

# Network-Coding-Based Coded Cooperation

Suwen Wu, Jinkang Zhu, Ling Qiu, and Ming Zhao

**Abstract:** Coded cooperation is a promising user cooperation scheme. In this paper, we first propose a novel network-coding-based coded cooperation scheme. When a user decodes its partner's information correctly in the first frame, it transmits the combination of the partner's parity bits and its own parity bits through network coding in the second frame. This is distinct from the classical scheme, where the user only transmits the partner's parity bits during cooperation. We analyze the outage probability of the proposed scheme, and show that it achieves a full diversity order. Numerical evaluations reveal that the proposed scheme outperforms the classical scheme when the inter-user channel is poor, yet is worse when the inter-user channel is strong. Also, the results show that the proposed scheme always outperforms that of no cooperation in various channel conditions while the performance of classical scheme is worse than that of no cooperation with the poor inter-user channels. This means that the performance of the proposed scheme is more stable than the classical scheme and the proposed scheme is more tolerant to the poor inter-user channels. To combine the advantages of the proposed scheme and the classical scheme under different inter-user channel conditions, we propose an adaptive solution. This adaptive scheme enhances the system performance considerably in all channel conditions in spite of the inter-user channel quality, at the expense of only one acknowledgement or non-acknowledgement bit.

**Index Terms:** Coded cooperation, cooperative communication, diversity order, network coding, outage probability.

## I. INTRODUCTION

Cooperative communication, in which each user transmits not only its own information but also its partner's information, has attracted more and more attention [1]–[3]. Coded cooperation is a cooperation scheme proposed by Hunter *et al.* in 2002 [3]. In this paper, we call it the classical coded cooperation (CCC) scheme. In [4], it was shown that the outage performance of the CCC scheme was better than the amplify-and-forward (AF) [5] scheme and the decode-and-forward (DF) [6] scheme. Hunter *et al.* analyzed the outage probability of the CCC scheme and showed that the outage performance was generally superior to that of space-time cooperation [7].

Network coding [8], which is firstly proposed in the noiseless wired networks, has been extended to the wireless networks recently, to improve the system performance [9]–[15]. In [9],

the authors compared the outage probability of user cooperation system with/without network coding, and the results showed that the performance of system with network coding is better than that of system without network coding. In [10] and [11], Woldegebreal *et al.* analyzed the outage performance of the network-coding-based DF scheme. The authors in [12] proposed a network coding-based user cooperation scheme to improve the system performance and generalized the scheme from two-user cooperation to multi-user cooperation. A network-coding scheme was used in the bi-direction cooperation model [13], [14]. Joint network-coding and channel coding was considered in [15]. In the above works, it considered the DF scheme in [10] and [11] and a bi-direction transmission in [13] and [14]. In [12], the user generated new parity bits protecting both its own and the partner's bits with network coding. In this scheme, it needs new more redundancy bits. While in our paper, we consider a new network-coding scheme in general coded cooperation system proposed by Hunter *et al.*, and it doesn't need anymore redundancy bits. Note that in the previous works [9], error-free or good inter-user channel was assumed, whereas we will explicitly consider noisy inter-user channels, as the inter-user channel would go bad from time to time and affect considerably the overall system performance [16].

Specifically, we propose a novel network-coding-based coded cooperation (NCCC) scheme. In the CCC scheme, when one user decodes its partner's information correctly in the first frame, it will transmit its partner's additional parity bits in the second frame. While in our proposed NCCC scheme, when one user decodes its partner's first frame correctly, it transmits the combination of its own and its partner's additional parity bits through network coding in the second frame. We carry out a thorough analysis on the outage probability of the NCCC scheme. Moreover, we derive the approximate outage probability of the NCCC scheme in the high signal to noise ratio (SNR) regime and prove that the proposed scheme achieves full diversity order. Numerical evaluations reveal that the performance of the NCCC scheme is superior to the CCC scheme when the inter-user channel is worse than user-destination channel, yet inferior to the CCC scheme when the inter-user channel is better than user-destination channel. The results are similar to [12]. Also, it is shown that the performance of the NCCC scheme is always better than no cooperation in various channel conditions while the CCC scheme is inferior to no cooperation when the inter-user channels are poor. This means that the NCCC scheme is more tolerant to the poor inter-user channels.

To combine the strengths of the CCC and NCCC scheme at different channel conditions, we further propose an adaptive solution. In the proposed adaptive network-coding-based coded cooperation (ANCCC) scheme, only when one user decodes its partner's information correctly, the NCCC scheme will be used. When each user decodes each other's information correctly or none of the users decodes its partner's information cor-

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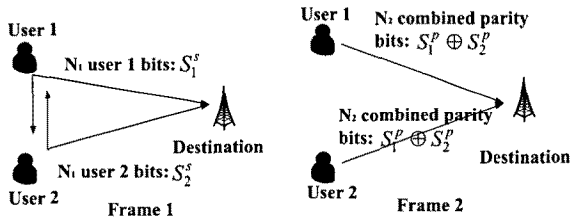


Fig. 1. System model of coded cooperation.

rectly, the CCC scheme will be chosen. Although the ANCCC scheme needs one acknowledgement or non-acknowledgement (ACK/NACK) bit for each user to inform its partner whether it decodes its partner's first frame correctly, the simulation results show that the ANCCC scheme outperforms the CCC and NCCC scheme in different channel conditions.

The remainder of this paper is organized as follows. In Section II, we introduce system model of the proposed NCCC scheme. Then the performance analysis of the NCCC scheme is given in Section III. In Section IV, with the CCC and NCCC scheme, we propose an adaptive solution to enhance the system performance and SNR regime. Conclusions are drawn in Section V.

## II. SYSTEM MODEL AND THE PROPOSED NCCC

In this section, we introduce the system model of the NCCC scheme. As illustrated in Fig. 1, there are two users and one destination in the system. The inter-user channels and user-destination channels are modeled as follows

$$y_{i,j} = h_{i,j}x_{i,j} + n_{i,j}, \quad i = 1, 2; j = 1, 2, d; i \neq j \quad (1)$$

where  $h_{i,j}$  denotes the channel fading coefficient,  $x_{i,j}$  and  $y_{i,j}$  are the input and output of the channel, respectively,  $n_{i,j}$  is the corresponding additive white Gaussian noise (AWGN). In (1), we use 1, 2, and  $d$  to denote user 1, user 2, and the destination, respectively. For slow fading channel, the instantaneous SNR is  $\gamma_{i,j} = |h_{i,j}|^2 \Gamma_i$ , where  $\Gamma_i$  is the transmission SNR at user  $i$ . Here, we assume  $\Gamma_1 = \Gamma_2 \triangleq \Gamma_T$ .

