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DEA에서 전체 효율의 측정을 위한 새로운 접근

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New Approach to Measure Overall Efficiency in DEA

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DEA 관련 연구에서 전체 효율을 측정하는 것의 중요성은 강조되어져 왔다. 그 연구는 주로 그것이 오직 가격과 비용의 정보를 정확히 알 때 측정될 수 있다는 제한이 있었다. 그러나 실제 사례에서 이 정확한 정보를 얻는 것은 쉽지 않다. 이런 한계를 일부 극복하기 위해 이 논문에서는 Cone-ratio 제약을 이용하여 전체 효율을 측정하기 위한 새로운 모델들을 제안했다. 제안된 모델들을 기반으로 DEA에서 전체 효율 모델들을 적용하기 위해 우리가 알아야 할 필요가 있는 모든 것은 정확한 비용 또는 가격이 아니라, 비용 또는 가격의 비율이라는 사실을 증명하였다.

Keywords : Data Envelopment Analysis, Overall Efficiency, Cone-Ratio Constraint

1. Introduction

The importance of measuring overall efficiency has been emphasized in DEA literature [1-5] indicated on the importance of measuring allocative efficiency (AE) as follows. “While the conventional measure of performance is expressed by the form of technical efficiency (TE), ‘effectiveness-the level of achieving a goal’ can be measured by allocative efficiency (AE) when the goal of each DMU is to achieve the lowest input prices. And the achievement of effectiveness is much more important for many organizations than achieving efficiency.” However the research for measuring overall (allocative) efficiency (OE, AE) has been rather limited. This is mainly due to the belief that it can be measured only when the information on prices and costs are exactly known, but to get this exact information in real applications is not easy. Cooper, et al. (2000) [1] mentioned the problems for measuring overall efficiency in real applications such that many companies are unwilling to disclose their unit costs and unit prices may also be a problem when these values are subject to large fluctuations. Also when the decision maker’s interest is not limited only to cost or price, there are many factors that cannot be easily quantified, for example variables represented by quality or customer service parameters. The previous models for measuring overall efficiency uses a two-step approach. For instance, to measure the cost efficiency, it tries to find the optimal quantities of each input of DMU j with objective function which minimizes the actual total cost in step 1 and in step 2 it calculates the overall

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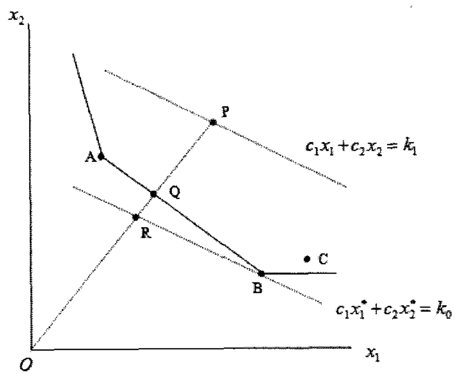
efficiency by the ratio of total optimal costs to total actual costs of DMU j. Therefore it is believed that when we don't have exact information on prices or costs, we cannot perform the calculation in step 1, so we cannot perform the calculation in step 2 either. This belief made even harder to use previous models effectively in the case that we need to do some efficiency analysis according to possible ranges of prices. In this paper, we developed the models that can measure the overall efficiency in a single step. The only difference between the proposed and CCR model is the added cost vector constraints, which results in the DEA models with cone-ratiomodel can directly measure the overall efficiency score of DMU j.

In previous DEA literature, we couldn't find any prior reference in which the relationship between two models is explored. Thus these two models have been treated separately. However, through the suggested models, we can show the relationship between two models such that the models for measuring overall efficiency can be considered as a subset of the models with general cone-ratio restrictions.

2. Concepts of Three Efficiency Measures

<Figure 1> shows the concepts of three efficiency measures originated by Farrell. Let us assume that 1) each DMU uses two inputs (x_1, x_2) in order to yield a single output (y), under the condition of constant returns to scale 2) two inputs and one output are assumed to be all positive. P is a point in the interior of the production possibility set representing the activity of a DMU which produces this same amount of output but with greater amounts of both inputs than any point on the production frontier. Then three efficiency measures can be defined as follows. First, technical efficiency (TE) of DMU P can be measured by $TE = OQ/OP$ since DMU Q exists on the frontier that use less input quantities than P to yield the same quantity of output. Second, allocative efficiency(AE) of DMU P can be measured by $AE = OR/OQ$. The budget (cost) line, which has the slope equal to the ratio of two input prices of DMU P, is $c_1x_1 + c_2x_2 = k_1$ and that of DMU B is $c_1x_1^* + c_2x_2^* = k_0(k_0 < k_1)$. Therefore the cost of $(k_1 - k_0)$ can be reduced by moving this line in parallel fashion until it intersects with the isoquant at B. The corresponding measure of $(1 - OR/OQ)$ indicates the allocative inefficiency that denotes a possible reduction in cost by using appropriate input mixes. Third, overall efficiency (OE) denotes a possible reduction in cost due to changing from P(observed input quantities) to B (cost minimizing input quantities) and it can be measured by $OE = OR/OP$. Therefore we have the relation to each of three efficiency measures

