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PVAJT 모션플래너를 이용한 Cubic Spline 보간기의 설계

신동원#

Design of Cubic Spline Interpolator using a PVAJT Motion Planner

Dong-Won Shin#

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ABSTRACT

A cubic spline trajectory planner with arc-length parameter is formulated with estimation by summing up to the 3rd order in Taylor's expansion. The PVAJT motion planning is presented to reduce trajectory calculation time at every cycle time of servo control loop so that it is able to generate cubic spline trajectory in real time. This method can be used to more complex spline trajectory. Several case studies are executed with different values of cycle time and sampling time, and showed the advantages of the PVAJT motion planner. A DSP-based motion controller is designed to implement the PVAJT motion planning.

Key Words : PVAJT Motion, Cubic Spline, Trajectory, Arc Length, Cycle Time, Sampling Time

1. Introduction

Employing only linear and circular interpolation techniques to motion control systems, more complex tasks have serious limitations in terms of achieving the desired path and productivity. To address these problems, a significant amount of effort has been put in the recent years, in developing new spline trajectory generation algorithms that provide smooth velocity profile. Moreover, modern motion control systems need to operate at high speeds with high accelerations in order to deliver the rapid feed motion. At such high speeds, small discontinuities in the reference path can result in undesirable high frequency harmonics in the reference trajectory, which end up exciting the natural modes of the mechanical structure and the servo control systems. The small discontinuities in the reference position mean small fluctuation errors of velocity. Therefore, the spline trajectory planner that can generate complex geometrical shape and contain small discontinuities is needed in the motion control area.

It has been reported that the accuracy and efficiency of the motion control system can be improved, using a parametric interpolator[1,2]. Several researchers have proposed parametric interpolators for Bezier/B-spline and implicit curves[3-5]. Erkokmaz and Altintas[6] introduced a new quintic spline interpolation technique

교신저자 : Department of Mechanical System Engineering
Kumoh National Institute of Technology
E-mail : shindw@kumoh.ac.kr

for more improvement of relationship between the parameter to the actual arc-length. S.-S Yeh et al. introduced an algorithm for parametric curves, which predicted the actuator saturation and suitably adjusted the interpolation parameter to maintain the tracking accuracy of a high-speed motion control system[7]. However, all studies deal with PT(Position Time) trajectory generation which has the limitation of calculation if the cycle time of servo controller becomes shorter.

This paper presents a cubic spline interpolator with arc-length parameter which is estimated by summing up to the 3rd order in Taylor's expansion, which can decrease velocity fluctuations associated with the approximation truncation errors. However, the process of obtaining parameter and position data for spline trajectory at every update time gives inevitably some burden to main CPU, thus implementation of real time trajectory calculation can be difficult at every cycle time of servo control loop.

Therefore, this paper presents the new method of generating the trajectory at every cycle using PVAJT data, where P means the position, V the velocity, A the acceleration, J the jerk, and T is the multiplied cycle time. This can reduce the calculation time in the trajectory planner so that it is able to generate more complex trajectory in real time.

2. Cubic spline

2.1 Cubic spline interpolating data points

The parametric equation of cubic degree is Eq. (1) where a_0, a_1, a_2, a_3 are the vector coefficients of the parametric equation. For simplicity, the formulation will be presented for only two dimensions (x and y). but it can easily be extended to the three dimensional case.

$$P(u) = [x(u)\ y(u)] = a_0 + a_1 u + a_2 u^2 + a_3 u^3 \tag{1}$$

The cubic curve has advantage that varying the magnitude of the tangent vectors offers a way to modify the interior shape of a curve without changing the position the endpoints.