The coded cooperation process consists of two frames. In frame 1, each user transmits its own  $N_1$  bits ( $S_i^s$ ,  $i = 1, 2$  for user 1 and user 2, respectively) to the partner and the destination, and receives its partner's information, where the transmissions of different users could be in time division multiple access (TDMA) scheme. If a user cannot decode the partner's information correctly, which is indicated by the cyclic redundancy check (CRC) code, the user will transmit its own  $N_2$  additional parity bits ( $S_i^p$ ,  $i = 1, 2$  for user 1 and user 2, respectively). Otherwise, the user will compute and transmit the combination of its partner's and its own  $N_2$  additional parity bits through network coding such as XOR which is a simple network coding function. As in [7], we define the cooperative level parameter  $\rho = N_1/(N_1 + N_2)$ .

In the coded cooperation process, due to the inter-user channels conditions, there are four possible cooperative cases for the transmission in frame 2, illustrated in Fig. 2. In case 1, both users decode each other correctly. In case 2, neither user decodes

Table 1. The transmission difference among CCC, NCCC, and ANCCC in the frame 2 ( $\oplus$  denotes XOR).

		Case 1	Case 2	Case 3	Case 4
CCC	User 1	$S_2^p$	$S_1^p$	$S_1^p$	$S_2^p$
	User 2	$S_1^p$	$S_2^p$	$S_1^p$	$S_2^p$
NCCC	User 1	$S_1^p \oplus S_2^p$	$S_1^p$	$S_1^p$	$S_1^p \oplus S_2^p$
	User 2	$S_1^p \oplus S_2^p$	$S_2^p$	$S_1^p \oplus S_2^p$	$S_2^p$
ANCCC	User 1	$S_2^p$	$S_1^p$	$S_1^p$	$S_1^p \oplus S_2^p$
	User 2	$S_1^p$	$S_2^p$	$S_1^p \oplus S_2^p$	$S_2^p$

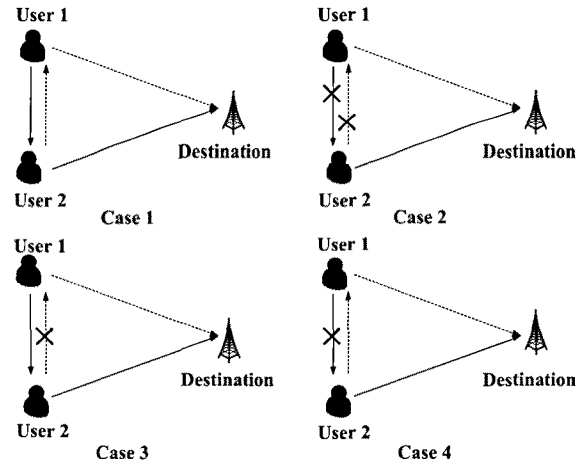


Fig. 2. Four cases of the cooperation.

its partner's first frame successfully. In case 3, user 2 decodes user 1's first frame correctly, but user 1 doesn't decode user 2's first frame successfully. Case 4 is identical to case 3 with roles of user 1 and user 2 reversed. For different cases, the transmission strategies are different for each user. For comparison, we list the transmission strategies of the CCC, our proposed NCCC, and the ANCCC scheme (see Section IV) in Table 1.

## III. PERFORMANCE ANALYSIS OF NCCC

In this paper, we consider the channel capacity outage probability. The channel capacity is expressed by the Shannon formula,  $C(\gamma_{i,j}) = \log_2(1 + \gamma_{i,j})$  bits/s/Hz. An outage event occurs if the capacity falls below the desired rate  $R$ . The corresponding outage event can be expressed as  $\{C(\gamma_{i,j}) < R\}$ . Then the outage probability is

$$\begin{aligned} p &= \Pr(C(\gamma_{i,j}) < R) = \Pr(\gamma_{i,j} < g(R)) \\ &= \int_0^{g(R)} p_{\gamma_{i,j}}(\gamma_{i,j}) d\gamma_{i,j} \end{aligned} \quad (2)$$

where  $g(R) = 2^R - 1$  and  $p_{\gamma_{i,j}}(\gamma_{i,j})$  denotes the probability density function of random variable  $\gamma_{i,j}$ . For the Rayleigh fading case,  $p_{\gamma_{i,j}}(\gamma_{i,j}) = \frac{1}{\Gamma_T \hat{\Gamma}_{i,j}} \exp(-\frac{\gamma_{i,j}}{\Gamma_T \hat{\Gamma}_{i,j}})$ , where  $\hat{\Gamma}_{i,j}$  denotes the average channel power gain over the fading channel, i.e.,  $\hat{\Gamma}_{i,j} = E\{|h_{i,j}|^2\}$ . For simplicity, we define  $\Gamma_{i,j} = \Gamma_T \hat{\Gamma}_{i,j}$ .

$$p_{u1,NCCC}^1 = \Pr(C_{1,2}(\gamma_{1,2}) > R) \Pr(C_{2,1}(\gamma_{2,1}) > R) [\Pr(C_{2,d}^1(\gamma_{2,d}) > R) \Pr(C_{1,d}^1(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^1(\gamma_{2,d}) > R) < R) \\ + \Pr(C_{2,d}^1(\gamma_{2,d}) < R) \Pr(C_{1,d}^1(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^1(\gamma_{2,d}) < R) < R)] \quad (7)$$

$$p_{u1,NCCC}^2 = \Pr(C_{1,2}(\gamma_{1,2}) < R) \Pr(C_{2,1}(\gamma_{2,1}) < R) \Pr(C_{1,d}^2(\gamma_{1,d}) < R) \quad (9)$$

$$p_{u1,NCCC}^3 = \Pr(C_{1,2}(\gamma_{1,2}) > R) \Pr(C_{2,1}(\gamma_{2,1}) < R) [\Pr(C_{2,d}^3(\gamma_{2,d}) > R) \Pr(C_{1,d}^3(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^3(\gamma_{2,d}) > R) < R) \\ + \Pr(C_{2,d}^3(\gamma_{2,d}) < R) \Pr(C_{1,d}^3(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^3(\gamma_{2,d}) < R) < R)] \quad (12)$$

$$p_{u1,NCCC}^4 = \Pr(C_{1,2}(\gamma_{1,2}) < R) \Pr(C_{2,1}(\gamma_{2,1}) > R) [\Pr(C_{2,d}^4(\gamma_{2,d}) > R) \Pr(C_{1,d}^4(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^4(\gamma_{2,d}) > R) < R) \\ + \Pr(C_{2,d}^4(\gamma_{2,d}) < R) \Pr(C_{1,d}^4(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^4(\gamma_{2,d}) < R) < R)] \quad (15)$$

