SNR=1dB (**Fig. 3**). When SNR=15dB and $N = 2$, the JDF scheme with diversity still maintains the same trend that plays better than its counterpart. However, when $N = 5$ and 10, the JDF scheme with diversity performs worse than the AF based scheme. Moreover, the outage performance of the JDF scheme with diversity is inferior to that of its counterpart when SNR=30dB.



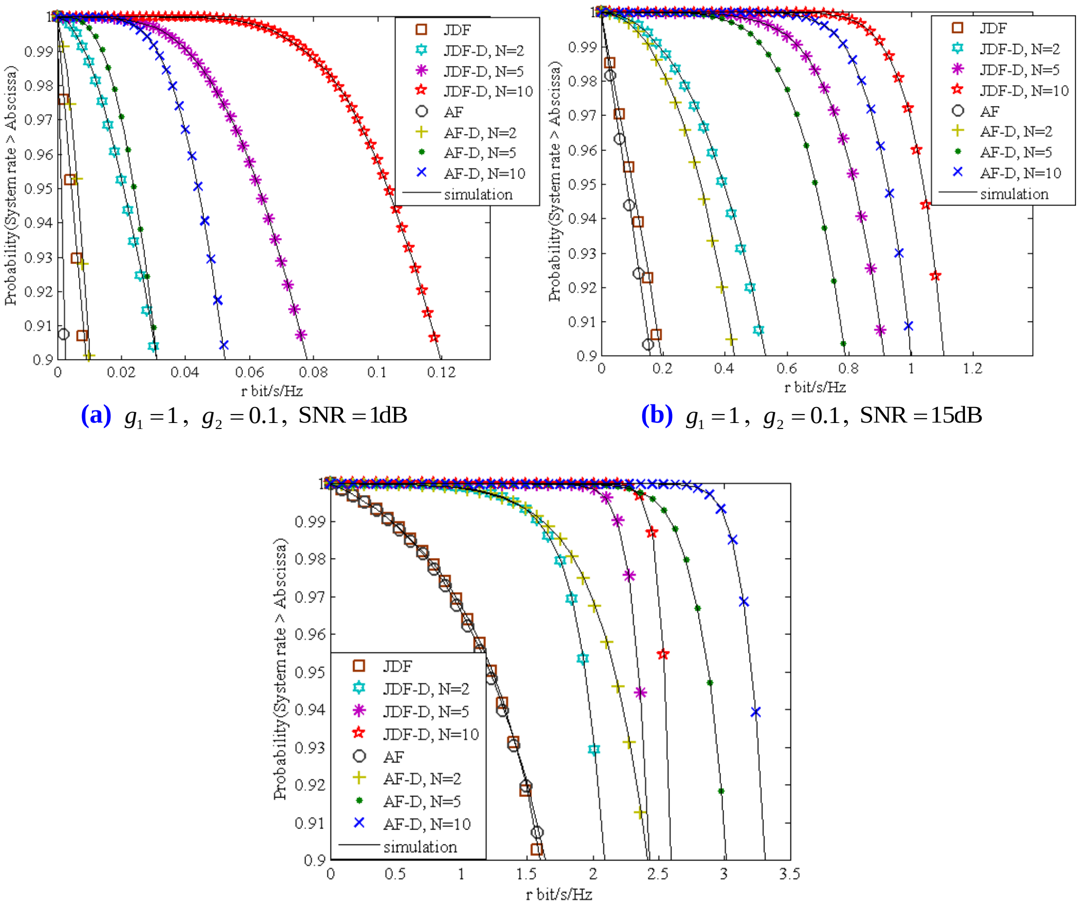
(a) $g_1 = g_2 = 1$, $\text{SNR} = 1\text{dB}$ **(b)** $g_1 = g_2 = 1$, $\text{SNR} = 15\text{dB}$

(c) $g_1 = g_2 = 1$, $\text{SNR} = 30\text{dB}$

Fig. 3. Complementary curves of outage probability under symmetric channels

Next, we explore the impact of channel asymmetry on the overall outage performance of the investigated network scenario. It can be seen from **Fig. 4** that we plot the complementary values of outage probability as functions of r when the channels are moderately asymmetric, i.e., $g_1 = 1$, $g_2 = 0.1$. Likewise, the three SNR cases considered in **Fig. 3** are adopted. First, we observe that the theoretic values are in excellent agreement with the simulation results, thereby validateing the accuracy of the analyses. Moreover, **Fig. 4** shows that the JDF scheme performs slightly better than the AF scheme when SNR is low. However, the outage probabilities of the two schemes are almost identical at the moderate and high SNR regions. The overall outage performance also improves significantly when the proposed relay selection strategy is applied. Furthermore, we can see that the JDF scheme with diversity performs better than the AF based scheme at low and moderate SNRs, but performs worse than its counterpart at high SNRs.