$$OE = \frac{OR}{OP} = \frac{OQ}{OP} \times \frac{OR}{OQ} = TE \times AE \qquad (1)$$



<Figure 1> Technical, Allocative and Overall Efficiency

3. Previous Models for Measuring Overall Efficiency

The concept of overall efficiency has been researched focusing on each preferable interest by several ways, i.e. 1) cost efficiency, 2) revenue efficiency, 3) profit efficiency and 4) ratio efficiency. This standard approach to determine overall efficiency and its components is due to [4] and they suggested the following model.

$$Max \sum_{r=1}^{s} p_r y_r - \sum_{i=1}^{m} c_i x_i$$

subject to

$$y_{ro} = \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+, \qquad r = 1, \cdots, s$$

$$x_{io} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_r^-, \qquad r = 1, \cdots, m$$

where,

$$y_r = \sum_{j=1}^{n} y_{rj}\lambda_j \qquad r = 1, \cdots, s$$

$$x_i = \sum_{j=1}^{n} x_{ij}\lambda_j \qquad i = 1, \cdots, m$$

$$\lambda_j, s_r^+, s_i^- \geq 0, \qquad p_r, c_i > 0$$

Here p_r, c_i, represent the unit prices and costs respectively and y_{ro}, x_{io} represent the amount of outputs and inputs of DMU 0 respectively. This model differs from the models that employ radial measures. Therefore in this paper, we begin with the following models that have been used to me asure overall efficiencies in DEA literature. To calculate each overall efficiency, we have to perform the following two-step procedures.

Step 1 : From the following LP models, we can find the optimal x_i^* and y_r^* for each DMU j. Therefore in step 1, we have to repeat each of the following models j times. Where, c_{io} is the unit cost of the input x_i of DMU0, and p_{ro} is the unit price of the output y_r of DMU0, which may vary from one DMU to another.

(Cost-E) $\quad Min \ \sum_{i=1}^{m} c_{io} x_i$

subject to

$$x_i \geq \sum_{j=1}^{n} x_{ij} \lambda_j, \qquad i = 1, \cdots, m$$

$$y_{ro} \leq \sum_{j=1}^{n} y_{rj} \lambda_j, \qquad r = 1, \cdots, s \qquad (2\text{-a})$$

$$\lambda_j \geq 0, \quad \forall j$$

(Revenue-E) $\quad Max \ \sum_{r=1}^{s} p_{ro} y_r$

subject to

$$x_{io} \geq \sum_{j=1}^{n} x_{ij} \lambda_j, \qquad i = 1, \cdots, m$$

$$y_r \leq \sum_{j=1}^{n} y_{rj} \lambda_j, \qquad r = 1, \cdots, s \qquad (3\text{-a})$$

$$\lambda_j \geq 0, \quad \forall j$$

(Profit-E) $\quad Max \ \sum_{r=1}^{s} p_{ro} y_r - \sum_{i=1}^{m} c_{io} x_i$

subject to

$$x_i = \sum_{j=1}^{n} x_{ij} \lambda_j \leq x_{io}, \qquad i = 1, \cdots, m$$

$$y_r = \sum_{j=1}^{n} y_{rj} \lambda_j \geq y_{ro}, \qquad r = 1, \cdots, s \qquad (4\text{-a})$$

$$\lambda_j \geq 0, \quad \forall j$$

(Ratio-E) $\quad Max \ \sum_{r=1}^{s} p_{ro} y_r \ / \ \sum_{i=1}^{m} c_{io} x_i$

subject to

Same as (4-a) (5-a)

Step 2 : Using the optimal value of x_i^* and y_r^*, in step 2, we can calculate each overall efficiency for DMU j. In step 2, we also have to repeat each of the following models j times to find out all DMU's overall efficiencies.

