3체 역학 방정식을 이용한 위성 임무 궤도 설계

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Mission Trajectory Design using Three-Body Dynamics

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요 약

이제까지 수행된 우주 탐사 임무에서 임무 궤도의 설계는 행성 혹은 위성과 인공위성의 2체 문제 (two-body problem)에 기초한 Hohmann transfer를 기반으로 하는 Patched Conic Approximation 방식이 주로 사용되어져 왔다. Hohmann transfer는 원 궤도에서 다른 원 궤도로 천이할 수 있는 타원 천이 궤도의 설계 방식으로서, Patched Conic Approximation은 태양계를 여러 개의 2체 문제로 분해하고 각기 분해된 2체 시스템 사이의 Hohmann 천이 궤도를 설계하여 조합함으로써 행성 간의 임무 궤도를 설계하는 방식이다. 이 방식은 하나의 행성만을 고려했을 때, 즉 행성과 인공위성의 2체 문제일 때, 가장 효율적인 천이 방식으로 알려져 있고 현재까지의 우주 탐사 임무 설계에 주로 이용되고 있다. 하지만, 우주 탐사 임무가 점차 다양화되고 소형 위성을 이용한 임무 수행의 필요성이 증가함에 따라 기존의 Patched Conic Approximation은 요구되는 연료의 양이 크다는 점과 원뿔꼴(conic) 특성을 가지는 궤도만을 표현할 수 있다는 점에서 한계점을 보이기 시작하고 있다. 이에 반해 3체 동역학의 기하학적 특성은 기존의 태양계의 패러다임을 획기적으로 변화시킨다. 개념적으로는 요구되는 에너지가 매우 적은 에너지로 태양계를 모두 연결하는 궤도를 구성할 수 있기 때문이다. 본 논문에서는2체문제 기반의 임무 궤도 설계 기술의 한계성에서 벗어나 유연하고 효율적인 탐사 임무를 설계한다.

Key Words : Space, Mission, three-body dynamics

ABSTRACT

Most mission trajectory design technologies for space exploration have been utilized the Patched Conic Approximation which is based on Hohmann transfer in two-body problem. The Hohmann transfer trajectory is basically an elliptic trajectory, and Patched Conic Approximation consists of Hohmann transfer trajectories in which each trajectory are patched to the next one. This technology is the most efficient method when considering only one major planet at each patch trajectory design. The disadvantages of the conventional Patched Conic Approach are more fuel (or mass) needed and only conic trajectories are designed.

Recent space exploration missions need to satisfy more various scientific or engineering goals, and mission utilizing smaller satellites are needed for cost reduction. The geometrical characteristics of three-body dynamics could change the paradigm of the conventional solar system. In this theoretical concept, one can design a trajectory connecting around the solar system with comparably very small energy. In this paper, the basic three-body dynamics are introduced and a spacecraft mission trajectory is designed utilizing the three-body dynamics.

Ⅰ. Introduction

Recently, the spacecraft trajectory to the Moon has become a topic of increasing attention. The conventional patched conic method has been used by many missions, including the Apollo program. Its advantage and disadvantage are the short transfer time and high fuel cost, since it is based on the two body dynamics, a celestial body and a spacecraft.

When considering another celestial body, the problem becomes three body problem, two celestial bodies and one spacecraft. This concept can change the paradigm of the conventional solar system[2].

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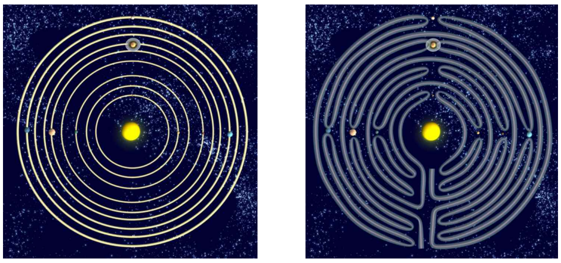

(*Courtesy of NASA*)