The outage probability for the Rayleigh fading channel can be evaluated as follows

$$p = \int_0^{g(R)} \frac{1}{\Gamma_{i,j}} \exp\left(-\frac{\gamma_{i,j}}{\Gamma_{i,j}}\right) d\gamma_{i,j} = 1 - \exp\left(-\frac{g(R)}{\Gamma_{i,j}}\right). \quad (3)$$

#### A. Outage Analysis of NCCC

According to the four cases discussed in Section II, we analyze the conditional outage events of user 1 for each case as follows.

**Case 1:** Both users decode each other's information correctly. It means the following events will occur

$$\begin{aligned} C_{1,2}(\gamma_{1,2}) &= \rho \log_2(1 + \gamma_{1,2}) > R, \\ C_{2,1}(\gamma_{2,1}) &= \rho \log_2(1 + \gamma_{2,1}) > R \end{aligned} \quad (4)$$

where  $C_{i,j}$  denotes the channel capacity from  $i$  ( $i = 1, 2$ ) to  $j$  ( $j = 1, 2, d; j \neq i$ ). In this case, both users transmit the combination parity bits. For user 1, the outage event should be analyzed in two aspects. On the one hand, if the destination can correctly decode user 2's first frame, the combination bits transmitted by both users can be used to decode the information of user 1. Assuming the signals transmitted by both users in frame 2 can be combined with maximal ratio combining (MRC), and the transmissions of the two frames are parallel channels [7], the corresponding outage events can be written as

$$\begin{aligned} C_{2,d}^1(\gamma_{2,d}) &= \rho \log_2(1 + \gamma_{2,d}) > R, \\ C_{1,d}^1(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^1(\gamma_{2,d}) > R) \\ &= \rho \log_2(1 + \gamma_{1,d}) + (1 - \rho) \log_2(1 + \gamma_{1,d} + \gamma_{2,d}) < R \end{aligned} \quad (5)$$

where the superscript 1 denotes case 1. On the other hand, if the destination cannot decode user 2's first frame successfully, it can only get the information transmitted by user 1 in frame 1. And the outage events are

$$\begin{aligned} C_{2,d}^1(\gamma_{2,d}) &= \rho \log_2(1 + \gamma_{2,d}) < R, \\ C_{1,d}^1(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^1(\gamma_{2,d}) < R) &= \rho \log_2(1 + \gamma_{1,d}) < R. \end{aligned} \quad (6)$$

The outage probability of case 1 is given as (7), where the superscript 1 in  $p_{u1,NCCC}^1$  denotes case 1 and subscript  $u1$  denotes user 1.

**Case 2:** Neither user decodes its partner's first frame correctly. This corresponds to the events  $C_{1,2}(\gamma_{1,2}) < R$  and  $C_{2,1}(\gamma_{2,1}) < R$ . We can get the outage event of user 1 as follows

$$C_{1,d}^2(\gamma_{1,d}) = \log_2(1 + \gamma_{1,d}) < R. \quad (8)$$

The outage probability of case 2 is given as (9).

**Case 3:** User 2 decodes user 1's first frame successfully, but user 1 does not decode user 2's first frame correctly. This happens when  $C_{1,2}(\gamma_{1,2}) > R$  and  $C_{2,1}(\gamma_{2,1}) < R$ . We analyze the outage event of user 1 in two aspects as the case 1 and get the similar outage events as follows

$$\begin{aligned} C_{2,d}^3(\gamma_{2,d}) &= \rho \log_2(1 + \gamma_{2,d}) > R, \\ C_{1,d}^3(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^3(\gamma_{2,d}) > R) &= \max(\log_2(1 + \gamma_{1,d}), \\ &\rho \log_2(1 + \gamma_{1,d}) + (1 - \rho) \log_2(1 + \gamma_{2,d})) < R \end{aligned} \quad (10)$$

and

$$\begin{aligned} C_{2,d}^3(\gamma_{2,d}) &= \rho \log_2(1 + \gamma_{2,d}) < R, \\ C_{1,d}^3(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^3(\gamma_{2,d}) < R) &= \log_2(1 + \gamma_{1,d}) < R. \end{aligned} \quad (11)$$

The outage probability of case 3 is given as (12).

**Case 4:** Case 4 is identical to case 3 with the roles of user 1 and user 2 reversed. The corresponding outage events are  $C_{1,2}(\gamma_{1,2}) < R$  and  $C_{2,1}(\gamma_{2,1}) > R$ . Then the outage events are listed in two aspects

$$\begin{aligned} C_{2,d}^4(\gamma_{2,d}) &= \log_2(1 + \gamma_{2,d}) > R, \\ C_{1,d}^4(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^4(\gamma_{2,d}) > R) &= \log_2(1 + \gamma_{1,d}) < R \end{aligned} \quad (13)$$

and

$$\begin{aligned} C_{2,d}^4(\gamma_{2,d}) &= \log_2(1 + \gamma_{2,d}) < R, \\ C_{1,d}^4(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^4(\gamma_{2,d}) < R) &= \rho \log_2(1 + \gamma_{1,d}) < R. \end{aligned} \quad (14)$$

The outage probability of case 4 is given as (15).