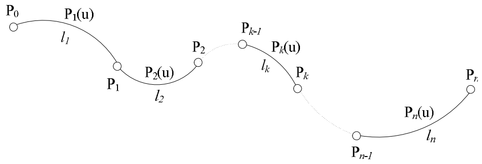

Fig. 1 Cubic spline interpolating n+1 data points

The data points P_0, P_1, P_n are given, the cubic spline interpolating these n+1 data points is shown in Fig. 1. The kth cubic curve in the spline can be expressed as following equation where the parameter u ranges from 0 to 1, and P_k' are the tangent vectors at the data point P_k.

$$P_k(u) = P_{k-1} + P'_{k-1}u + \left[3(P_k - P_{k-1}) - 2P'_{k-1} - P'_k\right]u^2 + \left[2(P_{k-1} - P_k) + P'_{k-1} + P_k\right]u^3 \tag{2}$$

From the following second-order continuity condition across the curve segments, we can evaluate the values of P_k'.

$$\left.\frac{d^2 P_k(u)}{du^2}\right]_{u=1} = \left.\frac{d^2 P_{k+1}(u)}{du^2}\right]_{u=0} \quad (k = 1, 2, ..., n-1) \tag{3}$$

Substituting Eq. (3) into Eq. (2) gives following matrix equation for the unknown variables P_1', $P_2' \cdots$, P_{n-1}' whereas the values of P_0' and P_n' are known

$$\begin{bmatrix} 4 & 1 & 0 & . & . & . & 0 & 0 \\ 1 & 4 & 1 & 0 & . & . & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & . & . & \\ & & & & & & & \\ & & & & & & & \\ 0 & 0 & . & . & & 1 & 4 & 1 \\ 0 & 0 & . & . & & 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} P_1' \\ P_2' \\ . \\ . \\ . \\ . \\ P_{n-1}' \end{bmatrix} = \begin{bmatrix} 3P_2 - 3P_0 - P_0' \\ 3P_3 - 3P_1 \\ 3P_4 - 3P_2 \\ . \\ . \\ . \\ 3P_n - 3P_{n-2} - P_n' \end{bmatrix} \tag{4}$$

2.2 Relationship between parameter and arc length

Motion planning generates the trajectory from the velocity profile formed from the values the initial velocity, acceleration, target velocity and deceleration. At every sampling time of the trajectory, the arc length of the curve can be computed by integrating the velocity profile. The parameters u of line and circle are directly proportional to the arc length s, whereas the parameter of the cubic curve is not.

The differential of s of the parameterized curve P(u) is as follows.

$$ds = |d\mathrm{P}| = \left|\frac{d\mathrm{P}}{du}\right| du = \sqrt{\frac{d\mathrm{P}}{du} \cdot \frac{d\mathrm{P}}{du}} du = \sqrt{\mathrm{P}' \cdot \mathrm{P}'} du \tag{5}$$

Therefore, differential of u can be expressed as Eq. (6~7) using $Q(u) = \sqrt{P' \cdot P'}$, where v is the velocity and t is time. The expression of $\mathrm{P}' \cdot \mathrm{P}'$ can be derived from Eq. (1).

$$du = \frac{ds}{\sqrt{\mathrm{P}' \cdot \mathrm{P}'}} \tag{6}$$

$$du = dsQ(u) = vdtQ(u) \tag{7}$$

The relationship between the parameter and the arc length in the cubic curve is the differential relationship while in the other curves such as line and circle the linear relationship. Thus, the value of parameter u can be determined by simply integrating Eq. (6). The ways of obtaining u are described in details in the next section. The differentials of u with respect to time can be derived from Eq. (7) as follows.

$$\dot{u} = vQ \tag{8}$$

$$\ddot{u} = aQ + vQ'\dot{u} \tag{9}$$

$$\dddot{u} = jQ + 2aQ'\dot{u} + vQ''\dot{u}^2 + vQ'\ddot{u} \tag{10}$$

In Eq. (8-10), the velocity v, acceleration and jerk j are determined from the velocity profile, and the values of Q' and Q'' are obtained from algebraic expressions rather than from numerical differentiation because of improving accuracy.