The left is the conventional solar system and the right is the new paradigm utilizing the three body dynamics. Conceptually, trajectories connecting all the celestial bodies in solar system can be found with small energy. When applying this three body dynamic characteristics, one can design a spacecraft trajectories with very small energy. Japanese spacecraft HITEN could reach to the Moon with 10% of designed propellant, when its normal operation disabled by malfunction. USA's spacecraft GENISIS could reach to its mission orbit with only one orbit insertion and return to the Earth. These conventionally impractical (or impossible) orbits are only possible in multi-body dynamics. Cost efficiency obtained by three body dynamics can extend applications of small satellites like STSAT series to the interplanetary mission. In this paper, simple three body dynamics and useful characteristics such as libration points, periodic orbits, and stable/unstable manifolds are introduced, and a Earth to Moon transfer orbit is designed based on these dynamics characteristics.

Ⅱ. Circular Restricted Three Body Problem - CRTBP

1. Problem Description

Consider the motion of a particle of negligible mass moving under the gravitational influences of two masses and as the primary masses. Assume that and have circular orbits about their common center of mass. The particle is free to move the space defined by the circular orbits of the primary masses, but cannot affect their motion.

The coordinate of the CRTBP is centered at the barycenter of the primaries and set to rotate with the motion of the primaries about the origin. When viewed from $+z$ axis, the synodic frame rotates in a counter-clockwise compared to the inertial frame. The x axis extends from the origin through the m_2, the z axis extends in the direction of the angular momentum of the system, and y axis completes the right-hand coordinate frame. When dealing with CRTBP, the useful convention is to applying unit normalization as following.

$$
\begin{aligned}
&\text{Distance :} && d_A = Ld \\
&\text{Time :} && t_A = T/(2\pi)s \\
&\text{Velocity :} && s_A = Vs
\end{aligned}
$$

Where L (in km) is distance between m_1 and m_2, T (in seconds) is orbital period of m_1 and m_2 and V (in km/s) is orbital velocity of m_1. Then the only parameter of the system is the mass parameter.

$$\mu = \frac{m_2}{m_1 + m_2} \tag{1}$$

Table 1 provides a table of mass parameters and dimensional values for Sun/Earth and Earth/Moon system.

Table 1 Parameters of Sun/Earth & Earth/Moon systems

System	μ	L (km)	V (km/s)	T (s)
Sun/Earth	3.036E-6	1.496E8	29.784	3.147E7
Earth/Moon	1.215E-2	3.850E5	1.025	2.361E6

2. Equation of Motion

The gravitational potential which the particle experiences due to m_1 and m_2 is

$$U(x,y,z) = -\frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} - \frac{1}{2}\mu_1\mu_2 \tag{2}$$

where

$$
\begin{aligned}
r_1^2 &= (x+\mu_1)^2 + y^2 + z^2 \\
r_2^2 &= (x-\mu_2)^2 + y^2 + z^2
\end{aligned}
$$

and

$$
\begin{aligned}
\mu_1 &= 1-\mu \\
\mu_2 &= \mu
\end{aligned}
$$

Then, Euler-Lagrange equations is given by

$$
\begin{aligned}
(d/dt)(\dot{x}-y) &= \dot{y} + x - U_x \\
(d/dt)(\dot{y}+x) &= -(\dot{x}-y) - U_y \\
(d/dt)(\dot{z}) &= -U_z
\end{aligned} \tag{3}
$$

After simplification, we have

$$\begin{aligned} \ddot{x} - 2\dot{y} &= -\overline{U}_x \\ \ddot{y} + 2\dot{x} &= -\overline{U}_y \\ \ddot{z} &= -\overline{U}_z \end{aligned} \tag{4}$$

where the effective potential is

$$\overline{U}(x,y,z) = -\frac{1}{2}(x^2+y^2) + U(x,y,z) \tag{5}$$

and U_i denotes $(\partial/\partial i)U$. Since the Lagrangian system can be transformed to Hamiltonian form, there is energy integral of motion and it is a function of positions and velocities.

$$E(x,y,z,\dot{x},\dot{y},\dot{z}) = \frac{1}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2) + \overline{U}(x,y,z) \tag{6}$$