$$E_c = \frac{\sum_{i=1}^{m} c_{io} x_i^*}{\sum_{i=1}^{m} c_{io} x_{io}}, \qquad (2\text{-b})$$

$$E_c = \frac{\sum_{r=1}^{s} p_{ro} y_{ro}}{\sum_{r=1}^{s} p_{ro} y_{r^*}}, \qquad (3\text{-b})$$

$$E_p = \frac{\sum_{r=1}^{s} p_{ro} y_{ro} - \sum_{i=1}^{m} c_{io} x_{io}}{\sum_{r=1}^{s} p_{ro} y_{r^*} - \sum_{i=1}^{m} c_{io} x_{i^*}}, \qquad (4\text{-b})$$

$$E_{Ratio} = \frac{\sum_{r=1}^{s} p_{ro} y_{ro} \ / \ \sum_{i=1}^{m} c_{io} x_{io}}{\sum_{r=1}^{s} p_{ro} y_{r^*} \ / \ \sum_{i=1}^{m} c_{io} x_{i^*}}, \qquad (5\text{-b})$$

Example 1 : Here we suggest an example 1, which is excerpted from [1] for the purpose of explaining the above models and afterward comparing the result with those of our models. <Table 1> shows the data for 4 DMUs with two inputs and two outputs, along with the unit cost for each input and unit price for each output. And <Table 2> shows the results of cost, revenue, profit and ratio efficiencies of the 4 DMUs.

<Table 1> Example 1 data

DMU	Input		Output		Input Cost		Output Price	
	x_1	x_2	y_1	y_2	c_1	c_2	p_1	p_2
1	2	3	5	8	2	2	5	5
2	1	5	2	6	2	4	4	5
3	3	8	4	8	3	3	6	4
4	2	7	1	2	4	2	7	4

According to the above models, for example, cost efficiency for DMU 2 is calculated by the following procedure.

Step 1 : Solve the model (2-a)

(Cost-E) $Min \quad 2x_1 + 4x_2$

subject to

$$x_1 - 2\lambda_1 - 1\lambda_2 - 3\lambda_3 - 2\lambda_4 \geq 0$$
$$x_2 - 3\lambda_1 - 5\lambda_2 - 8\lambda_3 - 7\lambda_4 \geq 0$$
$$5\lambda_1 - 2\lambda_2 - 4\lambda_3 - 1\lambda_4 \geq 2$$
$$8\lambda_1 - 6\lambda_2 - 8\lambda_3 - 2\lambda_4 \geq 6$$
$$\lambda_j \geq 0, \qquad \forall j$$

Then we get the optimal solution such that $x_1 = 1.5$, $x_2 = 2.25$, $\lambda_1 = 0.75$ with objective function value 12. All the other λ variables are 0.

Step 2 : Solve the model (2-b)

$$E_c = \frac{\sum_{i=1}^{m} c_{io} x_i^*}{\sum_{i=1}^{m} c_{io} x_{io}} = \frac{(2\times 1.5)+(4\times 2.25)}{(2\times 1)+(4\times 5)}$$
$$= \frac{12}{22} = 0.545$$

<Table 2> Results of example 1

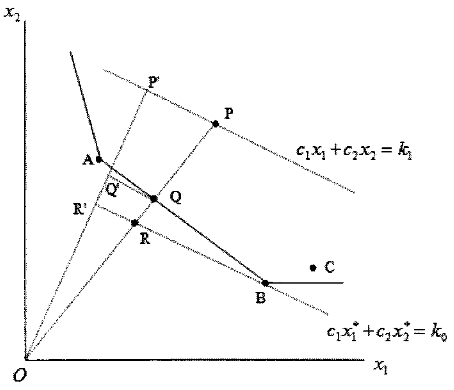
DMU	CCR	Cost	Revenue	Profit	Ratio
1	1	1	1	1	1
2	1	0.545	1	1	0.461
3	0.571	0.455	0.571	0.326	0.411
4	0.214	0.159	0.208	0	0.142

The revenue, profit and ratio efficiencies are also can be obtained by solving sequentially the model (3-a, 3-b), (4-a, 4-b) and (5-a, 5-b) respectively. When the actual profit appears to be negative, the profit efficiency score is assigned the value 0.

