A Novel Multiple Access Scheme via Compressed Sensing with Random Data Traffic

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Abstract: **The problem of compressed sensing (CS) based multiple access is studied under the assumption of random data traffic. In many multiple access systems, i.e., wireless sensor networks (WSNs), data arrival is random due to the bursty data traffic for every transmitter. Following the recently developed CS methodology, the technique of compressing the transmitter identities into data transmissions is proposed, such that it is unnecessary for a transmitter to inform the base station its identity and its request to transmit. The proposed compressed multiple access scheme identifies transmitters and recovers data symbols jointly. Numerical simulations demonstrate that, compared with traditional multiple access approaches like carrier sense multiple access (CSMA), the proposed CS based scheme achieves better expectation and variance of packet delays when the traffic load is not too small.**

Index Terms: **Bursty traffic, compressed sensing (CS), multiple access.**

I. INTRODUCTION

In many wireless communication systems, e.g., wireless sensor networks or cellular networks, multiple transmitters need to transmit their data to a base station, thus requiring the technique of multiple access, such as time division multiple access (TDMA), code division multiple access (CDMA), or orthogonal frequency division multiple access (OFDMA). We suppose that the channels are vectorized, either in frequency or in time, and assume that the dimension of the vector channel is smaller than the number of transmitters. Quite often, the data traffic at a transmitter is bursty, i.e., in one time slot, only a portion of the transmitters have data to transmit. For example, in a sensor network, there could be hundreds of sensors associated with one base station; however, in many applications, there are only several sensors reporting to the base station simultaneously. Therefore, the base station needs to know the identities of the active transmitters. Moreover, the data packet could be very small, e.g., just a record of local temperature. Thus the identity information may cause significant overhead. The identification problem could be solved using the following three different ways:

1. Add the identity information into data packets explicitly, i.e., adding an identity field in the packet header. If the receiver can decode a data packet, it can determine the owner of the packet.
2. Set a preamble before each data transmission. In this preamble, active transmitters send out requests containing their identity information.

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Table 1. Typical multiple access approaches.

Typical multiple access approaches		
Category	Identifying method	Cost
1	Decode packet header	Identity field
2	Send request in preamble	Preamble period
3	Assign different signature waveforms	Signature waveforms and their projection

3. Similar to CDMA systems, assign different signature waveforms to different transmitters and project the received signal onto all signature waveforms. Only transmitters with sufficiently large projections are considered as being active.

However, all three approaches have drawbacks. Approach 1 includes overhead to the data packet. In approach 2, it may require a long preamble if the requests of different transmitters are kept orthogonal, when the number of transmitters is large. If the orthogonality constraint in the preamble is removed (e.g., using contention based multiple access), there could be collisions of request signals. In approach 3, when the dimension of the vector channel is smaller than the number of transmitters (like an overloaded CDMA system), the signature waveforms cannot be orthogonal; therefore it is difficult to determine a threshold for the active user selection.

Besides the identification problem, multiple access scheme, i.e., how to separate the signals from different transmitters, is also an important problem. In approach 2, the receiver can allocate different time slots to the transmitters in a TDMA fashion. However, the feedback signaling of time slot allocation induces overhead to the system. In approach 3, CDMA can be used to separate the signals from different transmitters; whereas it suffers from multiuser interference when nonorthogonal spreading codes are used.

In this paper, we tackle the multiple access problem by employing the compressed sensing [1]–[3], a signal processing technique developed in recent years. Based on the assumption that the signal is sparse, i.e., most elements of the signal in a transformation domain are zero or have small amplitudes, compressed sensing reconstructs original signal from observations. Efficient algorithms like basis pursuit (BP) [4], [5], orthogonal matching pursuit (OMP) [6], [7], and stagewise OMP (StOMP) [8] have been proposed and applied in fields like data compression [9], sensor networks, and image processing [12].

It is easy to find the analogy between the multiple access and compressed sensing since the received signal at base station is also given by $\mathbf{\Phi x}$, where $\mathbf{x}$ is the vector of transmitted signal and the columns in $\mathbf{\Phi}$ are the signature waveforms of different transmitters. Therefore, we can allow the transmitters having data to transmit directly without the stage of request in approach 2. When there is no noise, the equation can be perfectly solved, thus avoiding the threshold in approach 3. The identities of the active transmitters are simply a by-product of the solution, i.e., the locations of the non-zero elements in $\mathbf{x}$, thus avoiding the overhead of explicit identity in approach 1. We coin the scheme proposed in this paper *Compressed Multiple Access* since the identity information is "compressed" into the data transmission. Moreover, the sparsity required by compressed sensing is assured by the assumption that most transmitters do not have data to transmit. Therefore, *the identification and multiple access problems are solved jointly.* Numerical simulation results will show that, compared with the traditional carrier sense multiple access (CSMA), the proposed multiple access scheme achieves better performance for the expectation and variance of packet delays when the traffic load is not too small.

