# Outage Analysis of Cooperative Transmission in Two-Dimensional Random Networks over Rayleigh Fading Channels 

Tran Trung Duy • Hyung-Yun Kong


#### Abstract

In this paper, we evaluate the outage performance of cooperative transmission in two-dimensional random networks. Firstly, we derive the joint distributions of the source-relay and the relay-destination links. Secondly, the outage probability for the decode-and-forward relaying system is derived when selection combining (SC) is employed at the destination. Finally, we calculate the average outage probability of the system and then attempt to express it by a simple approximate expression. The simulation results are presented to verify the accuracy of the derivations. Similar to deterministic networks, the cooperative transmission in random networks outperforms direct transmission at a high signal-to-noise ratio (SNR).


Key words: Cooperative Communication, Uniform Distribution, Selection Combining, Rayleigh Fading Channels, Outage Probability.

## I. Introduction

Cooperative wireless communication has gained much attention as an efficient method to mitigate the effects of fading channels. Currently, various cooperative transmission protocols have been researched in several studies [1]~[5] from the viewpoint of implementation issues and performance evaluations using outage analysis. So far, most work related to cooperative communication has been developed on deterministic networks, in which the distances are assumed to be constant. However, in practice, nodes in wireless networks may be randomly and independently distributed over the entire area. Thus, the distance between any selected node pair is a random variable, and the performance of the system should be evaluated following the distribution of the link distance. This motivates us to study the performance evaluation of the cooperative transmission in random networks.

In this paper, we consider the data transmission between a deterministic source and a deterministic destination, with the help of relay nodes that are uniformly distributed in a two-dimensional random network. The cumulative density function (CDF) and probability density function (PDF) of the distance between randomly selected nodes are derived for various deployment models [6], [7]. However, when a common terminal such as the common relay is involved, the distance between the source and the relay and the distance between the relay and the destination are not independent. In [8], [9], the
authors proposed the alternative method of finding the joint CDF of the distances between nodes and the reference terminal. Similarly, we first derive the CDF and PDF of the source-relay distance and the relay-destination distance. Next, we derive the joint CDF and PDF of the source-relay and relay-destination distances. Applying this joint PDF to the outage probability derived over Rayleigh fading channels, we calculate the average outage probability of cooperative communication in twodimensional random networks. To reduce the complexity of the numerical calculations, we propose a simple method to approximate this outage probability. Finally, simulation results are presented to verify the accuracy of the analyses.

The rest of this paper is organized as follows. The system model is described in section II and the theoretical analyses are presented in section III. The simulation results are presented in section IV and section $V$ concludes this paper.

## II . System Model

Fig. 1 illustrates a two-dimensional network in which the positions of the source $S$ and destination $D$ are fixed at points $(0,1 / 2)$ and $(1,1 / 2)$, respectively. Each relay node $R_{j}(j \in\{1,2, \ldots, N\})$ has a coordinate $\left(x_{j}, y_{j}\right)$, where $x_{j}$ and $y_{j}$ are uniformly and independently random variables in interval $(0,1)$. It is noted that in Fig. 1, we choose the positions of the sour-

[^0]

Fig. 1. Two-dimensional network including a deterministic source, a deterministic destination, and random relays.
ce and the destination to simplify the calculations. For their other positions, the mathematical derivations can be determined in the same manner.

In this paper, it is assumed that all nodes have a single antenna and hence, for medium access, the time division multiple access (TDMA) technique is used. In the first time slot, the source S broadcasts its signal to the destination and relay nodes. At the end of this time slot, relays attempt to decode the signal received from the source. If the channel between a relay and the source is good enough, it decodes and forwards the signal to the destination. Let us denote $C_{D}$ as a decoding set whose members are relays that successfully decode the signal. The remaining relays are assumed to belong to set $C_{W}$. In the following $N$ time slots, members of the set $C_{D}$ forward the signal to the destination in a predetermined order [2].