This energy integral defines energy surface and Hill's region, where the velocity becomes zero on the boundary of this region.

$$M(\mu,e) = \{(x,y,z,\dot{x},\dot{y},\dot{z}) | E(x,y,z,\dot{x},\dot{y},\dot{z}) = e\} \tag{7}$$

$$M_s(x,y,z) = \left\{(x,y) | \overline{U}(x,y) \le e\right\} \tag{8}$$

Therefore, the motion of particle is only possible in the side of region for which kinetic energy is positive. Also, note that there is five peak points in , and each point only can be reachable by appropriate energy level,. These five points are known as Libration or Equilibrium points of CRTBP, and plays very important roles in analyzing and designing the particle's motion (in other word, trajectory)

3. Libration Points and Geometry of Solutions

The equilibrium points $(L_1, L_2, L_3, L_4, L_5)$ of the CRTBP can be found by solving

$$(\partial/\partial x)\overline{U} = (\partial/\partial y)\overline{U} = (\partial/\partial z)\overline{U} = 0 \tag{9}$$

The behavior of particle trajectories near the two collinear libration points (L_1, L_2) can be analyzed by linear approximation near the point, since there is no known closed-form analytical solution. These linear approximation analysis reveals that the libration points act as (saddle)x(center)x(center), and there is families of periodic orbits, such as Lyapunov orbits, Halo orbits, and Lissajous orbits. In PCTBP, particle trajectories utilize Halo or Lissajous orbits for its own mission orbit or for transfer orbits to reach to mission orbits. These transfer dynamics is provided by the local stabilities near the periodic orbits.

Linearized analysis near the particular points of the periodic orbits shows that the stat transition matrices have stable eigenvalue and unstable eigenvalue, and the corresponding eigenvectors indicate that the direction of stable flow and unstable flow in the nonlinear CRTBP. The set of these flows are called manifolds (stable manifolds and unstable manifolds). Therefore, a small perturbation of the periodic orbits can make the flows in-going and out-going trajectories from the periodic orbits.

4. Periodic Orbits and Manfolds

It is critical to find a suitable periodic orbit for the particle's trajectory design. One of the approaches are Richardson's Third order Approximation and Differential Correction. The linearized form of the equations s.t the origin is located at one of the collinear libration points

$$\begin{aligned} \ddot{x} - 2\dot{y} - (1+c_2)x &= 0 \\ \ddot{y} + 2\dot{x} + (c_2-1)y &= 0 \\ \ddot{z} + c_2 z &= 0 \end{aligned} \tag{10}$$

And its solutions are

$$\begin{aligned} x &= -A_x \cos(\lambda t + \phi) \\ y &= kA_x \sin(\lambda t + \phi) \\ z &= A_z \cos(\lambda t + \psi) \end{aligned} \tag{11}$$

A_z, A_x, k, and λ can be calculated following procedure in [4]. Now, set the initial conditions for halo orbits s.t.

$$X_{0,Linear} = [x_L - A_x, 0, A_z, 0, \lambda k A_x, 0] \tag{12}$$

Then, we can find a initial point of the desired periodic orbit. Let X_0 is a point on a halo orbit intersecting XZ plane. Since a halo orbit in CR3BP is symmetric with respect to the XZ plane, when integrating the X_0 until the trajectory intersecting XZ plane, $X(T/2)$ shall also be in the form of

$$X(\frac{T}{2}) = [x, 0, z, 0, \dot{y}, 0] \tag{13}$$

However, integrating with the initial condition, $X_{0,Linear}$, will not produce $X(T/2)$ in the above form because it is not a solution of nonlinear CR3BP problem. Therefore, $X_{0,Linear}$ shall be adjusted using Differential Correction Algorithm. In Differential Correction Algorithm, the transition matrix at $(T/2)$ can be used to adjust the initial value s.t.

$$\delta X(\frac{T}{2}) = \Phi\left(\frac{T}{2}, 0\right)\delta X(0) \tag{14}$$

and $\Phi(T/2, 0)$ can be computed by the following equation.

$$\dot{\Phi}(t, t_0) = Df(x)\Phi(t, t_0) \tag{15}$$

where, $Df(x)$ is the Jacobian Matrix. By repeating this procedure until $|\dot{x}| < \epsilon$ and $|\dot{z}| < \epsilon$, we can find initial value for the halo orbit.