The remainder of this paper is organized as follows. In Section II, we present our system model. The design of the compressed multiple access is presented in Section III, followed by the simulation based performance evaluation in Section IV. The conclusions are provided in Section V.

The following mathematical notations are used throughout the paper.

- $\circ$ denotes Hadamard product. For two matrices $\mathbf{A}$ and $\mathbf{B}$ having the same size, $(\mathbf{A} \circ \mathbf{B})_{ij} = A_{ij}B_{ij}$.
- For matrix $\mathbf{A}$, $\mathbf{A}^T$ means the transpose of $\mathbf{A}$.
- For an n-vector $\mathbf{x}$, its 1-norm equals $\sum_{k=1}^{n} |x_k|$ and its 0-norm means the number of nonzero elements.

II. SYSTEM MODEL

In a wireless system, suppose that a base station receives signals from m transmitters (e.g., sensors in wireless sensor networks or mobile phones in cellular systems) via vector channels of dimension n. In general case, the vector channel could be in either time or frequency. In this paper, we assume that the vector is in frequency, i.e., each dimension corresponds to a subcarrier in the frequency domain. For simplicity, we assume that the received signal is real. It is straightforward to extend the real signal to complex signal case.

At time slot t, the received signal is given by an n-vector:

$$\mathbf{r}(t) = \sum_{k=1}^{m}[x_k(t)\mathbf{h}_k \circ \mathbf{s}_k(t)] + \mathbf{n}(t) \quad (1)$$

where $\mathbf{h}_k$ is the vector of channel amplitude gains of transmitter k with upper bound $h_{\max}$ and lower bound $h_{\min}$, $x_k(t)$ is the information symbol of transmitter k. $\mathbf{s}_k(t)$ is the vector of signature waveforms of transmitter k at time slot t. $\mathbf{n}(t)$ is the received noise vector at time slot t. We also place the following assumptions on the system model.

1. We assume that a transmitter does not always have data to transmit. The data burst is random which means that averagely $\rho m (0 < \rho < 1)$ transmitters generate a new data to transmit at a time slot. When transmitter k has no data, it does not transmit, namely $x_k = 0$. We also assume that the receiver does not have *a priori* information about which transmitters have data.
2. The channel gain vector $\mathbf{h}_k$ does not change in time. The receiver knows the channel gains perfectly by letting the transmitter send out pilot signals periodically. However, the transmitters do not know the channel gains perfectly.
3. Equation (1) implicitly assumes that the transmitters are perfectly synchronized in time. This assumption will be addressed in details and relaxed later.
4. We assume that $\mathbf{s}_k(t)$, the signature waveform of transmitter k, is a vector randomly chosen on the unit sphere in $\mathbf{R}^n$. The signature waveforms are known at both the transmitters and receiver.
5. A buffer is used for each transmitter to store untransmitted data packets.

Applying the assumption on signature waveforms, we can rewrite (1) as

$$\mathbf{r}(t) = \mathbf{\Phi}(t)\mathbf{x}(t) + \mathbf{n}(t) \quad (2)$$

where

$$\mathbf{x}(t) \triangleq (x_1(t), \cdots, x_m(t))^T \quad (3)$$

and

$$\mathbf{\Phi} = \mathbf{H} \circ \mathbf{S}(t) \quad (4)$$

where $\mathbf{S}(t) \triangleq (\mathbf{s}_1(t), \cdots, \mathbf{s}_m(t))$ and $\mathbf{H} \triangleq (\mathbf{h}_1, \cdots, \mathbf{h}_m)$.

When there are multiple time slots, say from time slot 1 to time slot t, during which the transmitted symbols do not change (will be called a frame later), we can stack the received signals together and obtain the following expression

$$\mathbf{r}(1:t) = \mathbf{\Phi}(1:t)\mathbf{x}(1) + \mathbf{n}(1:t) \quad (5)$$

where

$$\mathbf{r}(1:t) = \left(\mathbf{r}(1)^T, \cdots, \mathbf{r}(t)^T\right)^T, \quad (6)$$

$$\mathbf{n}(1:t) = \left(\mathbf{n}(1)^T, \cdots, \mathbf{n}(t)^T\right)^T, \quad (7)$$

and

$$\mathbf{\Phi}(1:t) = \left(\mathbf{H}^T, \cdots, \mathbf{H}^T\right)^T \circ \mathbf{S}(1:t) \quad (8)$$

and $\mathbf{S}(1:t) \triangleq (\mathbf{S}(1)^T, \cdots, \mathbf{S}(t)^T)^T$.

III. COMPRESSED MULTIPLE ACCESS

In this section, we propose a novel scheme of multiple access based on compressed sensing. We first explain the procedure of the multiple access. Then, we provide an illustrative example, as well as a proposition about the equivalence between 0-norm and 1-norm optimization conditioned on the signal sparsity.