## III. Outage Analysis

## 3-1 The Distribution of Link Distance

For ease of presentation, let us denote $d_{j}$ and $l_{j}(j \in$ $\{1,2, \ldots, N\})$ as the distance between the source $S$ and relay $R_{j}$ and the distance between the relay $R_{j}$ and destination $D$, respectively. The link distance between the source $S$ and relay $R_{j}$ is determined as:

$$
\begin{equation*}
d_{j}=\sqrt{x_{j}^{2}+\left(1-y_{j}\right)^{2}} . \tag{1}
\end{equation*}
$$

Now, we can define the CDF of $d_{j}$ as:

$$
\begin{equation*}
F_{d_{j}}(z)=\operatorname{Pr}\left[d_{j}<z\right] . \tag{2}
\end{equation*}
$$

Since $x_{j}$ and $y_{j}$ are uniformly and independently ran-


Fig. 2. The CDF $F_{d_{j}}(z)$ when $0<z \leq 1 / 2$.
dom variables, the CDF $F_{d_{i}}(z)$ is the overlapping area of the circle with radius $z$ centered at $(0,1 / 2)$ and the $1 \times 1$ square.

Fig. 2 presents the case where $0<z \leq 1 / 2$; the CDF $F_{d_{i}}(z)$ in this case is equal to the area of a half of circle with radius $z$. Hence, it is calculated as:

$$
\begin{equation*}
F_{d_{i}}(z)=\pi z^{2} / 2 \tag{3}
\end{equation*}
$$

If $1 / 2<z \leq 1$, as illustrated in Fig. 3, the overlapping area represents the compositions of a rectangle and a sector. Hence, we obtain the $\operatorname{CDF} F_{d_{i}}(z)$ as:

$$
\begin{equation*}
F_{d_{j}}(z)=\frac{\pi z^{2}}{2}+\frac{\sqrt{4 z^{2}-1}}{4}-z^{2} \tan ^{-1}\left(\sqrt{4 z^{2}-1}\right) . \tag{4}
\end{equation*}
$$

Similarly, if $1<z \leq \sqrt{5} / 2$, we have:

$$
\begin{align*}
F_{d_{j}}(z) & =\frac{\sqrt{4 z^{2}-1}}{4}+\sqrt{z^{2}-1} \\
& +z^{2}\left(\tan ^{-1}\left(\frac{1}{\sqrt{z^{2}-1}}\right)-\tan ^{-1}\left(\sqrt{4 z^{2}-1}\right)\right) \tag{5}
\end{align*}
$$

In the last case, when $z>\sqrt{5} / 2$, it is obvious that $F_{d_{j}}(z)=1$.

Now, differentiating $F_{d_{j}}(z)$ with respect to $z$, we obtain the PDF $f_{d_{j}}(z)$ as follows:


Fig. 3. The CDF $F_{d_{j}}(z)$ when $1 / 2<z \leq 1$.

$$
\begin{align*}
& f_{d_{j}}(z)= \\
& \begin{cases}0 ; & \mathrm{z} \leq 0 \text { or } z>\frac{\sqrt{5}}{2} \\
\pi z ; & 0<z \leq \frac{1}{2} \\
\pi z-2 z \tan ^{-1}\left(\sqrt{4 z^{2}-1}\right) ; & \frac{1}{2}<z \leq 1 \\
2 z\left(\tan ^{-1}\left(\frac{1}{\sqrt{z^{2}-1}}\right)-\tan ^{-1}\left(\sqrt{4 z^{2}-1}\right)\right) ; 1<\mathrm{z} \leq \frac{\sqrt{5}}{2} .\end{cases} \tag{6}
\end{align*}
$$

In addition, due to the symmetrical characteristic, the CDF and PDF of the link distance $l_{j}$ are same as those of $d_{j}$.