Then the outage probability of user 1 can be calculated as follows

$$p_{u1,NCCC} = p_{u1,NCCC}^1 + p_{u1,NCCC}^2 + p_{u1,NCCC}^3 + p_{u1,NCCC}^4. \quad (16)$$

Due to the symmetry, we can obtain a similar expression for user 2 which reverses the roles of user 1 and user 2. For the sake of simplicity, we only give the results of user 1 in the remainder of the paper.

For the Rayleigh fading case, (16) can be rewritten as follows [cf. (3)]

$$\begin{aligned}
p_{u1, \text{NCCC}} &= A_1 A_2 ((1 - B_2)(1 - B_1) + F_1) \\
&\quad + (1 - A_1)(1 - A_2)(1 - C_1) \\
&\quad + A_1(1 - A_2)((1 - B_2)(1 - C_1) + F_2) \\
&\quad + (1 - A_1)A_2((1 - C_2)(1 - B_1) + C_2(1 - C_1))
\end{aligned} \quad (17)$$

where

$$\begin{aligned}
A_1 &= \exp\left(-\frac{g(R/\rho)}{\Gamma_{1,2}}\right), \quad A_2 = \exp\left(-\frac{g(R/\rho)}{\Gamma_{2,1}}\right), \\
B_1 &= \exp\left(-\frac{g(R/\rho)}{\Gamma_{1,d}}\right), \quad B_2 = \exp\left(-\frac{g(R/\rho)}{\Gamma_{2,d}}\right), \\
C_1 &= \exp\left(-\frac{g(R)}{\Gamma_{1,d}}\right), \quad C_2 = \exp\left(-\frac{g(R)}{\Gamma_{2,d}}\right).
\end{aligned} \quad (18)$$

$F_1$  and  $F_2$  can be expressed by (19) and (20),

$$\begin{aligned}
F_1 &= \Pr(C_{2,d}^1(\gamma_{2,d}) > R) \\
&\quad \times \Pr(C_{1,d}^1(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^1(\gamma_{2,d}) > R) < R) \\
&= \iint_{\text{Area1}} \frac{1}{\Gamma_{1,d}} \exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}\right) \frac{1}{\Gamma_{2,d}} \exp\left(-\frac{\gamma_{2,d}}{\Gamma_{2,d}}\right) d\gamma_{1,d} d\gamma_{2,d},
\end{aligned} \quad (19)$$

$$\begin{aligned}
F_2 &= \Pr(C_{2,d}^3(\gamma_{2,d}) > R) \\
&\quad \times \Pr(C_{1,d}^3(\gamma_{1,d}, \gamma_{2,d} | C_{2,d}^3(\gamma_{2,d}) > R) < R) \\
&= \iint_{\text{Area2}} \frac{1}{\Gamma_{1,d}} \exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}\right) \frac{1}{\Gamma_{2,d}} \exp\left(-\frac{\gamma_{2,d}}{\Gamma_{2,d}}\right) d\gamma_{1,d} d\gamma_{2,d}
\end{aligned} \quad (20)$$

where

$$\begin{aligned}
\text{Area1} &\equiv \{(\gamma_{1,d}, \gamma_{2,d}) \mid \rho \log_2(1 + \gamma_{2,d}) > R, \\
&\quad \rho \log_2(1 + \gamma_{1,d}) + (1 - \rho) \log_2(1 + \gamma_{1,d} + \gamma_{2,d}) < R\},
\end{aligned} \quad (21)$$

$$\begin{aligned}
\text{Area2} &\equiv \{(\gamma_{1,d}, \gamma_{2,d}) \mid \rho \log_2(1 + \gamma_{2,d}) > R, \\
&\quad \rho \log_2(1 + \gamma_{1,d}) + (1 - \rho) \log_2(1 + \gamma_{1,d}) < R, \\
&\quad \rho \log_2(1 + \gamma_{1,d}) + (1 - \rho) \log_2(1 + \gamma_{2,d}) < R\}.
\end{aligned} \quad (22)$$

Through the results of Appendix A, we can get the outage probability of NCCC.

### B. Diversity Order Analysis of NCCC

In this subsection, we derive the approximate expression of outage probability in high SNR regime to analyze the diversity order of the NCCC scheme.

With the approach in Appendix B in [7], we can get the approximate outage probability of the NCCC scheme as follows

$$\begin{aligned}
p_{u1, \text{NCCC}} &= \frac{(g(R/\rho))^2}{\Gamma_T^2 \hat{\Gamma}_{1,d} \hat{\Gamma}_{2,d}} + \frac{g(R/\rho)g(R)}{\Gamma_T^2 \hat{\Gamma}_{1,2} \hat{\Gamma}_{1,d}} \\
&\quad + \frac{\Theta(R, \rho) - \tilde{\gamma}_1 g(R/\rho)}{\Gamma_T^2 \hat{\Gamma}_{1,d} \hat{\Gamma}_{2,d}} + O\left(\frac{1}{\Gamma_T^3}\right)
\end{aligned} \quad (23)$$

where

$$\begin{aligned}
\Theta(R, \rho) &= \\
&\begin{cases} \frac{1 - \rho}{1 - 2\rho} \left( (1 + \tilde{\gamma}_1)^{\frac{1-2\rho}{1-\rho}} 2^{R/(1-\rho)} - 2^{R/(1-\rho)} \right) - \tilde{\gamma}_1 - \frac{\tilde{\gamma}_1^2}{2}, & \rho \neq 1/2, \\ 2^{2R} \ln(1 + \tilde{\gamma}_1) - \tilde{\gamma}_1 - \frac{\tilde{\gamma}_1^2}{2}, & \rho = 1/2 \end{cases}
\end{aligned} \quad (24)$$

where  $\tilde{\gamma}_1$  is the root of equation (34). In (23),  $O(\frac{1}{\Gamma_T^3})$  denotes high order terms in from the Taylor's series. From (23), we can see that when  $\Gamma_T \rightarrow \infty$ , the outage probability is proportional to  $\Gamma_T^{-2}$ , which means that the diversity order is 2. Because there are only two users, full diversity order is achieved.

### C. Numerical Results and Discussions

In this subsection, we use Monte-Carlo simulation to evaluate the performance of our proposed NCCC scheme.