2.3 Estimating parameter from differential equation

The result after collecting similar terms in Eq. (6) and integrating is expressed as following equations.

$$\int_0^l ds = \int_0^w \sqrt{\mathrm{P}'(\mathrm{u}) \cdot \mathrm{P}'(\mathrm{u})} du \tag{11}$$

$$l = \int_0^w \sqrt{\mathrm{P}'(\mathrm{u}) \cdot \mathrm{P}'(\mathrm{u})} du \tag{12}$$

In the above equation, the value of arc length l is assigned from the velocity profile at the specified time so that the parameter u can be evaluated by solving Eq. (12). Solving this equation needs much time for calculating the right integral term, so that it is not fitted well to real time trajectory calculation.

Therefore, the value of parameter u should be estimated by Taylor expansion. Because the parameter u is the function of time t, the u_{k+1} can be represented by following Taylor expansion with respect to time, where the values of $\dot{u}_k$, $\ddot{u}_k$ and $\dddot{u}_k$ are determined from Eq. (8-10).

$$u_{k+1} = u_k + \dot{u}_k \Delta t + \frac{\dot{u}_k}{2}(\Delta t)^2 + \frac{\ddot{u}_k}{6}(\Delta t)^3 + \cdots \tag{13}$$

For the more accurate value of u, the sum up to higher order term in Eq. (13) is required. Considering the calculation time for real-time motion control, summming up to third order term is chosen.

3. PVAJT motion planning of cubic spline

The PVAJT means position, velocity, acceleration, jerk, segment time of the trajectory. Let the position vector of cubic curve be P(u), then the velocity vector

V(u), the acceleration vector A(u) and the jerk vector J(u) can be expressed as below.

$$\mathrm{V}(u) = \mathrm{P}'(u)\frac{du}{dt} = \mathrm{P}'\dot{u} \tag{14}$$

$$\mathrm{A}(u) = \mathrm{P}''\dot{u}^2 + \mathrm{P}'\ddot{u} \tag{15}$$

$$\mathrm{J}(u) = \mathrm{P}'''\dot{u}^3 + 2\mathrm{P}''\dot{u}\ddot{u} + \mathrm{P}''\dot{u}\ddot{u} + \mathrm{P}'\dddot{u} \tag{16}$$

The sets of these 4 values are specified for a given motion at multiple points of trajectory as shown Table 1, where ″No″ denotes the order of the set and ″SegT″ the segment time. Segment time means the time interval of the corresponding set and is usually expressed as the multiple number of the cycle time of servo controller. The row of the table represents each PVAJT set.

Table 1 The series of PVAJT sets

No	P	V	A	J	SegT
0	P_0	V_0	A_0	J_0	T_0
1	P_1	V_1	A_1	J_1	T_1
…	…	…	…	…	…
j	P_j	V_j	A_j	J_j	T_j
…	…	…	…	…	…
n	P_n	V_n	A_n	J_n	T_n

Let the segment time be denoted as SegT which is integer value, and the real time t_k is increased by the cycle time T_c of servo controller at every step, i.e. $t_k=T_c k$ (k=0,⋯, SegT-1). Let P, V, A and J at some moment be given, then the position P_k at next time step $t_k=T_c k$ (k=0,⋯, SegT-1) can be derived as following by integrating V, A and J with respect to time.

$$\begin{aligned} &\mathrm{P}_j^{(k)} = \mathrm{P}_j + \mathrm{V}_j t_k + \frac{\mathrm{A}_j}{2}t_k^2 + \frac{\mathrm{J}_j}{6}t_k^3 \\ &\text{where } t_k = \mathrm{T}_c k\,(k = 0,\cdots,\mathrm{T}_j - 1) \end{aligned} \tag{17}$$

For example, in case of SegT=3 and T_c=1, $P_j^{(k)}$ from k=0 to k=2 is as below.

$$\begin{aligned} \mathrm{P}_j^{(0)} &= \mathrm{P}_j \\ \mathrm{P}_j^{(1)} &= \mathrm{P}_j + \mathrm{V}_j + \mathrm{A}_j/2 + \mathrm{J}_j/6 \\ \mathrm{P}_j^{(2)} &= \mathrm{P}_j + 2\mathrm{V}_j + 2\mathrm{A}_j + 4\mathrm{J}_j/3 \end{aligned}$$

Let the current target position P be given, and V, A, J, and SegT be given, then the future target can be easily estimated by using Eq. (17) until SegT-1 step.