Next, we derive the joint distribution of the link distances $d_{j}$ and $l_{j}$. The joint CDF is defined as:

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=\operatorname{Pr}\left[d_{j}<z, l_{j}<t\right] . \tag{7}
\end{equation*}
$$

Given different values of $z$ and $t$, we have a total of 12 cases, as follows:

- Case 1: $z>\sqrt{5} / 2$

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=F_{l_{j}}(t) \tag{8.1}
\end{equation*}
$$

- Case 2: $t>\sqrt{5} / 2$

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=F_{d_{j}}(z) \tag{8.2}
\end{equation*}
$$

- Case 3: $z \leq 0$ or $t \leq 0$ or $z+t \leq 1$

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=0 \tag{8.3}
\end{equation*}
$$

- Case 4: $(0<z \leq 1 / 2$ and $1-z<t \leq 1)$ or

$$
\begin{align*}
& \left(1 / 2<z \leq 1 \text { and } 1-z \leq t \leq \sqrt{1+z^{2}-\sqrt{4 z^{2}-1}}\right) \\
& F_{d_{j}, l_{j}}(z, t)=K_{1}(z, t)+K_{1}(t, z) \tag{8.4}
\end{align*}
$$

- Case 5: $0<z \leq 1 / 2$ and $1<t \leq \sqrt{1+z^{2}}$

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=K_{1}(z, t)+K_{2}(t, z) \tag{8.5}
\end{equation*}
$$

- Case 6: $0<z \leq 1 / 2$ and $\sqrt{1+z^{2}}<t \leq \sqrt{5} / 2$

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=\frac{\pi z^{2}}{2} \tag{8.6}
\end{equation*}
$$

- Case 7: $\binom{1 / 2<z \leq(\sqrt{3}-1) / \sqrt{2}$ and }{$\sqrt{1+z^{2}-\sqrt{4 z^{2}-1}}<t \leq \sqrt{5} / 2}$
or $((\sqrt{3}-1) / \sqrt{2}<z \leq 1$ and $1 \leq t \leq \sqrt{5} / 2)$

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=G_{1}(t)+G_{2}(z)+K_{3}(z, t) \tag{8.7}
\end{equation*}
$$

- Case $8: \frac{\sqrt{3}-1}{\sqrt{2}}<z \leq 1$ and $\sqrt{1+z^{2}-\sqrt{4 z^{2}-1}}<t \leq 1$

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=G_{2}(z)+G_{2}(t)+K_{3}(z, t) \tag{8.8}
\end{equation*}
$$

- Case 9: $1<z \leq \sqrt{5} / 2$ and $t \leq \sqrt{z^{2}-1}$

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=\frac{\pi t^{2}}{2} \tag{8.9}
\end{equation*}
$$

- Case 10: $1<z \leq \sqrt{5} / 2$ and $\sqrt{z^{2}-1}<t \leq 1 / 2$

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=K_{1}(t, z)+K_{2}(z, t) \tag{8.10}
\end{equation*}
$$

- Case 11: $1<z \leq \sqrt{5} / 2$ and $1 / 2 \leq t<1$

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=G_{1}(z)+G_{2}(t)+K_{3}(z, t) \tag{8.11}
\end{equation*}
$$

- Case 12: $1<z \leq \sqrt{5} / 2$ and $1<t \leq \sqrt{5} / 2$

$$
\begin{equation*}
F_{d_{j}, l_{j}}(z, t)=G_{1}(z)+G_{1}(t)+K_{3}(z, t), \tag{8.12}
\end{equation*}
$$