Fig. 3 compares the outage probabilities of the NCCC and CCC scheme [7]. Both the simulation results (dashed) and analytical results (solid) are drawn. We also show the results of no cooperation [7] as a reference. We consider a symmetric case where  $\Gamma_{1,2} = \Gamma_{2,1}$  and  $\Gamma_{1,d} = \Gamma_{2,d}$ . The system outage probability is defined as the average outage probabilities of two users, i.e.,  $p_{\text{sys}} = (p_{u1} + p_{u2})/2$ . The cooperative level  $\rho = 0.5$ . The desired rate  $R = 1$  bit/s/Hz. We depict the results of three scenarios:

- The inter-user channels are worse than the user-destination channels, i.e.,  $\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d} - 10$  dB ( $\Gamma_{1,d} = \Gamma_{2,d}$ ).
- The inter-user channels are similar with the user-destination channels, i.e.,  $\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d}$  ( $\Gamma_{1,d} = \Gamma_{2,d}$ ).
- The inter-user channels are better than the user-destination channels, i.e.,  $\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d} + 10$  dB ( $\Gamma_{1,d} = \Gamma_{2,d}$ ).

From the results, in scenario (a), the NCCC scheme outperforms the CCC scheme and the gain is about 2 dB. We see that in scenario (b), the performance of the two schemes are nearly the same. In scenario (c), the NCCC scheme is inferior to the CCC scheme. It is similar to results in [12] where the network-coding cooperation scheme was superior to the coded cooperation scheme in the poor inter-user channel. Moreover, the NCCC scheme is always superior to no cooperation in all scenarios, while the performance of the CCC scheme is worse than that of no cooperation when the inter-user channels are poor, i.e., in scenario (a). It is shown that the performance of the NCCC scheme is more stable than CCC scheme. Also, our proposed NCCC scheme can be more tolerant to the poor inter-user channels. Moreover, we can see that the analytical results and simulation results are nearly similar.

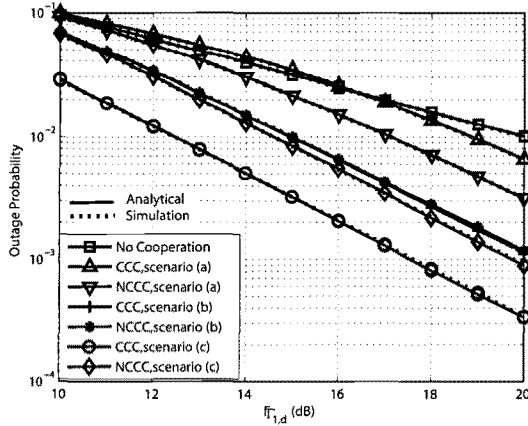


Fig. 3. The outage probability comparison of the NCCC scheme and the CCC scheme in three scenarios.

To analyze why system performance of the NCCC scheme is inferior under scenario (c), we draw the outage probability of each case in Fig. 4. From the results, it is shown that  $p_{\text{sys,NCCC}}^1 > p_{\text{sys,CCC}}^1$ ,  $p_{\text{sys,NCCC}}^2 = p_{\text{sys,CCC}}^2$ ,  $p_{\text{sys,NCCC}}^3 < p_{\text{sys,CCC}}^3$ , and  $p_{\text{sys,NCCC}}^4 < p_{\text{sys,CCC}}^4$ . And  $p_{\text{sys,NCCC}}^2 = p_{\text{sys,CCC}}^2$  is due to the same transmission strategies of the two schemes in the second frame in case 2. In case 1, there are *two* parallel channels for user 1 in the CCC scheme [7]. In the NCCC scheme, if  $C_{2,d}^1(\gamma_{2,d}) > R$ , it is found that user 1 has *one* channel in frame 1 and *two* channels in frame 2 from (5). But if  $C_{2,d}^1(\gamma_{2,d}) < R$ , it can be seen that user 1 has *only one* channel from (6). We can see that the outage event is more easily to occur if  $C_{2,d}^1(\gamma_{2,d}) < R$ , so the second part of the outage probability  $\Pr(C_{2,d}^3(\gamma_{2,d}) < R)\Pr(C_{1,d}^3(\gamma_{1,d}, \gamma_{2,d})|C_{2,d}^3(\gamma_{2,d}) < R) < R$  is dominant. Similar analysis can be given for user 2. Then  $p_{\text{sys,NCCC}}^1 > p_{\text{sys,CCC}}^1$ , where  $p_{\text{sys,NCCC}}^1$  and  $p_{\text{sys,CCC}}^1$  denote system outage probability of the NCCC scheme and the CCC scheme in case 1, respectively. In case 3 of the CCC scheme, for user 1, there are *one* channel in frame 1 and *two* channels in frame 2. For user 2, there is *only one* channel in frame 1. So the destination decodes the information of user 1 easily. The outage probability of user 2 is dominant in the system outage probability of the CCC scheme. In case 3 of the NCCC scheme, there are *at least two* parallel channels for user 1, and if the destination decodes user 1 correctly, there are two parallel channels for user 2. So in this case,  $p_{1,\text{NCCC}}^3$  is higher than  $p_{1,\text{CCC}}^3$ , but  $p_{2,\text{NCCC}}^3$  is much lower than  $p_{2,\text{CCC}}^3$ . It means that  $p_{\text{sys,NCCC}}^3 < p_{\text{sys,CCC}}^3$ . The case 4 is identical with case 3 with the roles of user 1 and user 2 reversed.

Through the results in Fig. 4, we can explain the results in Fig. 3. The better the inter-user channels are, the higher the probability of case 1 is. Then the outage probability of case 1 of the CCC scheme is much lower than the NCCC scheme. Although the outage probability of case 3 and case 4 of the CCC scheme is higher than the NCCC scheme, the total outage probability of the CCC scheme is lower than the NCCC scheme. When the inter-user channels are poor, the analysis process is similar. When the inter-user channels are similar to the user-destination channels, for the NCCC scheme, the performance gain in case

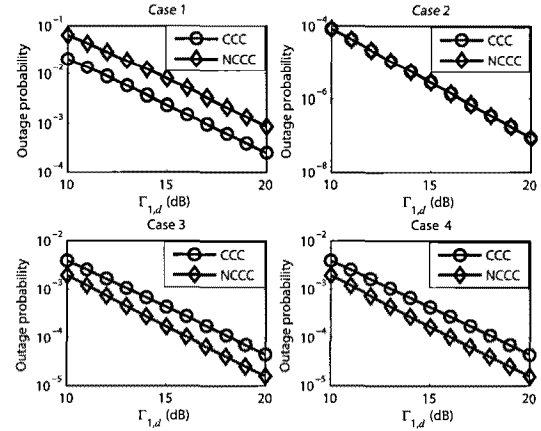


Fig. 4. The outage probability of 4 cases comparison between the NCCC scheme and the CCC scheme, where  $\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d} + 10$  dB ( $\Gamma_{1,d} = \Gamma_{2,d}$ ).