In **Fig. 5**, the asymmetric level of the channel is further increased (i.e., $g_1 = 1,\ g_2 = 0.01$). Similar to the previous results, overall outage performance is improved significantly when the proposed relay selection strategy is used. Besides, it is shown that the JDF scheme with diversity performs slightly better than the AF scheme using relay selection at moderate SNRs. Particularly, the outage probabilities of the two schemes using relay selection are almost identical when SNR is high. Also, the theoretic values are extremely close to the simulation results, further validating the accuracy of the derived expressions.

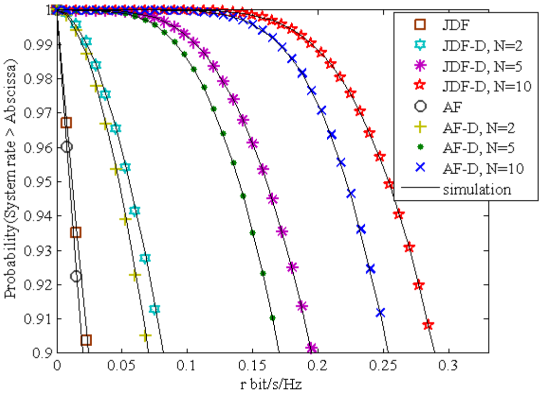


(a) $g_1 = 1,\ g_2 = 0.1,\ \mathrm{SNR} = 1\mathrm{dB}$ **(b)** $g_1 = 1,\ g_2 = 0.1,\ \mathrm{SNR} = 15\mathrm{dB}$

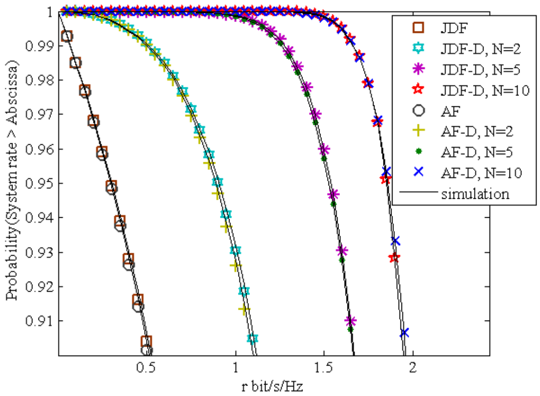
(c) $g_1 = 1,\ g_2 = 0.1,\ \mathrm{SNR} = 30\mathrm{dB}$

Fig. 4. Complementary curves of outage probability under moderately asymmetric channels

We can conclude that the conventional JDF scheme and that with diversity are more efficient than their counterparts at low SNRs regardless of the symmetry or asymmetry of the channels. For moderate SNRs, the conventional JDF scheme performs better than the AF scheme under symmetric channels, and the JDF scheme with diversity can outperforms its counterpart when the channels are asymmetric. For a high SNR region, the conventional JDF scheme and that with diversity perform worse than their counterparts when the channels are symmetric. Moreover, with the increase in the level of channel asymmetry, the outage probabilities of the two pairs tend to be the same. Compared with the conventional JDF scheme, the JDF scheme with diversity can achieve significant performance gains in terms of outage probability across the entire range of SNRs irrespective of the symmetry or asymmetry of the channels.



(a) $g_1 = 1$, $g_2 = 0.01$, $\text{SNR} = 15\text{dB}$



(b) $g_1 = 1$, $g_2 = 0.01$, $\text{SNR} = 30\text{dB}$

Fig. 5. Complementary curves of outage probability under strong asymmetric channels

7. Conclusions

In this paper, we have investigated the information theoretic metric of outage probability for the JDF based two-way relaying with network coding. Through analytical analysis, we find that the JDF scheme performs better than the AF scheme for symmetric channels when SNR is low and moderate. However, it shows a similar performance to that of the AF scheme under asymmetric channels, especially when SNR is high. In particular, at high SNRs, the JDF scheme performs worse than the AF scheme under symmetric channels. Moreover, to make a good use of the available diversity degrees of the channel, opportunistic relay selection has been examined for multi-relay networks. The results show that the use of additional relays can significantly improve overall performance. Finally, performance comparisons have been conducted between the JDF scheme with diversity and the AF scheme using relay selection. The results indicate that the JDF scheme with diversity is more efficient than its counterpart when SNR is low.