where $\Delta(a, b)=\sqrt{4 a^{2}-\left(1+a^{2}-b^{2}\right)^{2}}$,

$$
\begin{aligned}
& K_{1}(a, b)=\frac{\pi a^{2}}{2}-a^{2} \tan ^{-1}\left(\frac{1+a^{2}-b^{2}}{\Delta(a, b)}\right)-\frac{\left(1+a^{2}-b^{2}\right) \Delta(a, b)}{4}, \\
& K_{2}(a, b)=K_{1}(a, b)+\sqrt{a^{2}-1}-a^{2} \tan ^{-1}\left(\sqrt{a^{2}-1}\right), \\
& K_{3}(a, b)=\frac{\sqrt{4 a^{2}-1}}{2}+\frac{\sqrt{4 b^{2}-1}}{2}-1, \\
& G_{1}(a)=\sqrt{a^{2}-1}+a^{2}\left(\tan ^{-1}\left(\frac{1}{\sqrt{a^{2}-1}}\right)-\tan ^{-1}\left(\sqrt{4 a^{2}-1}\right)\right), \\
& \quad-\frac{\sqrt{4 a^{2}-1}}{4}
\end{aligned}
$$

and $G_{2}(a)=\pi a^{2}-a^{2} \tan ^{-1}\left(\sqrt{4 a^{2}-1}\right)-\frac{\sqrt{4 a^{2}-1}}{4}$.
In order to find $F_{d_{j}, l_{j}}(z, t)$ as above, we note that the joint CDF $F_{d_{j}, l_{j}}(z, t)$ equals the overlapping area of the circle with radius $z$ centered at $(0,1 / 2)$, the circle with radius $t$ centered at $(1,1 / 2)$, and the $1 \times 1$ square. For example, in Fig. 4, we consider the case where $0<z$


Fig. 4. The $\operatorname{CDF} F_{d_{j}, l_{j}}(z, t)$ when $0<z \leq 1 / 2$ and $1-z$ $<t \leq 1$.
$<1 / 2$ and $1-z<t<1$. In this figure, we denote A and $B$ as the intersection points of these two circles. In addition, it is easy to obtain the $x$-coordinate of the two points A and B as $x_{\mathrm{A}}=x_{\mathrm{B}}=\left(1+z^{2}-t^{2}\right) / 2$. Therefore, the overlapping area includes two sectors that are separated by the line $x=\left(1+z^{2}-t^{2}\right) / 2$. Considering the sectors on the left of the separating line, the area of this sector is given as:

$$
\begin{align*}
K_{1}(t, z) & =\int_{1-t}^{\left(1+z^{2}-t^{2}\right) / 2} \int_{\frac{1}{2}-\sqrt{t^{2}-(x-1)^{2}}}^{\frac{1}{2}+\sqrt{t^{2}-(x-1)^{2}}} d y d x \\
& =\frac{\pi t^{2}}{2}-t^{2} \tan ^{-1}\left(\frac{1+t^{2}-z^{2}}{\Delta(t, z)}\right)-\frac{1+t^{2}-z^{2}}{4} \Delta(t, z) \tag{9}
\end{align*}
$$

Similarly, the area of the sectors on the right of the separating line is determined by:

$$
\begin{align*}
& K_{1}(z, t)=\int_{\left(1+z^{2}-t^{2}\right) / 2}^{z} \int_{\frac{1}{2}-\sqrt{t^{2}-x^{2}}}^{\frac{1}{2}+\sqrt{z^{2}-x^{2}}} d y d x \\
& =\frac{\pi z^{2}}{2}-z^{2} \tan ^{-1}\left(\frac{1+z^{2}-t^{2}}{\Delta(z, t)}\right)-\frac{1+z^{2}-t^{2}}{4} \Delta(z, t) \tag{10}
\end{align*}
$$

From (9) and (10), we obtain (8.4).
Now, differentiating $F_{d_{j}, l_{j}}(z, t)$ in (8) with respect to $z$ and $t$, we obtain the joint PDF of link distances $d_{j}$ and $l_{j}$ as $f_{d_{j}, l_{j}}(z, t)=\partial^{2} F_{d_{j}, l_{j}}(z, t) / \partial z \partial t$.