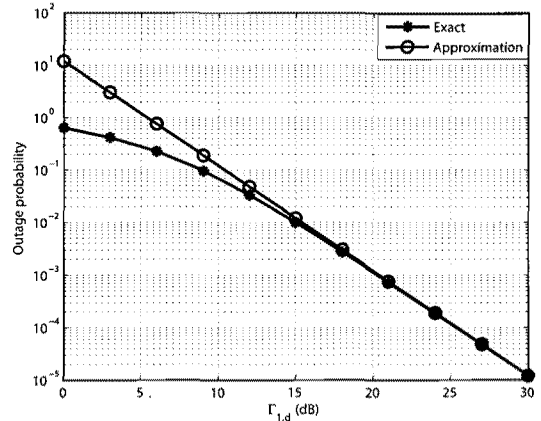


Fig. 5. Comparison of exact and approximate outage probability of the NCCC scheme where  $\rho = 0.5$ .

3 and case 4 is nearly the same as the performance loss in case 1 versus the CCC scheme, so the total outage probability of the NCCC scheme is similar to the CCC scheme.

The comparison of the exact and approximate outage probabilities of the NCCC scheme in high SNR regime is shown in Fig. 5. The cooperative level and desired rate are the same as Fig. 3. The channel condition is that all the channels are the same, i.e.,  $\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d} = \Gamma_{2,d}$ . From the results in Fig. 5, we can see that the high SNR approximate outage probability is nearly the same as the exact outage probability when the SNR is high enough such as  $\Gamma_{1,d} > 15$  dB.

#### IV. ADAPTIVE NETWORK-CODING-BASED CODED COOPERATION

With the analysis and numerical results in the above section, we propose an adaptive scheme to enhance the system performance in all channel conditions in this section.

### A. Transmission Strategies in Frame 2

In the ANCCC scheme, the process of frame 1 is the same as the CCC and NCCC scheme. While in frame 2, when one user decodes its partner's first frame, it will transmit one ACK/NACK bit to its partner to inform the partner whether it decodes information correctly. And at the same time each user listens to the ACK/NACK bits from its partner. If it decodes the partner's information incorrectly, it only transmits its own parity bits. If it finds that both its partner and itself decode each other correctly (case 1), it will transmit the additional parity bits for its partner which is the same as the CCC scheme. If it finds that only it decodes information of its partner correctly (i.e., case 4 for user 1), it will use the same transmission strategy as the NCCC scheme. The difference of transmission strategies among the three schemes in frame 2 is depicted in Table 1.

Noted that the ACK/NACK bits also can be received by the destination, so the destination can distinguish the transmission strategies in frame 2 for each user.

Obviously, the ANCCC scheme needs one ACK/NACK bit for each user. We will assume that the feedback channel is error-free in this paper.

### B. Outage Analysis of ANCCC

According to the outage analysis of the NCCC scheme in subsection III-B, we analyze the outage probability of the ANCCC scheme in a similar way.

**Case 1:** Both users decode each other correctly and it means  $C_{1,2}(\gamma_{1,2}) > R$  and  $C_{2,1}(\gamma_{2,1}) > R$  will occur. For user 1, follow the analysis in [7], we can get the outage event is  $\{\rho \log_2(1 + \gamma_{1,d}) + (1 - \rho) \log_2(1 + \gamma_{2,d}) < R\}$  and the corresponding outage probability is

$$p_{u1,ANCCC}^1 = \Pr(C_{1,2}(\gamma_{1,2}) > R) \Pr(C_{2,1}(\gamma_{2,1}) > R) \\ \times \Pr(\rho \log_2(1 + \gamma_{1,d}) + (1 - \rho) \log_2(1 + \gamma_{2,d}) < R). \quad (25)$$

**Case 2, case 3, and case 4** are the same as the corresponding cases in the NCCC scheme, so the outage events are the same as the NCCC scheme.

Then the outage probability of user 1 is given as

$$p_{u1,ANCCC} \\ = p_{u1,ANCCC}^1 + p_{u1,ANCCC}^2 + p_{u1,ANCCC}^3 + p_{u1,ANCCC}^4 \\ = p_{u1,ANCCC}^1 + p_{u1,NCCC}^2 + p_{u1,NCCC}^3 + p_{u1,NCCC}^4. \quad (26)$$

For the Rayleigh fading case, (26) can be rewritten as

$$p_{u1,ANCCC} = A_1 A_2 \mu + (1 - A_1)(1 - A_2)(1 - C_1) \\ + A_1(1 - A_2)((1 - B_2)(1 - C_1) + F_2) \\ + (1 - A_1)A_2((1 - C_2)(1 - B_1) + C_2(1 - C_1)) \quad (27)$$

where  $A_1, A_2, B_1, B_2, C_1, C_2$  and  $F_2$  are given by (18) and (39), and  $\mu$  can be expressed as (28) with the result in [7]

$$\mu = 1 - B_1 - \int_0^{g(R/\rho)} \frac{1}{\Gamma_{1,d}} \exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}} - \frac{a}{\Gamma_{2,d}}\right) d\gamma_{1,d} \quad (28)$$

