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Fuzzy (r, s)-S_1-pre-semicontinuous mappings

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Abstract

In this paper, we introduce the notion of fuzzy (r, s)-S_1-pre-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense, which is a generalization of S_1-pre-semicontinuous mappings by Shi-Zhong Bai. The relationship between fuzzy (r, s)-pre-semicontinuous mapping and fuzzy (r, s)-S_1-pre-semicontinuous mapping is discussed. The characterizations for the fuzzy (r, s)-S_1-pre-semicontinuous mappings are obtained.

Key Words: intuitionistic fuzzy topological space, (r, s)-S_1-pre-semicontinuous

1. Introduction

The concept of fuzzy set was introduced by Zadeh [1]. Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [3], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [4], and by Ramadan [5]. Shi-Zhong Bai [6] introduced the concept of fuzzy S_1-pre-semicontinuous mappings on Chang's fuzzy topological spaces.

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [7]. Recently, Çoker and his colleagues [8, 9] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [10] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. In the previous works, we also studied the structure of the category of intuitionistic fuzzy topological spaces, and investigated properties of fuzzy strongly (r, s)-precontinuous mappings in these spaces [11, 12].

In this paper, we introduce the notion of fuzzy (r, s)-S_1-pre-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense, which is a generalization of S_1-pre-semicontinuous mappings by Shi-Zhong Bai. The relationship between fuzzy (r, s)-pre-semicontinuous mapping and fuzzy (r, s)-S_1-pre-semicontinuous mapping is discussed. The characterizations for the fuzzy (r, s)-S_1-pre-semicontinuous mappings are obtained.

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2. Preliminaries

We will denote the unit interval $[0, 1]$ of the real line by I. A member μ of I^X is called a *fuzzy set* in X. For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively and $\mu_A + \gamma_A \leq 1$. Obviously every fuzzy set μ in X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

Definition 2.1 ([7]). Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets in X. Then

(1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.

(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

(3) $A^c = (\gamma_A, \mu_A)$.

(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.

(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.

(6) $\underline{0} = (\tilde{0}, \tilde{1})$ and $\underline{1} = (\tilde{1}, \tilde{0})$.

A *smooth topology* on X is a mapping $T : I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$.

(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.

(3) $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$.

The pair (X, T) is called a *smooth topological space*.

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $\underline{0}, \underline{1} \in T$.

(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.

(3) If $A_i \in T$ for each i, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space*.

Let $I(X)$ be the family of all intuitionistic fuzzy sets in X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.2 ([10]). Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a mapping $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties:

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Definition 2.3 ([13]). Let A be an intuitionistic fuzzy set in SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy (r, s)-open* if $\mathcal{T}_1(A) \geq r$ and $\mathcal{T}_2(A) \leq s$,

(2) *fuzzy (r, s)-closed* if $\mathcal{T}_1(A^c) \geq r$ and $\mathcal{T}_2(A^c) \leq s$.

Definition 2.4 ([13]). Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy (r, s)-interior* is defined by

$$\text{int}(A, r, s) = \bigcup \{B \in I(X) \mid B \subseteq A,\ B \text{ is fuzzy } (r, s)\text{-open}\}$$

and the *fuzzy (r, s)-closure* is defined by

$$\text{cl}(A, r, s) = \bigcap \{B \in I(X) \mid A \subseteq B,\ B \text{ is fuzzy } (r, s)\text{-closed}\}.$$

Lemma 2.5 ([13]). For an intuitionistic fuzzy set A in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$,

(1) $\text{int}(A, r, s)^c = \text{cl}(A^c, r, s)$.

(2) $\text{cl}(A, r, s)^c = \text{int}(A^c, r, s)$.

Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an intuitionistic fuzzy topological space in Šostak's sense. Then it is easy to see that for each $(r, s) \in I \otimes I$, the family $\mathcal{T}_{(r,s)}$ defined by

$$\mathcal{T}_{(r,s)} = \{A \in I(X) \mid \mathcal{T}_1(A) \geq r \text{ and } \mathcal{T}_2(A) \leq s\}$$

is an intuitionistic fuzzy topology on X.