4. New Approach to Measure Overall Efficiency

In this section, we suggest the models, which can measure the overall efficiency using cone-ratio constraints. Using <Figure 1>, we showed the concepts of three efficiency measures of DMU P are $TE = OQ/OP$, $AE = OR/OQ$, $OE = OR/OP$ And the relationship of three efficiency measures is

$$OE = \frac{OR}{OP} = \frac{OQ}{OP} \times \frac{OR}{OQ} = TE \times AE.$$



<Figure 2> Illustration of Suggested Model

Also $c_1x_1 + c_2x_2 = k_1$ and $c_1x_1^* + c_2x_2^* = k_0$ are isocost lines for DMU B and DMU P respectively which are parallel to each other. In <Figure 2>, OP' represents an orthogonal vector to the isocost lines, which passes through the origin. It is clear from above figure that we can find the unique vector, which is perpendicular to the isocost lines and passes through the origin. And Q', R' are the projection points which are projected perpendicular to the vector OP' from Q and R respectively. Since Q and Q', R and R', P and P' lie on each isocost line, it is clear that these points have the same costs respectively. Therefore, the following relation (6) should hold that is also obvious by the property of right-angled triangle in $\triangle OPP.'$

$$TE = \frac{OQ}{OP} = \frac{OQ'}{OP'},$$
$$AE = \frac{OR}{OQ} = \frac{OR'}{OQ'},$$
$$OE = \frac{OR}{OP} = \frac{OR'}{OP'} \qquad (6)$$

And the relationship of three efficiency measures also holds by (7)

$$OE = \frac{OR'}{OP'} = \frac{OQ'}{OP'} \times \frac{OR'}{OQ'} = TE \times AE \qquad (7)$$

Now we define the vector OP' as the cost vector with the following property (P1).

(P1) Cost vector $\vec{c}$ is a vector, which is perpendicular to the iso-cost lines (planes) of DMU j and passes through the origin. Similarly, price vector $\vec{p}$ is a vector, which is perpendicular to the iso-revenue lines (planes) of DMU j and passes through the origin.

Generally if there is a DMU j, which uses m inputs with the unit costs $(c_1, \cdots, c_m)$, then the iso-cost plane of DMU j can be expressed as $c_1x_1 + \cdots + c_mx_m = k$.

The equation of $c_1x_1 + \cdots + c_mx_m = k$ is the general form of the plane equation, which intersects each of m axes with the following points, i.e. $(k/c_1, 0, 0, \cdots, 0), (0, k/c_2, 0, \cdots, 0), \cdots, (0, 0, 0, \cdots, k/c_m)$ and $(c_1, \cdots, c_m)$ represents orthogonal (directional) vector to this plane which is passing through the origin. Therefore we have the following property on cost (price) vector (P2).

(P2) The cost vector of DMU j, which uses m inputs with the unit costs $(c_1, \cdots, c_m)$ and passes through the origin, is the $(c_1, \cdots, c_m)$.

Similarly, the price vector of DMU j, which produces r outputs with the unit prices $(p_1, \cdots, p_r)$ and passes through the origin, is the $(p_1, \cdots, p_r)$.

For simplicity, we assumed that unit costs of two inputs, c_1 and c_2 are the same for all DMUs in the case of <Figure 2>. Therefore we have all different isocost lines but parallel to each other to all DMUs and a unique cost vector. However in case that each DMU's unit cost for each input is different as shown in the example 1, each DMU has its own cost vector.

After all, when the cone-ratio weight restrictions (here, cost or price ratios) are applied to the general CCR model, all DMUs are projected to the cost vector along with the iso-cost lines, and the overall efficiency is measured by the following ratio (P3).

(P3) When the cost (price) vectors are applied to the CCR model, the overall efficiency of DMU j can be measured by the following ratio.

Overall efficiency score of DMU j =

$$\frac{\text{(Norms of orthogonalprojection of DMU } j \text{ to the cost(price) vector)}}{\text{(Norms of orthogonalprojection of DMU } j^* \text{ to the cost(price) vector)}}$$

where, DMU j^* has the largest norm (revenue maximization case) or smallest norm (cost minimization case) when projected to the price vector.

The weight vector for measuring overall efficiency can be represented such as

$$\text{cost vector : } \vec{c} = (v_1, \cdots, v_m) = (c_1, \cdots, c_m),$$
$$\text{price vector : } \vec{p} = (\mu_1, \cdots, \mu_r) = (p_1, \cdots, p_r).$$

Therefore when we consider the DEA models for measuring overall efficiency as one of general cone-ratio restrictions, we can replace the each of input, output multiplier to each corresponding cost and price. Therefore, the following two properties (8), (9) hold.

$$\frac{v_1}{v_2} = \frac{c_1}{c_2}, \cdots, \frac{v_1}{v_m} = \frac{c_1}{c_m}, \frac{v_2}{v_3} = \frac{c_2}{c_3}, \cdots\cdots, \frac{v_{m-1}}{v_m} = \frac{c_{m-1}}{c_m} \quad (8)$$

$$\frac{\mu_1}{\mu_2} = \frac{p_1}{p_2}, \cdots, \frac{\mu_1}{\mu_m} = \frac{p_1}{p_m}, \frac{\mu_2}{\mu_3} = \frac{p_2}{p_3}, \cdots\cdots, \frac{\mu_{m-1}}{\mu_m} = \frac{p_{m-1}}{p_m} \quad (9)$$

That is, the only difference of proposed model with CCR model is the added constraints according to the each case as follows.