The stable and unstable manifolds are represented by trajectories leading forward or away from the fixed point.

A particular point on the periodic orbit defines the beginning and end of the orbit period. So, this point can be a fixed point. By linearizing about this fixed point, we can find eigenvalues and corresponding eigenvectors. The eigenvectors corresponding to the stable eignevalue approximates stable manifolds and eigenvectors corresponding the unstable eigenvalue approximates unstable manifolds. The particular state transition matrix describing the state change for one period (T) of the orbit is used to calculate these eigen vectors.

Let's say Y^s and Y^u denote stable and unstable eigenvectors, respectively. Then initial guess of stable and unstable manifolds at the fixed point is

$$\begin{aligned} X_0^s &= X^* + dY^s \\ X_0^u &= X^* + dY^u \end{aligned} \tag{16}$$

where d is small displacement.

5. Trajectories Patch

The final mission orbits can be obtained by patching the end and start points of trajectories in stable/unstable manifolds. Flows in unstable manifolds go out from the periodic orbit and flows in stable manifolds go in to the periodic orbit. If there exist any intersection between these manifolds, a particle can move from a periodic orbit to another. These intersections can be found by Poincare section or Differential Correction algorithm.

Ⅲ. Earth to Moon Mission Trajectory Design

1. Problem Structure

The design of Earth to Moon trajectory considers the systems in Figure 1. Actually this system is a four body system, but as described in Section 2.5, we will utilizing trajectory patching techniques between Sun/Earth system and Earth/Moon system.

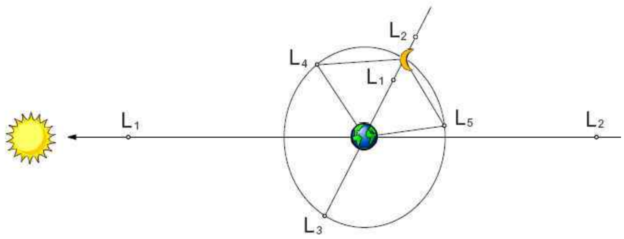

Figure 1. Sun/Earth and Earth/Moon Systems

Earth parking orbit is assumed to have 300km altitude and the desired transfer trajectory is near the Moon with arbitrary distance since the mission orbit can be changed depending on a particular mission. In the CRTBP, the transfer orbit can be designed utilizing Sun/Earth L2 halo orbits and Earth/Moon L2 halo orbits.

2. Sun/Earth Halo Orbit and Its Manfolds

Sun/Earth L2 halo orbit is designed following procedures in Section 2.4. After setting $A_z = 440,000(km)$, the initial guess of the halo orbit is

$$x_{0,guess} = (1.00705, 0.0, 0.00335, 0.0, 0.01409, 0.0)$$

Differential Correction with $x_{0,guess}$ calculates the initial point of the halo orbit as shown in Figure 2.

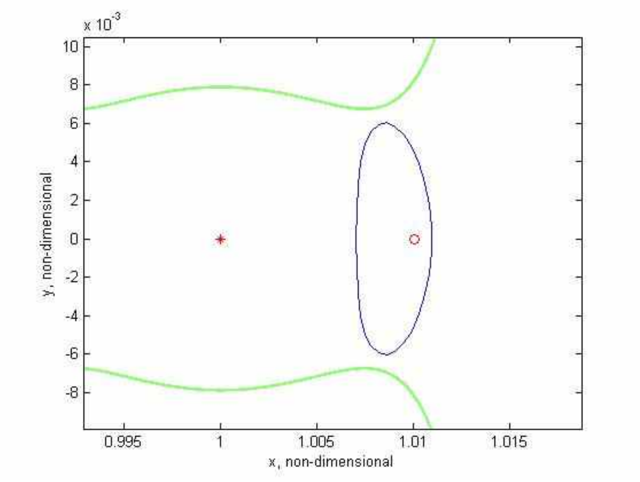

Figure 2a. Sun/Earth L2 Halo Orbit (2-dimensional view)