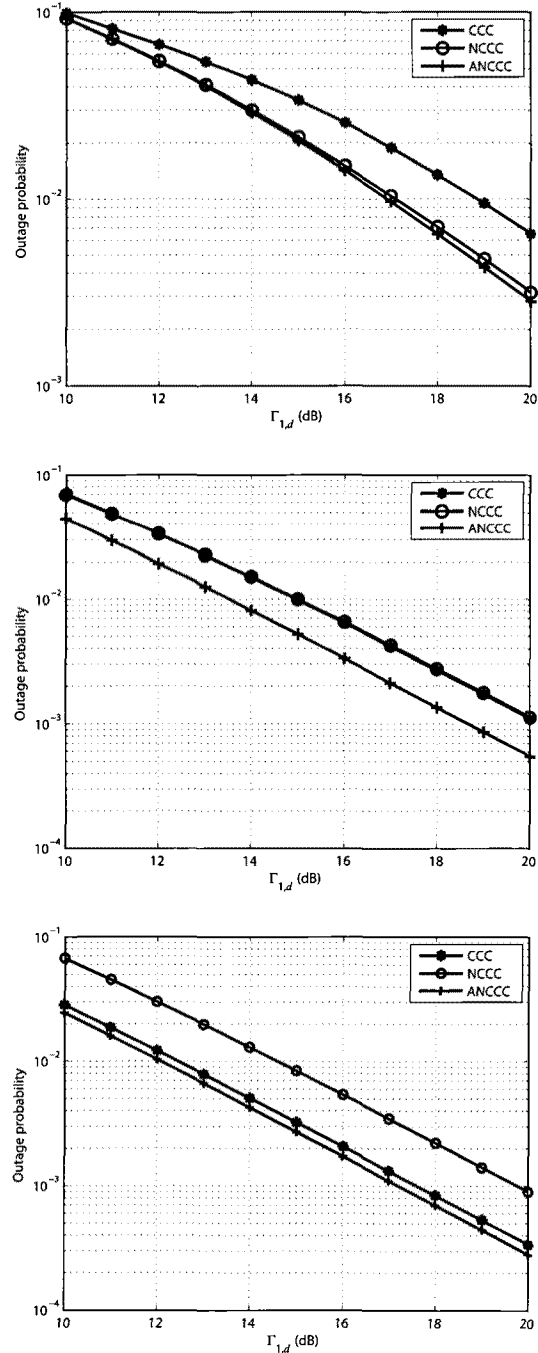


Fig. 6. Comparison of outage probability among the three schemes: (a)  $\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d} - 10$  dB ( $\Gamma_{1,d} = \Gamma_{2,d}$ ), (b)  $\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d}$  ( $\Gamma_{1,d} = \Gamma_{2,d}$ ), and (c)  $\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d} + 10$  dB ( $\Gamma_{1,d} = \Gamma_{2,d}$ ).

in which  $a$  is given by (36).

We use Monte-Carlo simulation to illustrate the outage performance of the ANCCC scheme. In Fig. 6, the simulation condition and channel scenarios are the same as Fig. 3. In scenario (a), when the outage probability is  $10^{-2}$ , the ANCCC scheme can get about 2 dB gain over the CCC scheme, and it is a little better than the NCCC scheme. In this scenario, because the inter-user channel is worse, so the probability of case 1 is low.

The transmission strategy of the ANCCC scheme is the same as the NCCC scheme in case 2, case 3, and case 4, so the outage performance of the ANCCC scheme is only a little better than the NCCC scheme. It is shown that in scenario (b), the performance of the ANCCC scheme is about 1.5 dB better than the CCC scheme and the NCCC scheme when the outage probability is  $10^{-2}$ . This is due to the fact that  $p_{\text{sys,ANCCC}}^1 < p_{\text{sys,NCCC}}^1$  and  $p_{\text{sys,ANCCC}}^3 < p_{\text{sys,CCC}}^3$  ( $p_{\text{sys,ANCCC}}^4 < p_{\text{sys,CCC}}^4$ ). In scenario (c), the ANCCC scheme gets about 2 dB gain over the NCCC scheme and a little gain over the CCC scheme when the outage probability is  $10^{-2}$ . The reason is identical to that of scenario (a) with the role of the CCC scheme and the NCCC scheme reversed. In summary, we can see that the outage performance of the ANCCC scheme is always the best.

### C. Diversity Order Analysis of ANCCC

We derive the approximate expression of outage probability of the ANCCC scheme in high SNR regime in this subsection.

Such as the analysis in subsection III-B, it is easy to get the approximate outage probability of the ANCCC scheme as follows

$$p_{u1,\text{ANCCC}} = \frac{\Lambda(R, \rho)}{\Gamma_T^2 \hat{\Gamma}_{1,d} \hat{\Gamma}_{2,d}} + \frac{g(R/\rho)g(R)}{\Gamma_T^2 \hat{\Gamma}_{1,2} \hat{\Gamma}_{1,d}} + O\left(\frac{1}{\Gamma_T^3}\right) \quad (29)$$

where

$$\Lambda(R, \rho) = \begin{cases} \left(\frac{\rho}{1-2\rho}\right) 2^{R/\rho} - \left(\frac{1-\rho}{1-2\rho}\right) 2^{R/(1-\rho)} + 1, & \rho \neq 1/2, \\ R2^{2R+1} \ln 2 - 2^{2R} + 1, & \rho = 1/2. \end{cases} \quad (30)$$

From (29), we can see that the diversity order is the same as the NCCC scheme which is the full diversity order.

The comparison of the exact and approximate outage probabilities of the ANCCC scheme in high SNR regime is shown in Fig. 7. The simulation condition is the same as Fig. 5. It is shown that the approximate outage probability in high SNR is nearly the same as the exact outage probability when the SNR is high enough such as  $\Gamma_{1,d} > 15$  dB.

In Fig. 8, we compare the approximate outage probability in high SNR regime of different schemes under the same channel condition as Fig. 5 and the cooperative level  $\rho = 0.5$ . And the approximate outage probability of CCC scheme is obtained by (20) in [7]. The result shows that the performance of the ANCCC scheme is about 2 dB better than the CCC and NCCC scheme.