1) Cost Efficiency (10) : add constraint (8)

(Cost-E) $\underset{u_r}{Max} \; h_{jo} = \sum_{r=1}^{s} \mu_r y_{rjo}$

subject to $\sum_{i=1}^{m} v_i x_{ijo} = 1$

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1, \cdots, n \qquad (10)$$

$$\frac{v_1}{v_2} = \frac{c_1}{c_2}, \cdots, \frac{v_1}{v_m} = \frac{c_1}{c_m}, \frac{v_2}{v_3} = \frac{c_2}{c_3}, \cdots\cdots, \frac{v_{m-1}}{v_m} = \frac{c_{m-1}}{c_m}$$

$\mu_r, v_i \ge 0, \quad \forall r$ and i

2) Revenue Efficiency (11) : add constraint (9)

(Revenue-E) $\underset{u_r}{Max} \; h_{jo} = \sum_{r=1}^{s} \mu_r y_{rjo}$

subject to $\sum_{i=1}^{m} v_i x_{ijo} = 1$

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1, \cdots, n \qquad (11)$$

$$\frac{\mu_1}{\mu_2} = \frac{p_1}{p_2}, \cdots, \frac{\mu_1}{\mu_m} = \frac{p_1}{p_m}, \frac{\mu_2}{\mu_3} = \frac{p_2}{p_3}, \cdots\cdots, \frac{\mu_{m-1}}{\mu_m} = \frac{p_{m-1}}{p_m}$$

$\mu_r, v_i \ge 0, \quad \forall r$ and i

(Ratio-E) $\underset{u_r}{Max} \; h_{jo} = \sum_{r=1}^{s} \mu_r y_{rjo}$

subject to $\sum_{i=1}^{m} v_i x_{ijo} = 1$

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1, \cdots, n \qquad (12)$$

$$\frac{v_1}{v_2} = \frac{c_1}{c_2}, \cdots, \frac{v_1}{v_m} = \frac{c_1}{c_m}, \frac{v_2}{v_3} = \frac{c_2}{c_3}, \cdots\cdots, \frac{v_{m-1}}{v_m} = \frac{c_{m-1}}{c_m}$$

$$\frac{\mu_1}{\mu_2} = \frac{p_1}{p_2}, \cdots, \frac{\mu_1}{\mu_m} = \frac{p_1}{p_m}, \frac{\mu_2}{\mu_3} = \frac{p_2}{p_3}, \cdots\cdots, \frac{\mu_{m-1}}{\mu_m} = \frac{p_{m-1}}{p_m}$$

$\mu_r, v_i \ge 0, \quad \forall r$ and i

From the property of (8), the following formulation (13) is equivalent to (10).

$$\text{(Cost-E)} \qquad \underset{p_r}{Max} \quad h_{jo} = \sum_{r=1}^{s} p_r y_{rjo}$$

$$\text{subject to} \qquad \sum_{i=1}^{m} c_i x_{ijo} = 1$$

$$\sum_{r=1}^{s} p_r y_{rj} - \sum_{i=1}^{m} c_i x_{ij} \leq 0, \qquad j = 1, \cdots, n \qquad (13)$$

$$\frac{c_1}{c_2} = \frac{v_1}{v_2}, \cdots, \frac{c_1}{c_m} = \frac{v_1}{v_m}, \frac{c_2}{c_3} = \frac{v_2}{v_3}, \cdots\cdots, \frac{c_{m-1}}{c_m} = \frac{v_{m-1}}{v_m}$$

$$p_r, c_i \geq 0, \qquad \forall r \text{ and } i$$

Similarly we can make formulations which is equivalent to (11) and (12) respectively with replacing v_i *to* c_i, $\forall i$ and μ_r *to* p_r, $\forall r$.