Appendix

Appendix A. Proof for (12) ~ (20)

(1) Proof for (12)~(15) :

Recall that $\mathbf{E}_1$ is defied as $\mathbf{E}_1 = \{x + y > \min[x(x+2), y(y+2)]\}$, where x and y are both exponentially distributed random variables with parameters $\lambda_1 = 1/\gamma g_1$ and $\lambda_2 = 1/\gamma g_2$, respectively, and $\mathbf{E}_2$ is the complementary event of $\mathbf{E}_1$. Considering the case where $x > y$, $\mathbf{E}_1$ can be written as $\mathbf{E}_1 = \left\{x + \frac{1}{4} > \left(y + \frac{1}{2}\right)^2\right\}$. Likewise, for the case where $x \le y$, $\mathbf{E}_1$ can be written as $\mathbf{E}_1 = \left\{y + \frac{1}{4} > \left(x + \frac{1}{2}\right)^2\right\}$. In addition, the maximal sum-rate r_0 can be written as $r_0 = \frac{1}{2}\min\left[\log_2(1+x), \log_2(1+y)\right]$ when event $\mathbf{E}_1$ occurs. To facilitate the analysis, Eq. (12) is calculated by separately considering the following two cases.

(a) Case: $x > y$

Eq. (12) can be written as

$$\begin{aligned} p_1 &= \mathrm{P}\left\{\min\left[\log_2(1+x), \log_2(1+y)\right] \le 2r, \mathbf{E}_1, x > y\right\} = \mathrm{P}\left\{y \le 2^{2r} - 1, x + \tfrac{1}{4} > \left(y + \tfrac{1}{2}\right)^2\right\} \\ &= \mathrm{P}\left\{y \le z, y < \sqrt{x + \tfrac{1}{4}} - \tfrac{1}{2}\right\} \end{aligned} \quad \text{(A1)}$$

In order to determine the integral region of Eq. (A1), two additional cases are examined. For the case where $z \ge \sqrt{x + \frac{1}{4}} - \frac{1}{2}$, i.e., $x \le \left(z + \frac{1}{2}\right)^2 - \frac{1}{4}$, Eq. (A1) can be expressed as follows

$$\begin{aligned} p_1 &= \int_0^{\left(z+\frac{1}{2}\right)^2 - \frac{1}{4}} \lambda_1 e^{-\lambda_1 x} dx \int_0^{\sqrt{x+\frac{1}{4}} - \frac{1}{2}} \lambda_2 e^{-\lambda_2 y} dy = \int_0^{\left(z+\frac{1}{2}\right)^2 - \frac{1}{4}} \lambda_1 e^{-\lambda_1 x}\left(1 - e^{\frac{\lambda_2}{2}} e^{-\lambda_2\sqrt{x+\frac{1}{4}}}\right) dx \\ &= 1 - \exp\left[-\lambda_1\left(z + \tfrac{1}{2}\right)^2 + \tfrac{\lambda_1}{4}\right] - \lambda_1 e^{\frac{\lambda_2}{2}} \int_0^{\left(z+\frac{1}{2}\right)^2 - \frac{1}{4}} \exp\left(-\lambda_1 x - \lambda_2\sqrt{x + \tfrac{1}{4}}\right) dx \\ &= 1 - \exp\left[-\lambda_1\left(z + \tfrac{1}{2}\right)^2 + \tfrac{\lambda_1}{4}\right] - 2\lambda_1 \exp\left[\tfrac{\lambda_1}{4} + \tfrac{\lambda_2}{2} + \tfrac{(\lambda_2)^2}{4\lambda_1}\right] \int_0^z \left(x + \tfrac{1}{2}\right) \exp\left[-\lambda_1\left(x + \tfrac{\lambda_1 + \lambda_2}{2\lambda_1}\right)^2\right] dx \end{aligned} \quad \text{(A2)}$$

From Eq. (3.326.4) in [22], Eq. (A2) can be given by

$$p_1 = 1 - \exp\left[-\lambda_1\left(z + \tfrac{1}{2}\right)^2 + \tfrac{\lambda_1}{4}\right] - 2\lambda_1 \exp\left[\tfrac{\lambda_1}{4} + \tfrac{\lambda_2}{2} + \tfrac{\lambda_2^2}{4\lambda_1}\right] \times \varphi(b_1, a_1), \quad \text{(A3)}$$

where $\varphi(u,v)$ is defined in Eq. (15).