In Fig. 5, we present the joint $\operatorname{CDF} F_{d_{j}, l_{j}}(z, t)$ as a function of the two variables $z$ and $t$. In this figure, the simulation results are obtained using the Monte Carlo method, while the theoretical results are drawn by mathematical expressions (8.1)~(8.12). It can be observed that the simulation and theoretical results match very well with each other.

## 3-2 Outage Probability



Fig. 5. The Monte Carlo simulation and theoretical results of the joint CDF $F_{d_{j}, l_{j}}(z, t)$.

Assuming that the channels between two nodes are subjected to flat Rayleigh fading plus additive white Gaussian noise (AWGN), the signal received at receiver $j$ due to the transmission of transmitter $i$ is given by:

$$
\begin{equation*}
r_{i, j}=\sqrt{P} h_{i, j} s+\eta_{j} \tag{11}
\end{equation*}
$$

where $P$ is the average transmit power of node $i$ (we assume that the source and relays have same transmit power $P$ ); $\eta_{j}$ is an AWGN noise sample with variance $N_{0} / 2$ per dimension at receiver $j ; h_{i, j}$ is a fading coefficient between nodes $i$ and $j$; and $s$ is the signal transmitted by the transmitter $i$.

Because $h_{i, j}$ has Rayleigh distribution, $\left|h_{i, j}\right|^{2}$ has exponential distribution with parameter $\lambda_{i, j}$. To take path loss into account, we can model the variance of the channel coefficient between nodes $i$ and $j$ as a function of distance between two nodes [10]. Therefore, the parameter $\lambda_{i, j}$ can be expressed by:

$$
\begin{equation*}
\lambda_{i, j}=d_{i, j}^{\beta}, \tag{12}
\end{equation*}
$$

where $\beta$ is a path loss exponent that varies from 2 to 6 and $d_{i, j}$ is the distance between node $i$ and node $j$.

Now, we introduce some notations used below as follows. Let us denote $h_{1, j}$ and $h_{2, j}$ as the channel coefficients between source S and relay $R_{j}$ and between relay $R_{j}$ and the destination, respectively; furthermore, we denote $\lambda_{1, j}$ and $\lambda_{2, j}$ as the parameters of $\left|h_{1, j}\right|^{2}$ and $\left|h_{2, j}\right|^{2}$, respectively. We also denote $h_{2,0}$ and $\lambda_{2,0}$ as the channel coefficient between the source S and the destination D and the parameter of $\left|h_{2,0}\right|^{2}$, respectively.

In the direct transmission scheme (DT), the source transmits the signal to the destination directly without the help of any relays. The mutual information between the source $S$ and the destination $D$ is given by:

$$
\begin{equation*}
I_{S, D}=\log _{2}\left(1+P\left|h_{2,0}\right|^{2} / N_{0}\right) . \tag{13}
\end{equation*}
$$

Now, we define the outage probability of the S-D link as:

$$
\begin{equation*}
P_{o u t}^{D T}=\operatorname{Pr}\left[I_{S, D}<R\right], \tag{14}
\end{equation*}
$$

where $R$ is the target rate of the system.
Using the CDF of $\left|h_{2,0}\right|^{2}$, we can easily calculate the outage probability of the direct transmission as:

$$
\begin{equation*}
P_{\text {out }}^{D T}=1-\exp \left(-\lambda_{2,0} \mathrm{SNR}\right), \tag{15}
\end{equation*}
$$

where $\mathrm{SNR}=P / N_{0}$ is the average transmit signal-tonoise ratio (SNR).
For cooperative communication, the mutual information between the source S and relay $R_{j}$ is determined
as:

$$
\begin{equation*}
I_{1, j}=\frac{1}{N+1} \log _{2}\left(1+P\left|h_{1, j}\right|^{2} / N_{0}\right), \tag{16}
\end{equation*}
$$

where the factor $1 /(N+1)$ accounts for the fact that the overall transmission is split into $N+1$ time slots.