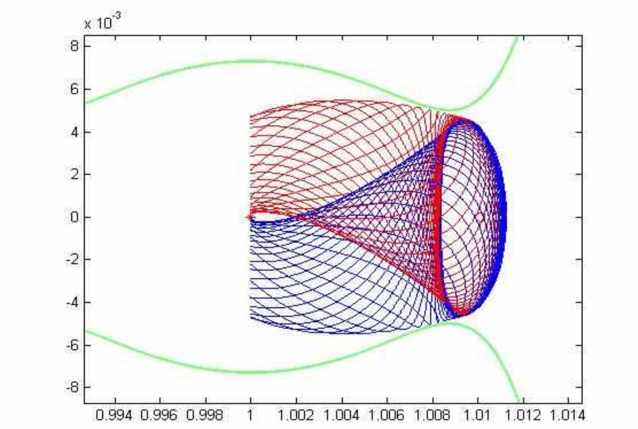

Figure 3a. Stable and Unstable Manifolds of Sun/Earth L2 Halo Orbit (2-dimensional view)

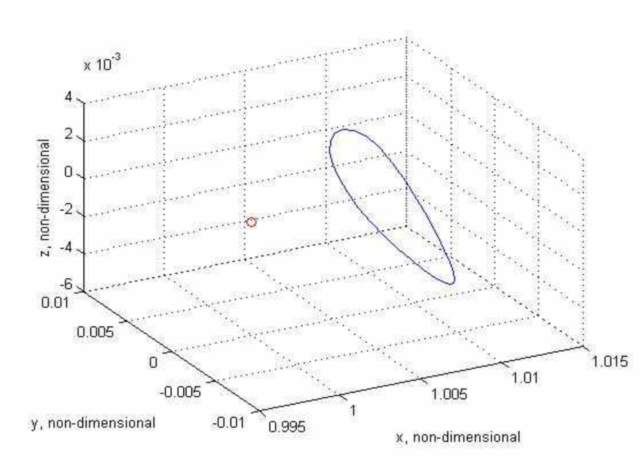

Figure 2b. Sun/Earth L2 Halo Orbit (3-dimensional view)

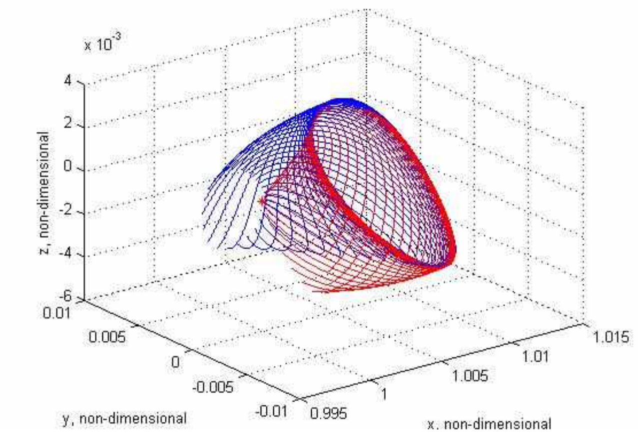

Figure 3b. Stable and Unstable Manifolds of Sun/Earth L2 Halo Orbit (3-dimensional view)

In Figure 2, $*$ and o is the position of Earth and L_2 and the blue line (upper & lower) is the boundary of Hill's region. Flows near this halo orbit defines manifolds, which are stable and unstable.