For the case where $z < \sqrt{x + \frac{1}{4}} - \frac{1}{2}$, i.e., $x_k > \left(z + \frac{1}{2}\right)^2 - \frac{1}{4}$, Eq. (A1) can be calculated as follows:

$$\begin{aligned} p_1 &= \mathrm{P}\left\{y \le z, y < \sqrt{x + \tfrac{1}{4}} - \tfrac{1}{2}\right\} = \int_{\left(z+\frac{1}{2}\right)^2 - \frac{1}{4}}^{+\infty} \lambda_1 e^{-\lambda_1 x} dx \int_0^z \lambda_2 e^{-\lambda_2 y} dy \\ &= \int_{\left(z+\frac{1}{2}\right)^2 - \frac{1}{4}}^{+\infty} \lambda_1 e^{-\lambda_1 x}\left(1 - e^{-\lambda_2 z}\right) dx = \left[1 - \exp\left(-\lambda_2 z\right)\right] \exp\left[\tfrac{\lambda_1}{4} - \lambda_1\left(z + \tfrac{1}{2}\right)^2\right] \end{aligned} \quad \text{(A4)}$$

Consequently, combining Eqs. (A3) and (A4) yields Eq. (13). The proof for Eq. (13) is completed.

(b) Case: $x \le y$

Eq. (14) can be obtained using a similar method. However, this operation is omitted due to the space limitations.

(2) Proof for (16)~ (20):

Recall that the maximal sum-rate r_0 can be written as $r_0 = \frac{1}{4}\log_2(1 + x + y)$ when event $\mathbf{E}_2$ occurs. Similar to the calculation of Eq. (13), Eq. (16) is calculated by separately considering the following two cases.

(a) Case: $x > y$

For this case, Eq. (16) can be written as follows:

$$\begin{aligned} p_{JDF}^{E_2} &= \mathrm{P}\left\{x+y \le z(z+2), \mathbf{E}_2\right\} = \mathrm{P}\left\{x+y \le z(z+2), x+\tfrac{1}{4} \le \left(y+\tfrac{1}{2}\right)^2\right\} \\ &= \int_0^{\frac{z^2+2z}{2}} \lambda_1 e^{-\lambda_1 x} dx \int_{\sqrt{x+\frac{1}{4}}-\frac{1}{2}}^{x} \lambda_2 e^{-\lambda_2 y} dy + \int_{\frac{z^2}{2}+z}^{z^2+z} \lambda_1 e^{-\lambda_1 x} dx \int_{\sqrt{x+\frac{1}{4}}-\frac{1}{2}}^{z^2+2z-x} \lambda_2 e^{-\lambda_2 y} dy \\ &= \lambda_1 \exp\left(\tfrac{\lambda_2}{2}\right) \int_0^{z^2+z} \exp\left(-\lambda_1 x - \lambda_2 \sqrt{x+\tfrac{1}{4}}\right) dx - \tfrac{\lambda_1}{\lambda_1+\lambda_2}\left\{1-\exp\left[-\tfrac{(\lambda_1+\lambda_2)(z^2+2z)}{2}\right]\right\} \\ &\quad - \lambda_1 \exp\left[-\lambda_2 (z^2+2z)\right] \int_{\frac{z^2+2z}{2}}^{z^2+z} \exp\left[\left(\lambda_2-\lambda_1\right)x\right] dx \end{aligned} . \quad \text{(A5)}$$

From Eq. (3.326.5) in [22], Eq. (A5) can be further written as

$$\begin{aligned} p_{JDF}^{E_2} &= 2\lambda_1 \exp\left(\tfrac{\lambda_1}{4}+\tfrac{\lambda_2}{2}+\tfrac{\lambda_2^2}{4\lambda_1}\right) \times \varphi\left(b_1, a_1\right) - \tfrac{\lambda_2}{\lambda_1+\lambda_2}\left\{1-\exp\left[-\tfrac{(\lambda_1+\lambda_2)(z^2+2z)}{2}\right]\right\} \\ &\quad - \lambda_1 \exp\left[-\lambda_2 (z^2+2z)\right] \int_{\frac{z^2+2z}{2}}^{z^2+z} \exp\left[\left(\lambda_2-\lambda_1\right)x\right] dx \end{aligned} . \quad \text{(A6)}$$

For the derivation of Eq. (A6), two additional cases should be considered. For the case where $\lambda_1 = \lambda_2$, the last integral term in Eq. (A6) can be given by

$$\lambda_1 \exp\left[-\lambda_2 (z^2+2z)\right] \int_{\frac{z^2+2z}{2}}^{z^2+z} \exp\left[\left(\lambda_2-\lambda_1\right)x\right] dx = \lambda_1 \tfrac{z^2}{2} \exp\left[-\lambda_2 (z^2+2z)\right]. \quad \text{(A7)}$$