If $I_{1, j}$ is higher than the system rate $R$, the relay $R_{j}$ successfully decodes the source's signal. Otherwise, the decoding status at relay $R_{j}$ is considered to fail. Hence, this outage probability can be formulated as:

$$
\begin{equation*}
P_{1, j}=\operatorname{Pr}\left[I_{1, j}<R\right]=1-\exp \left(-\lambda_{1, j} \rho\right), \tag{17}
\end{equation*}
$$

where $\rho=\left(2^{(N+1) R}-1\right) /$ SNR.
Because the decoding set $C_{D}$ is a random set, the number of relays in this set is also a random variable $k$, i.e., $\left|C_{D}\right|=k, k=0,1, \ldots, N$. For each $k$, there are $\binom{N}{k}$ possible subsets of size $k$. Assuming that $C_{D}=$ $\left\{R_{i_{1}}, R_{i_{2}}, \ldots, R_{i_{k}}\right\}$ and $C_{W}=\left\{R_{i_{k+1}}, R_{i_{k+2}}, \ldots, R_{i_{N}}\right\}$, the probability for the decoding set $C D$ can be obtained by:

$$
\begin{equation*}
P_{C_{D}}=\prod_{j=1}^{k}\left(1-P_{1, i_{j}}\right) \prod_{j=k+1}^{N} P_{1, i_{j}} \tag{18}
\end{equation*}
$$

where $P_{1, i_{m}}$ is calculated as in (17).
Using the total probability law, we can write the outage probability of the system as follows:

$$
\begin{equation*}
\mathrm{P}_{\text {out }}^{C C}=\sum_{C_{D}} P_{C_{D}} \operatorname{Pr}\left[I_{C_{D}}<R\right] . \tag{19}
\end{equation*}
$$

Because the SC technique is used at the destination, the mutual information $I_{C_{D}}$ is given by:

$$
\begin{equation*}
I_{C_{D}}=\frac{1}{N+1} \log _{2}\left(1+\frac{P}{N_{0}} \max _{m=1,2, k, k}\left(\left|h_{2,0}\right|^{2},\left|h_{2, i_{m}}\right|^{2}\right)\right) . \tag{20}
\end{equation*}
$$

From (20), the outage probability at the destination in each case of the decoding set $C_{D}$ is calculated as:

$$
\begin{align*}
\operatorname{Pr}\left[I_{C_{D}}<R\right] & =\operatorname{Pr}\left[\max _{j=1,2, \ldots, k}\left(\left|h_{2,0}\right|^{2},\left|h_{2, i_{j}}\right|^{2}\right)<\rho\right] \\
& =\operatorname{Pr}\left[\left|h_{2,0}\right|^{2}<\rho\right] \prod_{j=1}^{k} \operatorname{Pr}\left[\left|h_{2, i_{j}}\right|^{2}<\rho\right] \\
& =\left(1-\exp \left(-\lambda_{2,0} \rho\right)\right) \prod_{j=1}^{k}\left(1-\exp \left(-\lambda_{2, i_{j}} \rho\right)\right) \tag{21}
\end{align*}
$$

From (18), (19) and (21), the outage probability of the system can be given by:

$$
\begin{equation*}
\mathrm{P}_{\text {out }}^{C C}=\sum_{C_{D}}\binom{\left(1-\exp \left(-\lambda_{2,0} \rho\right)\right) \prod_{j=k+1}^{N}\left(1-\exp \left(-\lambda_{1, i, j} \rho\right)\right)}{\times \prod_{j=1}^{k} \exp \left(-\lambda_{1, i, j} \rho\right)\left(1-\exp \left(-\lambda_{2, i,} \rho\right)\right)} . \tag{22}
\end{equation*}
$$