If a spacecraft is on the stable manifold, it will move into the halo orbit. If a spacecraft is on the unstable manifold, it will move out from the halo orbit. Therefore, it is necessary to find a stable manifold to escape the Earth parking orbit. Figure 3 shows stable and unstable manifolds on the halo orbit. Note that the stable manifold can reach to the near region of the Earth and, if a manifold reaches to the Earth parking orbit, the spacecraft can escape the Earth parking orbit by changing its velocity (or by changing its energy level), called a ΔV. Usually the Earth escaping is provided by a launch vehicle.

The Earth escaping point of the stable manifolds can be found by looking at the manifold's cross section, called Poincare Section, on the x-z plane. Figure 4 shows this cross section. The left is the section of unstable manifold and the right is of stable manifold. The center line is the position of the Earth.

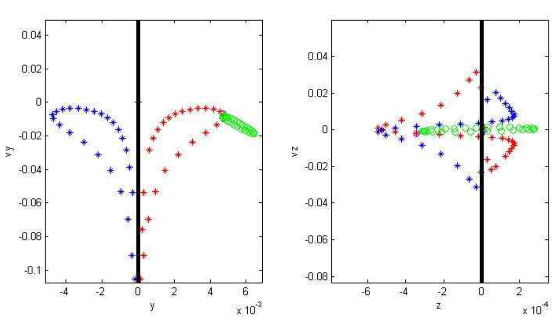

Figure 4. Poincare Section on X-Z Plane (center line: position of the Earth, * : cross sections of stable and unstable manfolds in of Sun/Earth system, o : cross section of stable manfold of Earth/Moon system)

If an initial point is selected on the * line of left side and integrated forward, the trajectory will converge onto the halo orbit. If selected on the * line of rigth side and integrated backward, its trajectory will also converged onto the halo orbit. The other regions defines points for transit and non-transit trajectories. So, by properly selecting an initial point, we can find out a Earth escaping trajectory which starts near the Earth parking orbit, flows into the halo orbit, and goes out from the halo orbit. The bold line in Figure 5 shows the Earth escaping trajectory with the initial point selected using the Poincare section in Figure 4.

The next step is to find a trajectory whose initial point is the same as the final point or near the final final point of this Earth escaping traejectory.

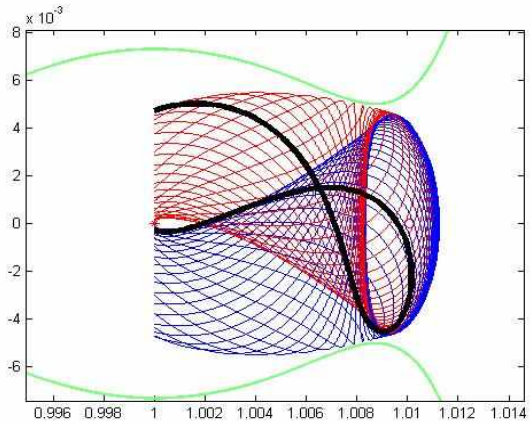

Figure 5. Earth Escapting Trajectory

3. Earth/Moon Halo Orbit and Its Manfolds

Earth/Moon L2 orbit and its manifolds can be calculated similar way. Figure 6 shows the Earth/Moon L2 halo orbit and its stable manifolds in Earth/Moon system. Any point on this stable manifold will converge on the the Earth/Moon halo orbit. So, the problem is to find a point on this manifold, which is connected to the Earth escaping trajectory, and can be found by Poincare section in Figure 4. The * section is for stable/unstable manifold in Sun/Earth system. When we transform Earth/Moon trajectory to the same one expressed in Sun/Earth system and calculate the same cross section, we can see if there is common regions, which eventually connect two trajectories. In Figure 4, o line is the cross section of stable manfold of Earth/Moon halo orbit in Sun/Earth system. There exist certainly comman region and we will select a point in this region to go to the Moon.