Let (X, T) be an intuitionistic fuzzy topological space and $(r, s) \in I \otimes I$. Then the map $T^{(r,s)} : I(X) \to I \otimes I$ defined by

$$T^{(r,s)}(A) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (r, s) & \text{if } A \in T - \{\underline{0}, \underline{1}\}, \\ (0, 1) & \text{otherwise} \end{cases}$$

becomes an intuitionistic fuzzy topology in Šostak's sense on X.

Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X is an intuitionistic fuzzy set in X defined by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta) & \text{if } y = x, \\ (0, 1) & \text{if } y \neq x. \end{cases}$$

In this case, x is called the *support* of $x_{(\alpha,\beta)}$, α the *value* of $x_{(\alpha,\beta)}$, and β the *nonvalue* of $x_{(\alpha,\beta)}$. An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to *belong* to an intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in X, denoted by $x_{(\alpha,\beta)} \in A$, if $\mu_A(x) \geq \alpha$ and $\gamma_A(x) \leq \beta$. An intuitionistic fuzzy set A in X is the union of all intuitionistic fuzzy points which belong to A.

Definition 2.6 ([13]). Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy (r, s)-semiopen* if there is a fuzzy (r, s)-open set B in X such that $B \subseteq A \subseteq \text{cl}(B, r, s)$,

(2) *fuzzy (r, s)-semiclosed* if there is a fuzzy (r, s)-closed set B in X such that $\text{int}(B, r, s) \subseteq A \subseteq B$.

Theorem 2.7 ([13]). Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) A is a fuzzy (r, s)-semiopen set.

(2) A^c is a fuzzy (r, s)-semiclosed set.

(3) $\text{cl}(\text{int}(A, r, s), r, s) \supseteq A$.

(4) $\text{int}(\text{cl}(A^c, r, s), r, s) \subseteq A^c$.

Definition 2.8 ([14, 15]). Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy* (r,s)*-pre-semiopen* if $A \subseteq \text{sint}(\text{cl}(A,r,s),r,s)$,

(2) *fuzzy* (r,s)*-pre-semiclosed* if $\text{scl}(\text{int}(A,r,s),r,s) \subseteq A$,

(3) *fuzzy strongly* (r,s)*-semiopen* if there is a fuzzy (r,s)-open set B in X such that $B \subseteq A \subseteq \text{int}(\text{cl}(B,r,s),r,s)$,

(4) *fuzzy strongly* (r,s)*-semiclosed* if there is a fuzzy (r,s)-closed set B in X such that $\text{cl}(\text{int}(B,r,s),r,s) \subseteq A \subseteq B$.

Theorem 2.9 ([15]). Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then the following statements are equivalent:

(1) A is a fuzzy strongly (r,s)-semiopen set.

(2) A^c is a fuzzy strongly (r,s)-semiclosed set.

(3) $A \subseteq \text{int}(\text{cl}(\text{int}(A,r,s),r,s),r,s)$.

(4) $A^c \supseteq \text{cl}(\text{int}(\text{cl}(A^c,r,s),r,s),r,s)$.

Definition 2.10 ([15]). Let $(X,\mathcal{T}_1,\mathcal{T}_2)$ be a SoIFTS. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$,

(1) the *fuzzy strongly* (r,s)*-semiinterior* is defined by

$$\text{ssint}(A,r,s) = \bigcup\{B \in I(X) \mid B \subseteq A,$$
$$B \text{ is fuzzy strongly } (r,s)\text{-semiopen}\},$$

(2) the *fuzzy strongly* (r,s)*-semiclosure* is defined by

$$\text{sscl}(A,r,s) = \bigcap\{B \in I(X) \mid A \subseteq B,$$
$$B \text{ is fuzzy strongly } (r,s)\text{-semiclosed}\}.$$

Definition 2.11 ([14, 15]). Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is called

(1) a *fuzzy* (r,s)*-pre-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy (r,s)-pre-semiopen set in X for each fuzzy (r,s)-open set B in Y,

(2) a *fuzzy strongly* (r,s)*-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy strongly (r,s)-semiopen set in X for each fuzzy (r,s)-open set B in Y.