At a high transmit SNR value, we can approximate $P_{C_{D}}$ in (18) and $\operatorname{Pr}\left[I_{C_{D}}<R\right]$ in (21) as:

$$
\begin{align*}
& \mathrm{P}_{C_{D}} \approx \rho^{N-k} \prod_{j=k+1}^{N} \lambda_{1, i_{j}} \prod_{j=1}^{k} \exp \left(-\lambda_{1, i_{j}} \rho\right),  \tag{23}\\
& \operatorname{Pr}\left[I_{C_{D}}<R\right] \approx \lambda_{2,0} \rho \prod_{j=1}^{k}\left(\lambda_{2, i_{j}} \rho\right)=\rho^{k+1} \prod_{j=1}^{k} \lambda_{2, i_{j}} . \tag{24}
\end{align*}
$$

Substituting (23), (24) into (22), we obtain:

$$
\begin{equation*}
\mathrm{P}_{\text {out }}^{C C} \approx \sum_{C_{D}} \rho^{N+1} \prod_{j=k+1}^{N} \lambda_{1, i_{j}} \prod_{j=1}^{k} \lambda_{2, i_{j}} \exp \left(-\lambda_{1, i_{j}} \rho\right) . \tag{25}
\end{equation*}
$$

Hence, the expected value of $P_{\text {out }}^{C C}$ can be approximately calculated in a two-dimensional network as follows:

$$
\begin{align*}
\overline{P_{\text {out }}^{C c}} & \approx \sum_{C_{D}}\left[\begin{array}{l}
\rho^{N+1} \prod_{j=k+1}^{N} \int_{0}^{1}\left(z_{i_{j}}\right)^{\beta} f_{d_{i_{j}}}\left(z_{i_{j}}\right) d z_{i_{j}} \\
\times \prod_{m=1}^{k}\binom{\left.\int_{0}^{+\infty} \int_{0}^{+\infty}\left(t_{i_{m}}\right)^{\beta} \exp \left(-\left(z_{i_{n}}\right)^{\beta} \rho\right)\right)}{\left.\times f_{d_{i_{m}, l_{m}}\left(z_{i_{m}}, t_{i_{m}}\right.}\right) d t_{i_{m}} d z_{i_{m}}} \\
\\
\\
\approx \sum_{C_{D}} \rho^{N+1}\left(\frac{1}{1+\beta}\right)^{N-k}[C(\beta)]^{k}
\end{array},\right.
\end{align*}
$$

where $f_{d_{i m}, l_{i m}}\left(z_{i_{m}}, t_{i_{m}}\right)$ is the joint PDF of the link distances $d_{i_{m}}$ and $i_{i_{m}}$, and $C(\beta)=\prod_{m=1}^{k} \int_{0}^{+\infty} \int_{0}^{+\infty}\left(t_{i_{m}}\right)^{\beta} \exp \left(-\left(z_{i_{m}}\right)^{\beta} \rho\right)$ $f_{i_{m}}\left(z_{i_{m}}, t_{i_{m}}\right) d t_{i_{m}} d z_{i_{m}}$.

## IV. Simulation Results

In this section, we provide some numerical results for the outage probabilities developed in section III and verify them with Monte Carlo simulations.


Fig. 6. Outage probability as a function of the average transmit $\operatorname{SNR}\left(P / N_{0}\right)$ in dB when $R=1$ and $\beta=3$.


Fig. 7. Outage probability as a function of the average transmit $\operatorname{SNR}\left(P / N_{0}\right)$ in dB when $R=1$ and $\beta=4$.

Fig. 6 shows the outage probability as a function of the average transmit $\mathrm{SNR}\left(P / N_{0}\right)$ in dB . In this figure, the target rate and the path loss exponent are set by 1 and 3 , respectively, while the number of cooperative nodes $N$ varies from 0 to 2 . For the direct transmission (DT) protocol, which corresponds to $N=0$, the source transmits the signal to the destination directly without the help of relays. We can see from Fig. 6 that the cooperative communication (CC) protocol outperforms the direct transmission protocol at the high SNR region. This is because the cooperative communication obtains a higher diversity order than that of direct transmission. In addition, it can be observed that the simulation and numerical results are in good agreement, and the approximate results (dotted lines) match very well with the exact results (solid lines) in the high SNR region.