Trajectory planning of motion controller usually generates the position data at every cycle time of servo controller, and when complex trajectory like a spline is required in some task the motion planner have much burden of calculating the position at every cycle time of servo controller.

Therefore, the strategy of PVAJT motion planning is required. The motion planner calculates the P, V, A and J at larger sampling time T_s = SegT·T_c and transfers these values to servo controller, and the servo controller calculates intermediate target positions between the sampling time using Eq. (17).

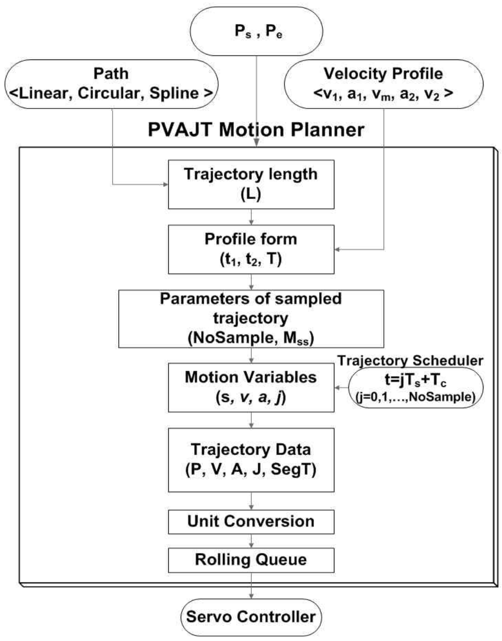

Fig. 2 Flow chart of PVAJT motion planner

The PVAJT motion planner receives input data as starting position(usually current position), ending position and path type, and generate P, V, A, J and SegT in the rolling queue which consist of DPRAM(dual port RAM). Then, the servo controller fetches the P, V, A, J, SegT from the rolling queue at every sample time, makes intermediate positions at every cycle time, and uses these positions as the reference command positions(see Fig. 2).

P_s means starting position coordinates of a path, and P_e means ending position coordinates. The type of a path can be linear, circular or spline. The path which is focused in this paper is a cubic spline interpolating several data points. Velocity profile is needed in order to calculate the time intervals. Starting velocity (v_1), acceleration (a_1), maximum velocity (v_m), deceleration (a_2) and ending velocities (v_2) are input data for building the velocity profile. Subsequently, we need a total length(L) of the given path. Time intervals are the output data of the velocity profile. They are accelerating time (t_1), decelerating time (t_2) and total traveling time (T) to pass the path. After defining the time intervals, we can go to the next step of the trajectory generation which is called motion scheduling. Number of samples NoSample and sub-sampling multiple Mss are determined here. s, v, a, j means respectively position, velocity, acceleration, and jerk variables along the path at every sampling time. Using the values obtained in the above, trajectory data of P, V, A, J and SegT are computed, and these values are written in the rolling queue which can be accessed by servo controller.

4. Case studies

Cubic spline trajectory using PVAJT motion planner are generated to verify the advantages of PVAJT motion planner by executing comparative sample study with 3 cases for different values of T_c and T_s.

Case 1: $T_c=0.256ms,\ T_s=0.256ms$

Case 2: $T_c=2.56ms,\ T_s=2.56ms$

Case 3: $T_c=0.256ms,\ T_s=2.56ms$

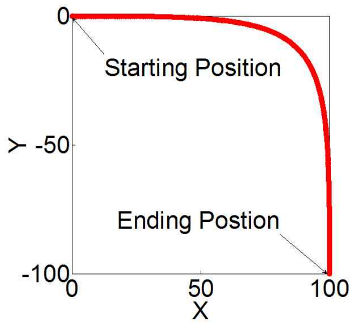

Fig. 3 The shape of cubic spline path

The data points of cubic spline are [0 0] and [100 -100], and tangent vectors are [300, 0] and [0 -300]. The starting and ending velocity are 0, the maximum velocity is 100mm/s, and the acceleration and deceleration are 200mm/s^2. The shape of path is shown as Fig. 3.