Example 2 : When the unit costs (prices) are the same for all DMUs

<Table 3> Example 2 data and results

DMU	x_1	x_2	c_1	c_2	DMU	CCR	Cost-E
1	4	2	4	2	1	1	0.8
2	2	4	4	2	2	1	1
3	4	6	4	2	3	0.6	0.571

It is clear in <Figure 3> that DMU 1 and 2 are CCR-efficient but only DMU 2 is overall efficient (here, cost efficient). Cost efficiency of DMU 3 is explained as a ratio $OR/O3$, but suggested model measures the ratio $OR'/O3'$. Both measures should be the same from the property of right-angled triangle. Model to find the cost efficiency of DMU 3 is

$$\text{(Cost-E)} \qquad Max \quad \mu$$

$$\text{subject to} \qquad 4v_1 + 6v_2 = 1$$

$$\mu - 4v_1 - 2v_2 \leq 0$$

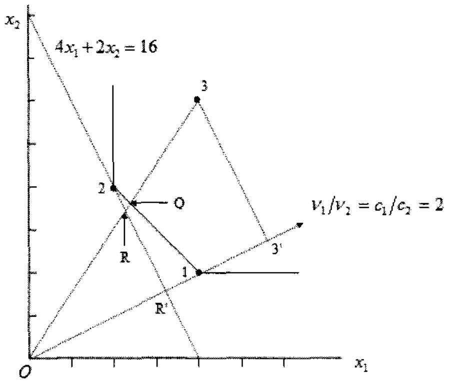
$$\mu - 2v_1 - 4v_2 \leq 0$$

$$\mu - 4v_1 - 6v_2 \leq 0$$

$$v_1 - 2v_2 = 0$$

$$\mu \geq 0, \ v_1 \geq 0, \ v_2 \geq 0$$

And it gives the solution $h^*_{j_o} = \theta^* = 0.57143$ Actually under the constant returns to scale assumption, it doesn't matter to use either one of input-oriented or output-oriented model. Therefore when we apply the model with output orientation, we can also get the same solution. When we multiply the efficiency score to the amount of each input of DMU 3, i.e. $\theta^*(x_1, x_2) = 0.57143 \times (4,6) = (2.2857, 3.42858)$, then this point is the coordinate of point R.



<Figure 3> Illustration of Example 2

Therefore we can see that the above model measures the ratio of $OR/O3 = OR'/O3'$.

On the other hand, when we use the model (2-a) and (2-b) to measure cost efficiency of DMU 3, we also can obtain the same score. That is,

$$\text{(Cost-E)} \qquad Min \quad 4x_1 + 2x_2$$

$$\text{subject to} \qquad x_1 - 4\lambda_1 - 2\lambda_2 - 4\lambda_3 \geq 0$$

$$x_2 - 2\lambda_1 - 4\lambda_2 - 6\lambda_3 \geq 0$$

$$\lambda_1 + \lambda_2 + \lambda_3 \geq 1$$

$$\lambda_1 \geq 0, \ \lambda_2 \geq 0, \ \lambda_3 \geq 0$$

And it gives the solution with $x_1^* = 2$, $x_2^* = 4$, $\lambda_2 = 1$ and $\lambda_1 = \lambda_3 = 0$, which is the same with the coordinate of DMU 2. Therefore

$$E_c = \frac{\sum_{i=1}^{m} c_{io} x_i^*}{\sum_{i=1}^{m} c_{io} x_{io}} = \frac{(4 \times 2) + (2 \times 4)}{(4 \times 4) + (2 \times 6)} = \frac{16}{28} = 0.57143$$

After all in this example 2, we can state the difference in the way of measuring overall efficiency between two models as follows. That is, to measure the cost efficiency of DMU 3, proposed model measures the ratio of $OR/O3 = OR'/O3'$ while previous model measures the ratio of total costs of DMU 2/total costs of DMU 3. If we assume another DMU 4 which has the coordinate of R in <Figure 3>, then the total costs of DMU2 and DMU 4 should be the same since both DMUs lie on the iso-cost line. Therefore it is clear that the results from both models are the same.

Then the allocative efficiency (AE) of DMU 3 can be obtained from equation (7)

$$AE = \frac{OE}{TE} = \frac{E_c}{TE} = \frac{0.571}{0.6} = 0.952$$

Example 3 : When the unit costs (prices) are not the same for all DMUs

When the unit costs (prices) are not the same for all DMUs like example 1, we can apply models (10), (11) and (12) replacing previous models (2), (3) and (5) respectively.