## V. CONCLUSION

In this paper, we have proposed a NCCC scheme which transmits combination of two users' parity bits instead of only its partner's parity bits in the CCC scheme during cooperation. Based on the analysis of the performance of the NCCC scheme, we have presented an adaptive scheme-the ANCCC scheme, which combines the advantage of two schemes. The outage probability of the ANCCC scheme has been derived

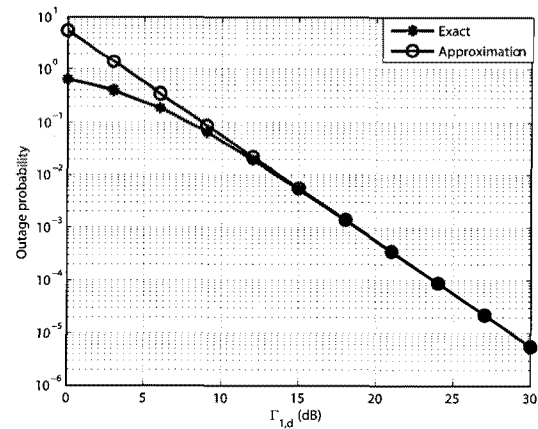


Fig. 7. Comparison of exact and approximate outage probability in high SNR regime where  $\rho = 0.5$ .

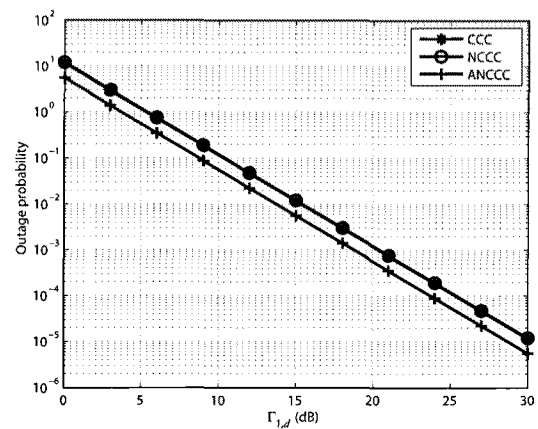


Fig. 8. Comparison of approximate outage probability in high SNR regime among different schemes where  $\rho = 0.5$ .

and simulation results have showed that outage performance of the ANCCC scheme was better than the other two schemes in all channel conditions. However, the ANCCC scheme needs one ACK/NACK bit. In future work, we will pursue alternative schemes to enhance the system performance over all channel conditions without the additional price.

## APPENDIX

### A. Derivation of the Outage Probability of the NCCC Scheme

Considering constraint Area1 in (21) we can obtain the following two inequalities

$$\gamma_{2,d} > 2^{R/\rho} - 1 = g(R/\rho), \quad (31)$$

$$\gamma_{2,d} < \frac{2^{R/(1-\rho)}}{(1 + \gamma_{1,d})^{\rho/(1-\rho)}} - 1 - \gamma_{1,d} \triangleq b. \quad (32)$$

So we can get the following requirement

$$g(R/\rho) < b. \quad (33)$$

From (32), we know that  $b$  is monotonically decreasing in  $\gamma_{1,d}$ . Let  $\tilde{\gamma}_1$  be the root of the follow equation

$$g(R/\rho) = b. \quad (34)$$

If  $\gamma_{1,d} < \tilde{\gamma}_1$ , then (33) can be satisfied.

Then we can rewrite (19) as

$$\begin{aligned} F_1 &= \int_0^{\tilde{\gamma}_1} \int_{g(R/\rho)}^b \frac{\exp(-\frac{\gamma_{2,d}}{\Gamma_{2,d}})}{\Gamma_{2,d}} d\gamma_{2,d} \frac{\exp(-\frac{\gamma_{1,d}}{\Gamma_{1,d}})}{\Gamma_{1,d}} d\gamma_{1,d} \\ &= B_2 \left( 1 - \exp(-\frac{\tilde{\gamma}_1}{\Gamma_{1,d}}) \right) - \int_0^{\tilde{\gamma}_1} \frac{\exp(-\frac{\gamma_{1,d}}{\Gamma_{1,d}} - \frac{b}{\Gamma_{2,d}})}{\Gamma_{1,d}} d\gamma_{1,d}. \end{aligned} \quad (35)$$

Next, considering the constraint Area2 in (22) we can get follow constraints

$$\begin{aligned} \gamma_{2,d} &> 2^{R/\rho} - 1 = g(R/\rho), \\ \gamma_{1,d} &< 2^R - 1 = g(R), \\ \gamma_{2,d} &< \frac{2^{R/(1-\rho)}}{(1 + \gamma_{1,d})^{\rho/(1-\rho)}} - 1 \triangleq a. \end{aligned} \quad (36)$$

From (36), we can obtain

$$g(R/\rho) < a. \quad (37)$$

Equation (37) can be rewritten as follows

$$\gamma_{1,d} < 2^{R(2\rho-1)/\rho^2} - 1 = g(R(2\rho-1)/\rho^2). \quad (38)$$

Note that  $\gamma_{1,d} > 0$ , which means that  $2^{R(2\rho-1)/\rho^2} - 1 > 0$ , so  $\rho > 1/2$ . Then if  $\rho \leq 1/2$ ,  $F_2 = 0$ . If  $\rho > 1/2$ , then  $2^{R(2\rho-1)/\rho^2} - 1 < 2^R - 1$ , (20) can be rewritten as

$$\begin{aligned} F_2 &= \int_0^{g(R(2\rho-1)/\rho^2)} \int_a^{g(R/\rho)} \frac{\exp(-\frac{\gamma_{2,d}}{\Gamma_{2,d}})}{\Gamma_{2,d}} d\gamma_{2,d} \frac{\exp(-\frac{\gamma_{1,d}}{\Gamma_{1,d}})}{\Gamma_{1,d}} d\gamma_{1,d} \\ &= B_2(1 - D_1) - \int_0^{g(R(2\rho-1)/\rho^2)} \frac{\exp(-\frac{\gamma_{1,d}}{\Gamma_{1,d}} - \frac{a}{\Gamma_{2,d}})}{\Gamma_{1,d}} d\gamma_{1,d} \end{aligned} \quad (39)$$

where  $D_1$  is defined as follows

$$D_1 = \exp\left(-\frac{g(R(2\rho-1)/\rho^2)}{\Gamma_{1,d}}\right). \quad (40)$$

So  $F_2$  can be expressed by

$$F_2 = \begin{cases} 0, & \rho \leq 1/2, \\ B_2(1 - D_1) - \int_0^{g(R(2\rho-1)/\rho^2)} \frac{\exp(-\frac{\gamma_{1,d}}{\Gamma_{1,d}} - \frac{a}{\Gamma_{2,d}})}{\Gamma_{1,d}} d\gamma_{1,d}, & \text{otherwise.} \end{cases} \quad (41)$$

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