There are no intermediate positions in the case 1 and 2 because T_c and T_s are same. This motion type is like PT motion planning, that is, there is only position information. The cycle time of the case 3 is larger that the case 1. Case 3 show typical PVAJT motion planning such that Ts is 10 times of T_c.

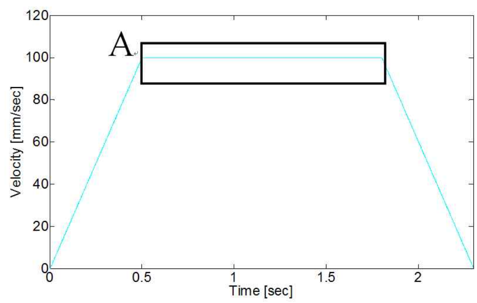

Fig. 4 Velocity profile for case 1, 2 and 3

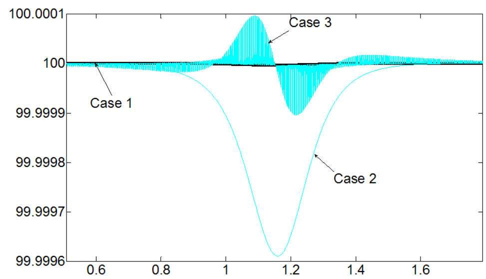

Fig. 5 Zoomed region A for case 1, 2 and 3 in velocity profile

The results of velocity profile of motion planning for 3 cases are shown in Fig. 4, where graph patterns of 3 cases seem almost same in magnitude. Looking inside the zoomed region of time from 0.5 to 1.7sec, the results of 3 cases appear to be differently pattern. The velocity of the case 1 has no velocity fluctuation in 0.1μm/s scale, whereas the case 2 shows maximum 0.4μm/s fluctuation error because it is 10 times larger sampling time than the case 1. The larger cycle time as the case 2 can also incur time delay in generated trajectory. The case 3 shows maximum 0.1μm/s fluctuation errors undulating in high frequency. This is why the graph of the case 3 is compactly drawn in Fig. 5. Though the case 3 calculates trajectory at large sampling time same as the case 2, it shows 4 times less fluctuation errors than the case 2. Actually the fluctuation of the case 3 is larger than the case 1, however, the magnitude 0.1μm/s can be negligible in motion control applications.

Consequently, it can be said that the case 3 adopting PVAJT motion planning has advantage of less burden of trajectory calculating and small fluctuation error. The PVAJT motion planning has both characteristics of the case 1(less cycle time and small fluctuation error) and the case 2(less burden of trajectory calculating).

The Fig. 6 shows P, V, A and J for X and Y coordinates, and the trajectory position data are generated based on these information at every cycle time.

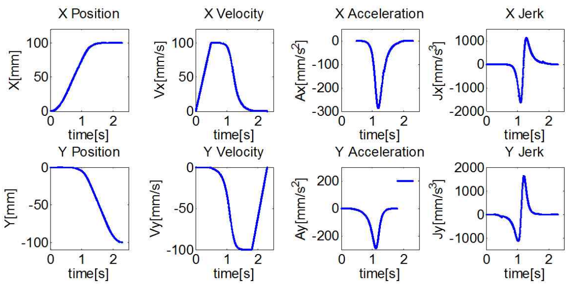

Fig. 6 P, V, A, J for X, Y coordinates for case 3

5. Conclusion

The design of the PVAJT motion planner is presented to reduce the burden of trajectory calculation time at every cycle time. This method is applied to the trajectory generation of complex trajectory such as the spline. Also, a cubic spline trajectory with arc-length parameter is formulated with estimation by summing up to the 3rd order in Taylor's expansion.

Several case studies with different parameters showed the advantages of PVAJT motion planner.

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