1) Cost efficiency of DMU 2 : objective function value = 0.54545

(Cost-E) $\quad Max \quad 2\mu_1 + 6\mu_2$

subject to

$$1v_1 + 5v_2 = 1$$
$$5\mu_1 + 8\mu_2 - 2v_1 - 3v_2 \le 0$$
$$2\mu_1 + 6\mu_2 - 1v_1 - 5v_2 \le 0$$
$$4\mu_1 + 8\mu_2 - 3v_1 - 8v_2 \le 0$$
$$1\mu_1 + 2\mu_2 - 2v_1 - 7v_2 \le 0$$

$2v_1 - v_2 = 0$ ▷ cost vector constraint

$\mu_1 \ge 0,\ \mu_2 \ge 0,\ v_1 \ge 0,\ v_2 \ge 0$

2) Revenue efficiency of DMU 3 : objective function value = 0.57143

(Revenue-E) $\quad Max \quad 4\mu_1 + 8\mu_2$

subject to

$$3v_1 + 8v_2 = 1$$
$$5\mu_1 + 8\mu_2 - 2v_1 - 3v_2 \le 0$$
$$2\mu_1 + 6\mu_2 - 1v_1 - 5v_2 \le 0$$
$$4\mu_1 + 8\mu_2 - 3v_1 - 8v_2 \le 0$$
$$1\mu_1 + 2\mu_2 - 2v_1 - 7v_2 \le 0$$

$4\mu_1 - 6\mu_2 = 0$ ▷ price vector constraint

$\mu_1 \ge 0,\ \mu_2 \ge 0,\ v_1 \ge 0,\ v_2 \ge 0$

3) Ratio efficiency of DMU 3 : objective function value = 0.41056

(Ratio-E) $\quad Max \quad 4\mu_1 + 8\mu_2$

subject to

$$3v_1 + 8v_2 = 1$$
$$5\mu_1 + 8\mu_2 - 2v_1 - 3v_2 \le 0$$
$$2\mu_1 + 6\mu_2 - 1v_1 - 5v_2 \le 0$$
$$4\mu_1 + 8\mu_2 - 3v_1 - 8v_2 \le 0$$
$$1\mu_1 + 2\mu_2 - 2v_1 - 7v_2 \le 0$$

$3V_1 - 3V_2 = 0$ ▷ cost vector constraint

$4\mu_1 - 6\mu_2 = 0$ ▷ price vector constraint

$\mu_1 \ge 0,\ \mu_2 \ge 0,\ v_1 \ge 0,\ v_2 \ge 0$

Even though we showed just one case in each of efficiency, all solutions are exactly equal to the result in <Table 2>. When the unit costs (prices) are not the same for all DMUs, we should use the DMU's own cost vector which is being analyzed.

5. Some Comments on the Models for Measuring Overall Efficiency

In this paper, we suggested three models (10)~(12) to find each of overall efficiency and the results were compared with those of previous models (2), (3), (5). In fact, the suggested models (10)~(12) turn out to be a CCR model with added cost cone-ratio weight restrictions and they make the same results with those from (2), (3) and (5). Previously, we mentioned that the research for measuring overall efficiency has been rather limited due to the belief that it can be measured only when the information on prices and costs are exactly known. However, this belief such that "overall efficiency can be measured only when the information on prices and costs are exactly known" is not true in some sense. All we need to know to measure the overall efficiency is not the exact prices and costs but the ratios of price or cost. This can be explained by the facts that the suggested models (10)~(12) are made simply adding cost (price) cone-ratio restrictions to CCR model and therefore even when the actual costs are changed while keeping the same cost ratio, it will make the same cost efficiency score since both have the same cost vector constraints. The other things we need to indicate on both models are 1) each overall efficiency score of DMU j depends on input, output quantities of sample DMUs and only the ratios of price or cost of DMU j. That is, other DMUs' prices and costs doesn't affect to the overall efficiency of DMU j. And this implies that the overall efficiency score of DMU j in both models is calculated with the assumption that all the other DMUs are assumed to use those of DMU j (i.e. all the other DMUs are projected to the cost (price) vector of DMU j). 2) DMU j can show its maximum overall efficiency, (which is equal to its TE, i.e. AE = 1) if the ratios of costs and prices are the same with those of input, output multipliers in CCR result respectively. However we have to consider that CCR result for DMU j may have multiple solutions and therefore it may not be a unique solution. Alternatively, we can verify the above description 1) by using the previous model (2-a) in example 1 data. To find cost efficiency of DMU 2, the previous model (2-a) used only DMU 2's input costs in the objective function and all the other DMU's input costs are not used at all. That is, DMU 2's cost efficiency is calculated with the assumption that all the other DMUs are assumed to use DMU 2's cost ratio. In fact, many calculations with keeping the same cost ratio but changed costs in (2-a), it made the same optimal quantities and therefore don't affect the result. For example, even if we change the objective function from $(2x_1+4x_2)$ to any of $(4x_1+8x_2)$ or $(15x_1+30x_2)$, it makes the same results as before such that $x_1 = 1.5$, $x_2 = 2.25$, $\lambda_1 = 0.75$ except objective function values. All the other objective functions while they have the input cost relation of $c_2 = 2c_1$ will make the same result. And different objective function values in step 1 don't affect to the result in step 2. When we let the cost efficiency score of DMU j as E_c^1, E_c^2 where E_c^2 represents the cost efficiency using the same cost ratio but different costs compared with E_c^1. And let the corresponding optimal input quantities for E_c^1 as x_1^{1*}, x_2^{1*} and for E_c^2 as x_1^{2*}, x_2^{2*}. Then,