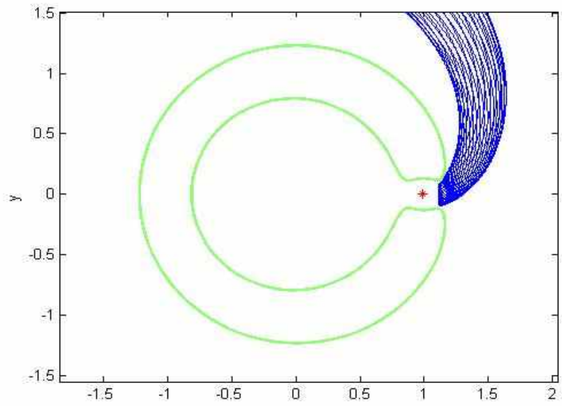

Figure 6. Stable Manfold in Earth/Moon System

4. Earth to Moon Trajectory

The remaining calculation of the trajectory design is to find a patch point between two trajectories. This calculation is to select a point in cross section in Sun/Earth xy plane, which satisfies the Earth parking and Moon reaching requirement. Usually this calculation is performed by iterating procedures.

Figure 7 shows the result. The Sun/Earth and Earth/Moon systems are expressed together in this Figure and the Earth/Moon system is rotating the Earth. The trajectory (sold line) which starts from the Earth parking orbit can reach onto the Earth/Moon L2 halo orbit. At the patching point, we needed small ΔV (about 22m/s).

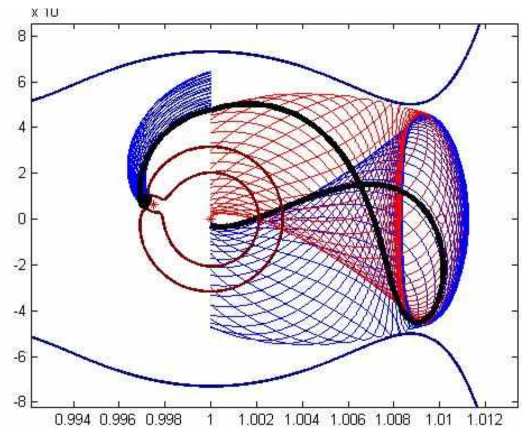

Figure 7. Trajectory from Earth Parking to Moon's L2 Halo Orbit

The bold line in Figure 8 shows the same trajectory. It shows that the trajectory converges onto the Earth/Moon L2 halo orbit. The final step is to find a small deviation from the halo orbit to reach to the Moon. This will be done with similar way (using unsable manifold) as described above and we can find lots of trajectories which reach to the Moon region with very wide range of application choices. We can impact on the Moon or we can reach to the Moon's circular orbit, elliptic orbit, etc. The red lines in Figure 8 are some of the trajectories we can obtain from Earth/Moon L2 halo orbit.

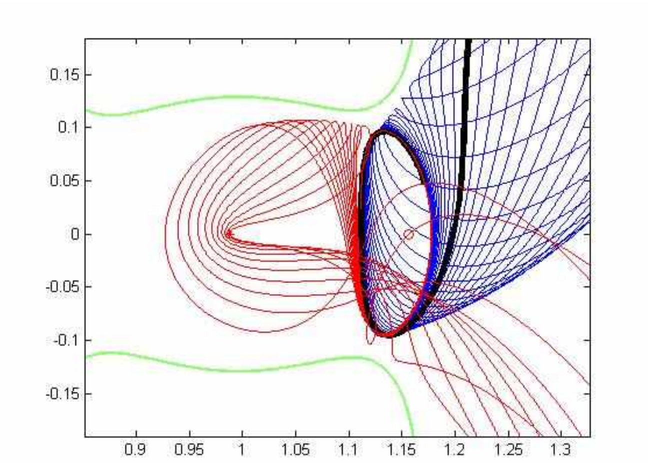

Figure 8. Trajectories reaching to the Moon

Ⅲ. Conclusions

In this paper, characteristics of three body dynamics was introduced for further complicated orbit design. Periodic orbit about libration points and stable/unstable manifolds could provide useful tools in cost efficient transfer trajectory design. In the designing Earth to Moon trasnfer, the energy change was required in two points and their magnitudes are very small (Note that these energy changes were used as small perturbation in CRTBP). As a result, we conclude that the CRTBP apporach can improve the applications of cost limited spacecraft program to interplanetary missions.

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