Definition 2.12 ([6]). Let $f : (X_1,\delta_1) \to (X_2,\delta_2)$ be a mapping from a fuzzy space X_1 to another fuzzy space X_2, f is called a *fuzzy* S_1*-pre-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy pre-semiopen set of X_1 for each fuzzy strongly semiopen set B of X_2.

3. Fuzzy (r,s)-S_1-pre-semicontinuous mappings

Now, we define the notion of fuzzy (r,s)-S_1-pre-semicontinuous mappings, and then we investigate some of their chacracteristic properties.

Definition 3.1. Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is called a *fuzzy* (r,s)*-*S_1*-pre-semicontinuous* mapping if $f^{-1}(B)$ is fuzzy (r,s)-pre-semiopen in X for each fuzzy strongly (r,s)-semiopen set B in Y.

Definition 3.2. Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is said to be *fuzzy* (r,s)*-*S_1*-pre-semicontinuous at an intuitionistic fuzzy point* $x_{(\alpha,\beta)}$ in X if for each fuzzy strongly (r,s)-semiopen set B in Y with $f(x_{(\alpha,\beta)}) \in B$, there is a fuzzy (r,s)-pre-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

Theorem 3.3. Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is fuzzy (r,s)-S_1-pre-semicontinuous if and only if f is fuzzy (r,s)-S_1-pre-semicontinuous for each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X.

Proof. Let f be a fuzzy (r,s)-S_1-pre-semicontinuous mapping, $x_{(\alpha,\beta)}$ an intuitionistic fuzzy point in X, and B a fuzzy strongly (r,s)-semiopen set in Y with $f(x_{(\alpha,\beta)}) \in B$. Since f is fuzzy (r,s)-S_1-pre-semicontinuous, $f^{-1}(B)$ is fuzzy (r,s)-pre-semiopen in X. Put $A = f^{-1}(B)$. Then A is fuzzy (r,s)-pre-semiopen such that $x_{(\alpha,\beta)} \in A$, and $f(A) = f(f^{-1}(B)) \subseteq B$.

Conversely, let B be a fuzzy strongly (r,s)-semiopen set in Y and $x_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(x_{(\alpha,\beta)}) \in B$. By the assumption, there is a fuzzy (r,s)-pre-semiopen set $A_{x_{(\alpha,\beta)}}$ in X such that $x_{(\alpha,\beta)} \in A_{x_{(\alpha,\beta)}}$ and $f(A_{x_{(\alpha,\beta)}}) \subseteq B$. Hence

$$\begin{aligned} f^{-1}(B) &= \bigcup\{x_{(\alpha,\beta)} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq \bigcup\{A_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq f^{-1}(B). \end{aligned}$$

Thus $f^{-1}(B) = \bigcup\{A_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\}$, which is a fuzzy (r,s)-pre-semiopen set in X. Therefore f is a fuzzy (r,s)-S_1-pre-semicontinuous mapping. □

Remark 3.4. It is clear that every fuzzy (r,s)-S_1-pre-semicontinuous mapping is fuzzy (r,s)-pre-semicontinuous. However, the following example shows that the converse need not be true.

Example 3.5. Let $X = \{x,y,z\}$ and let A_1, A_2, and A_3 be intuitionistic fuzzy sets in X defined as

$A_1(x) = (0.3, 0.6), A_1(y) = (0.2, 0.6), A_1(z) = (0.4, 0.5);$

$A_2(x)=(0.4,0.5), A_2(y)=(0.2,0.6), A_2(z)=(0.5,0.5)$; and

$$A_3(x)=(0.2,0.7), A_3(y)=(0.6,0.2), A_3(z)=(0.4,0.5).$$

Define $\mathcal{T}: I(X)\to I\otimes I$ and $\mathcal{U}: I(X)\to I\otimes I$ by

$$\mathcal{T}(A)=(\mathcal{T}_1(A),\mathcal{T}_2(A))=\begin{cases}(1,0) & \text{if } A=\underline{0},\underline{1},\\ (\frac{1}{2},\frac{1}{3}) & \text{if } A=A_1,A_2,\\ (0,1) & \text{otherwise;}\end{cases}$$

and

$$\mathcal{U}(A)=(\mathcal{U}_1(A),\mathcal{U}_2(A))=\begin{cases}(1,0) & \text{if } A=\underline{0},\underline{1},\\ (\frac{1}{2},\frac{1}{3}) & \text{if } A=A_3,\\ (0,1) & \text{otherwise.}\end{cases}$$