In Fig. 7, the parameters are fixed as follows: $R=1$ and $\beta=4$. In this figure, the results are plotted by numerical computations. As we can see, the cooperative communication can obtain full diversity order, which equals the number of cooperative nodes $(N)$ in the network plus 1 .

## V. Conclusion

In this paper, we evaluated the performance of cooperative communication when relays were placed randomly in two-dimensional networks. We proposed a simple approximation of the average outage probability for the decode-and-forward system to reduce numerical computations. The numerical and simulated results were presented to demonstrate the validity of the analytical results. The results presented that similar to determinis-
tic networks, the cooperative transmission in random networks not only outperforms direct transmission, but also achieves higher diversity gain.

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2011-0006043).

## References

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," IEEE Trans. Inform. Theory, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.
[2] J. N. Laneman, G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," IEEE Trans. Inform. Theory, vol. 49, no. 10, pp. 2415-2425, Nov. 2003.
[3] T. E. Hunter, A. Nosratinia, "Diversity through coded cooperation," IEEE Trans. on Wire. Commun., vol. 5, no. 2, pp. 283-289, Feb. 2006.
[4] A. Stefanov, E. Erkip, "Cooperative coding for wireless networks," IEEE Trans. on Commun., vol. 52, no. 9, pp. 1470-1476, Sep. 2004.
[5] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," IEEE Journal on Selected Areas in Commun., vol. 24, no. 3, pp. 659-672, Mar. 2006.
[6] L. E. Miller, "Distribution of link distances in a wireless network," NIST J. of Research, vol. 106, pp. 401-412, Mar.-Apr. 2001.
[7] B. Ghosh, "Random distances within a rectangle and between two rectangles," Bulletin Calcutta Math Soc., vol. 43, pp. 17-24, 1951.
[8] L. E. Miller, "Joint distribution of link distances," 2003 Conference on Information Sciences and Systems, Johns Hopkins University, Mar. 2003.
[9] C. -C. Tseng, K. -C. Chen, "Layerless design of a po-wer-efficient clustering algorithm for wireless ad hoc networks under fading," Journal of Wireless Personal Communications, vol. 44, no. 1, pp. 3-26, Jan. 2008.
[10] T. T. Duy, B. An, and H. -Y. Kong, "A novel co-operative-aided transmission in multi-hop wireless networks," IEICE Trans. on Commun., vol. E93.B, no. 3 , pp. 716-720, Mar. 2010.

## Tran Trung Duy


received Telecommunications Engineering from Ho Chi Minh City University of Te the B.E. degree in Electronics and technology, Vietnam, in 2007. He is currently working toward the Master degree in the Department of Electrical Engineering, University of Ulsan, Korea. His major research interests are mobile ad-hoc networks, wireless sensor networks, cooperative communications, cooperative routing, cognitive radio, combining techniques.

## Hyung-Yun Kong


received the M.E. and Ph.D. degrees in electrical engineering from Polytechnic University, Brooklyn, New York, USA, in 1991 and 1996, respectively, He received a B.E. in electrical engineering from New York Institute of Technology, New York, in 1989. Since 1996, he has been with LG electronics Co., Ltd., in the multimedia research lab developing PCS mobile phone systems, and from 1997 the LG chairman's office planning future satellite communication systems. Currently he is a professor in electrical engineering at the University of Ulsan, Korea. His research area includes channel coding, detection and estimation, cooperative communications, cognitive radio and sensor networks. He is a member of IEEK, KICS, KIPS, IEEE, and IEICE.


[^0]:    Manuscript received November 2, 2011 ; revised December 15, 2011. (ID No. 20111102-031J)
    Department of Electronic Engineering, University of Ulsan, Ulsan, Korea.
    Corresponding Author: Hyung-Yun Kong (e-mail : hkong@ulsan.ac.kr)