$$E_c^1 = \frac{\sum_{i=1}^{m} c_{io} x_i^*}{\sum_{i=1}^{m} c_{io} x_{io}} = \frac{(c_1 \times x_1^{1*}) + (c_2 \times x_2^{1*})}{(c_1 \times x_1) + (c_2 \times x_2)},$$

$$= \frac{\left(\frac{c_1}{c_2} \times x_1^{1*}\right) + x_2^{1*}}{\left(\frac{c_1}{c_2} \times x_1\right) + x_2} = \frac{kx_1^{1*} + x_2^{1*}}{kx_1 + x_2}$$

$$E_c^2 = \frac{\sum_{i=1}^{m} c_{io} x_i^*}{\sum_{i=1}^{m} c_{io} x_{io}} = \frac{kx_1^{2*} + x_2^{2*}}{kx_1 + x_2}$$

In order for two equations E_c^1, E_c^2 to make the same result,

$$E_c^1 - E_c^2 = \frac{k(x_1^{1*} - x_1^{2*}) + (x_2^{1*} - x_2^{2*})}{kx_1 + x_2} = 0,$$

and it means $x_1^{1*} = x_1^{2*}$, $x_2^{1*} = x_2^{2*}$

Therefore as long as we have the same cost ratio $(c_1/c_2 = k)$, (2-a) will produce the same values of x_1, x_2 and after all it doesn't change the overall efficiency.

<Table 4> shows the CCR result on the example 1 data in which final two columns represent the ratios of input, output multipliers.

DMU 2's CCR efficiency score is 1, but its cost efficiency is 0.545 (in <Table 2>) using $c_1/c_2 = 2/4 = 0.5$. When we change this cost ratio from 0.5 to 5.5, its cost efficiency becomes to 1 that is equal to its CCR efficiency score. This can be confirmed from two models by changing objective function such as $Min\ 5.5x_1 + x_2$ in (2-a) or by adding cost vector constraint such as $\nu_1 - 5.5\nu_2 = 0$ in (10). The managerial information we can get from cost efficiency in DMU B is it can obtain its maximum cost efficiency of 1.0 while keeping the cost ratio of $c_1/c_2 = 5.5$. However we have to be careful that it may not be a unique solution.

6. Conclusion

In this paper, we described the characteristics of previous DEA model for measuring overall efficiency and suggested new model.

The merits or contributions of suggested models we believe are 1) It showed that the overall efficiency (cost/revenue/ratio efficiency) model can be referred to be one of the general cone-ratio restriction type problem, which considers only prices of outputs or costs of inputs as applied weights. 2) From suggested models, we showed the fact that all we need to know to measure the overall efficiency is not the exact prices or costs but the ratios of prices or

<Table 4> CCR Multipliers of Example 2 data

DMU	CCR	ν_1	ν_2	μ_1	μ_2	ν_1/ν_2	μ_1/μ_2
1	1	0	0.333	0	0.125	0	0
2	1	0.524	0.095	0	0.167	5.50	0
3	0.571	0.224	0.041	0	0.071	5.50	0
4	0.214	0.5	0	0.143	0.036	-	4.00

costs. This contracts to the traditional belief on measuring overall efficiency. Therefore in case that we have insufficient information on prices or costs, we can expand the availability of these models applying possible ranges of price or cost ratios.

Although models (2)~(5) and (10)~(12) can give some useful information, there still needs a further research to overcome the followings. First, these models cannot tell the specific method to increase overall efficiency except telling general direction on quantities or costs (prices). Second, the overall efficiency of DMU j is calculated based on the assumption that all the other DMUs are assumed to use DMU j's cost or price vectors, and it may not appropriately reflect a variety of management strategies.

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