For case where $\lambda_1 \ne \lambda_2$, the last integral term in Eq. (A6) can be expressed as follows:

$$\begin{aligned} &\lambda_1 \exp\left[-\lambda_2 (z^2+2z)\right] \int_{\frac{z^2+2z}{2}}^{z^2+z} \exp\left[\left(\lambda_2-\lambda_1\right)x\right] dx \\ &= \tfrac{\lambda_1}{\lambda_2-\lambda_1} \exp\left[-\lambda_1 z - \lambda_1 \tfrac{z^2}{2} - \lambda_2 z)\right]\left[\exp\left(-\lambda_1 \tfrac{z^2}{2}\right) - \exp\left(-\lambda_2 \tfrac{z^2}{2}\right)\right] \end{aligned} . \quad \text{(A8)}$$

Consequently, substituting Eqs. (A7) and (A8) into Eq. (A6) yields Eqs. (17) and (19), respectively.

(b) Case $x \le y$

Eqs. (18) and (20) can be obtained using a similar method. However, this operation is omitted due to space limitations. Finally, using the law of total probability, Eq. (21) can be obtained, directly.

Appendix B. Outage probability of the AF scheme

Here, we present the outage probability expressions of AF based two-way relay transmission, which are used for the performance comparisons in section 6. It is well known that for a conventional three-node AF scenario, the mutual information achieved by terminals A and B can be respectively given by [9][11][12][18][23][24]

$$I_A = \tfrac{1}{2}\log_2\left(1+\tfrac{\gamma^2|h_{AR}|^2|h_{BR}|^2}{2\gamma|h_{AR}|^2+\gamma|h_{BR}|^2+1}\right), I_B = \tfrac{1}{2}\log_2\left(1+\tfrac{\gamma^2|h_{AR}|^2|h_{BR}|^2}{\gamma|h_{AR}|^2+2\gamma|h_{BR}|^2+1}\right). \quad \text{(A9)}$$

Thus, the outage probabilities of terminals A, B and the whole system can be depicted as $p_{AF-A} = \mathrm{P}\left(I_A < r\right)$, $p_{AF-B} = \mathrm{P}\left(I_B < r\right)$ and $p_{AF} = \mathrm{P}\left(I_A < r, I_B < r\right)$, respectively. For an AF system, p_{AF-A} and p_{AF-B} can be given by the following

$$p_{AF-A} = 1-\exp\left(-(\lambda_1+2\lambda_2)z\right)\sqrt{4\lambda_1\lambda_2 z(2z+1)}K_1\left(\sqrt{4\lambda_1\lambda_2 z(2z+1)}\right), \quad \text{(A10)}$$

$$p_{AF-B} = 1-\exp\left(-(2\lambda_1+\lambda_2)z\right)\sqrt{4\lambda_1\lambda_2 z(2z+1)}K_1\left(\sqrt{4\lambda_1\lambda_2 z(2z+1)}\right), \quad \text{(A11)}$$

where $K_1(\cdot)$ is the modified Bessel function of the second kind.

Moreover, the outage probability of the whole system can be shown as follows:

$$p_{AF} = 1 - \left[\lambda_1 \exp\left(-2\lambda_1 z - \lambda_2 z\right)\psi\left(\lambda_1, \lambda_2\right) + \lambda_2 \exp\left(-\lambda_1 z - 2\lambda_2 z\right)\psi\left(\lambda_2, \lambda_1\right)\right] + \exp\left[-\left(\lambda_1 + \lambda_2\right)\left(\tfrac{3Z}{2} + \tfrac{\sqrt{9z^2+4z}}{2}\right)\right], \tag{A12}$$

where $\psi(u,v) = \int_{-\frac{z}{2}+\frac{\sqrt{9z^2+4z}}{2}}^{+\infty} \exp\left[-ut - \frac{v(2z+1)z}{t}\right]dt$.

In order to make fair comparison, the AF scheme using relay selection is also considered. As the NCD (Network Coding with Diversity) strategy proposed in [9], for an AF system with N relay nodes, we presume that the best relay is chosen according to the max-min mutual information criterion. Thereby, when the max-min criterion is applied, the system outage probability can be expressed as

$$p_{AF-D}^{(N)} = \left(p_{AF}\right)^N . \tag{A13}$$

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