Then $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f:(X,\mathcal{T})\to(X,\mathcal{U})$ defined by $f(x)=x$, $f(y)=y$, and $f(z)=z$. Note that

$$\begin{aligned}A_1^c &\subseteq \text{int}(\text{cl}(\text{int}(A_1^c,\frac{1}{2},\frac{1}{3}),\frac{1}{2},\frac{1}{3}),\frac{1}{2},\frac{1}{3})\\ &= \text{int}(\text{cl}(A_3,\frac{1}{2},\frac{1}{3}),\frac{1}{2},\frac{1}{3})\\ &= \text{int}(\underline{1},\frac{1}{2},\frac{1}{3})=\underline{1}.\end{aligned}$$

Hence A_1^c is fuzzy strongly $(\frac{1}{2},\frac{1}{3})$-semiopen in $(X,\mathcal{U})$. Since

$$\begin{aligned}f^{-1}(A_3)=A_3 &\subseteq \text{sint}(\text{cl}(A_3,\frac{1}{2},\frac{1}{3}),\frac{1}{2},\frac{1}{3})\\ &= \text{sint}(A_2^c,\frac{1}{2},\frac{1}{3})=A_2^c,\end{aligned}$$

f is fuzzy $(\frac{1}{2},\frac{1}{3})$-pre-semicontinuous. But f is not fuzzy $(\frac{1}{2},\frac{1}{3})$-$S_1$-pre-semicontinuous, because

$$\begin{aligned}f^{-1}(A_1^c)=A_1^c &\not\subseteq \text{sint}(\text{cl}(A_1^c,\frac{1}{2},\frac{1}{3}),\frac{1}{2},\frac{1}{3})\\ &= \text{sint}(A_1^c,\frac{1}{2},\frac{1}{3})=A_2^c.\end{aligned}$$

Theorem 3.6. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s)\in I\otimes I$. Then the following statements are equivalent:

(1) f is fuzzy (r,s)-S_1-pre-semicontinuous.

(2) $f^{-1}(B)$ is fuzzy (r,s)-pre-semiclosed in X for each fuzzy strongly (r,s)-semiclosed set B in Y.

(3) For each intuitionistic fuzzy set A in X,

$$f(\text{scl}(\text{int}(A,r,s),r,s))\subseteq \text{sscl}(f(A),r,s).$$

(4) For each intuitionistic fuzzy set B in Y,

$$\text{scl}(\text{int}(f^{-1}(B),r,s),r,s)\subseteq f^{-1}(\text{sscl}(B,r,s)).$$

(5) For each intuitionistic fuzzy set B in Y,

$$f^{-1}(\text{ssint}(B,r,s))\subseteq \text{sint}(\text{cl}(f^{-1}(B),r,s),r,s).$$

Proof. (1) ⇔ (2) Trivial.

(2) ⇒ (3) Let $A\in I(X)$, then $f(A)\in I(Y)$. Since $\text{sscl}(f(A),r,s)$ is fuzzy strongly (r,s)-semiclosed in Y, $f^{-1}(\text{sscl}(f(A),r,s))$ is fuzzy (r,s)-pre-semiclosed in X from (2). Hence
$\text{scl}(\text{int}(A,r,s),r,s)$

$$\begin{aligned}&\subseteq \text{scl}(\text{int}(f^{-1}(f(A)),r,s),r,s)\\ &\subseteq \text{scl}(\text{int}(f^{-1}(\text{sscl}(f(A),r,s)),r,s),r,s)\\ &\subseteq f^{-1}(\text{sscl}(f(A),r,s)).\end{aligned}$$

Thus we have $f(\text{scl}(\text{int}(A,r,s),r,s))\subseteq\text{sscl}(f(A),r,s)$.

(3) ⇒ (4) Let $B\in I(Y)$, then $f^{-1}(B)\in I(X)$. By (3), we have

$$\begin{aligned}f(\text{scl}(\text{int}(f^{-1}(B),r,s),r,s)) &\subseteq \text{sscl}(f(f^{-1}(B)),r,s)\\ &\subseteq \text{sscl}(B,r,s).\end{aligned}$$

Thus $\text{scl}(\text{int}(f^{-1}(B),r,s),r,s)\subseteq f^{-1}(\text{sscl}(B,r,s))$.

(4) ⇒ (5) Let $B\in I(Y)$. By (4), we obtain

$$\text{scl}(\text{int}(f^{-1}(B^c),r,s),r,s)\subseteq f^{-1}(\text{sscl}(B^c,r,s)).$$

Hence we have

$$f^{-1}(\text{ssint}(B,r,s))\subseteq\text{sint}(\text{cl}(f^{-1}(B),r,s),r,s).$$

(5) ⇒ (1) Let B be a fuzzy strongly (r,s)-semiopen set in Y. Then by (5), we have

$$f^{-1}(B)=f^{-1}(\text{ssint}(B,r,s))\subseteq\text{sint}(\text{cl}(f^{-1}(B),r,s),r,s).$$

Thus $f^{-1}(B)$ is fuzzy (r,s)-pre-semiopen in X. Hence f is fuzzy (r,s)-S_1-pre-semicontinuous. □

Theorem 3.7. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a bijective mapping from a SoIFTS X to a SoIFTS Y and $(r,s)\in I\otimes I$. Then f is fuzzy (r,s)-S_1-pre-semicontinuous if and only if $\text{ssint}(f(A),r,s)\subseteq f(\text{sint}(\text{cl}(A,r,s),r,s)$ for each intuitionistic fuzzy set A in X.

Proof. Suppose that f is fuzzy (r,s)-S_1-pre-semicontinuous. Let $A\in I(X)$, then $f(A)\in I(Y)$. Since $\text{ssint}(f(A),r,s)$ is fuzzy strongly (r,s)-semiopen in Y, $f^{-1}(\text{ssint}(f(A),r,s))$ is fuzzy (r,s)-pre-semiopen in X. Hence we have
$\text{ssint}(f(A),r,s)$

$$\begin{aligned}&= f(f^{-1}(\text{ssint}(f(A),r,s)))\\ &\subseteq f(\text{sint}(\text{cl}(f^{-1}(\text{ssint}(f(A),r,s)),r,s),r,s))\\ &\subseteq f(\text{sint}(\text{cl}(f^{-1}(f(A)),r,s),r,s))\\ &= f(\text{sint}(\text{cl}(A,r,s),r,s)).\end{aligned}$$

Conversely, let B be a fuzzy strongly (r,s)-semiopen set in Y. Then $f^{-1}(B) \in I(X)$. By hypothesis, we have

$$\begin{aligned} B = \text{ssint}(B,r,s) &= \text{ssint}(f(f^{-1}(B)),r,s) \\ &\subseteq f(\text{sint}(\text{cl}(f^{-1}(B),r,s),r,s)). \end{aligned}$$

Thus $f^{-1}(B) \subseteq \text{sint}(\text{cl}(f^{-1}(B),r,s),r,s)$. Hence $f^{-1}(B)$ is fuzzy (r,s)-pre-semiopen in X. Therefore f is fuzzy (r,s)-S_1-pre-semicontinuous. □

Theorem 3.8. Let $f : (X,\mathcal{T}) \to (Y,\mathcal{U})$ and $g : (Y,\mathcal{U}) \to (Z,\mathcal{S})$ be mappings. If f is fuzzy (r,s)-S_1-pre-semicontinuous and g is fuzzy strongly (r,s)-semicontinuous, then $g \circ f$ is fuzzy (r,s)-pre-semicontinuous.

Proof. Let B be a fuzzy (r,s)-open set in Y. Since g is fuzzy strongly (r,s)-semicontinuous, $g^{-1}(B)$ is fuzzy strongly (r,s)-semiopen in Y. Because f is fuzzy (r,s)-S_1-pre-semicontinuous, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is fuzzy (r,s)-pre-semiopen in X. Thus $g \circ f$ is fuzzy (r,s)-pre-semicontinuous. □

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