A. *Procedure of Multiple Access*

A.1 Frame Structure

In contrast to conventional multiple access approaches (e.g., CSMA/CA), the proposed compressed multiple access scheme encourages transmission collisions. Actually, every single measurement is a mixture of received information symbols modulated by their own channel gains and signature waveforms plus noise. As illustrated in Fig.1, we define a *frame* with varying length (different numbers of time slots). A frame ends only when sufficiently many measurements have been obtained and two adjacent reconstructions generate the same results [13].

Here, we assume that a computationally efficient reconstructor, which can reconstruct the original signals exactly when enough samples have been received, is equipped at the base station. Therefore, multiple access is now possible using the smallest number of time slots, and without any *a priori* knowledge about how many transmitters are transmitting data in the current frame.

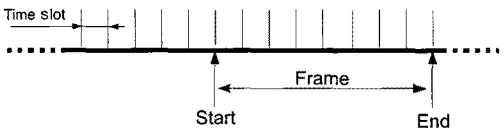

Fig. 1. Frame with varying length.

A.2 Data Transmission

In each frame, the number of transmitters allowed to transmit data is determined at the beginning of the frame and then fixed throughout the entire frame. Before starting a new frame, the base station broadcasts a very short beacon signal, e.g., a sinusoid, indicating the start of a new frame. Sensors having data in their buffers are legal for transmitting in the new frame. In the entire frame, sensors keep transmitting the same data of their own and only change signature waveforms $\mathbf{s}_k(t)$ in each time slot. If a new data is generated in the middle of a frame, then the transmitter will put the newly generated data into its buffer, and forms a first-in-first-out queue for the data waiting to be transmitted. When the base station finds that it has received sufficiently many observations and is able to distinguish the signals from different transmitters, it sends out a beacon signal to indicate the end of the current frame. Thus, the transmitters stop transmitting the current data. At this time, the received signal at the base station is given by (5) and the base station uses compressed sensing algorithm, e.g., OMP, to reconstruct the signals from different transmitters. In the next time slot, a new frame begins and the procedure is repeated.

Algorithm 1 Procedure at the transmitter side

if $start ==$ TRUE **then**
  $i \leftarrow 0$
end if
while $stop \neq$ TRUE **do**
  Send $x_k \times s_k[i]$
  $i \leftarrow i + 1$
  if New data generated **then**
    $n \leftarrow n + 1$
    Send Buffer[n]=New data
  end if
end while
for $j = 0$ to $n - 1$ **do**
  Send Buffer[j]=Send Buffer[$j + 1$]
end for
$n \leftarrow n - 1$

The described procedure of the compressed multiple access is summarized in algorithms 1 and 2 for transmitter and receiver, respectively. In the pseudo codes, *start* and *stop* are control signals broadcasted by base station in order to inform transmitters the start and end of one frame.

Algorithm 2 Procedure at the receiver side

if Previous frame ends **then**
  Send $start =$ TRUE
end if
repeat
  CS reconstruction
until two consequent recoveries are identical
Send $stop =$ TRUE

B. An Illustrative Example

In Fig.2, we provide an example to illustrate the procedure of compressed sensing based multiple access. For the first frame, transmitters Tx1 and Tx2 have data x_{10}, x_{20} to transmit (here additional subscript n in x_{kn} represents the nth information symbol sent by transmitter k). During the first frame, x_{11} and x_{30} are generated at Tx1 and Tx3 and are saved in their own buffers, respectively. For the second frame, Tx1 and Tx3 start transmitting. The detailed procedure is given below.

1. At time slot 1, a *start* signal is received by all transmitters in base station's coverage; the transmitters initialize signature waveform index i to 0;
2. within the following several time slots before receiving *frame 1*'s *stop* signal, transmitters Tx1, Tx2 send $x_{10} \times \mathbf{s}_1(i)$, $x_{20} \times \mathbf{s}_2(i)$ respectively; index i increases by 1; other transmitters keep silent and send nothing;
3. the base station receives the ith measurement $y[i] = x_{10} \times \mathbf{s}_1(i) \circ \mathbf{h}_1 + x_{20} \times \mathbf{s}_2(i) \circ \mathbf{h}_2 + \mathbf{n}(i)$; combined with the previously received measurements, information symbols are reconstructed using the OMP algorithm;
4. repeat the steps (2) and (3) until two consequent recoveries are the same [13]. Then, at time slot 7, the base station sends out the *stop* signal of *frame 1*, and this *stop* signal is also treated as the *start* signal of *frame 2*;
5. transmitters Tx1 and Tx2 stop current *frame 1*'s transmission and reset index i to 0;
6. starting from time slot 8, transmitters Tx1, Tx3 send $x_{11} \times \mathbf{s}_1(i)$, $x_{30} \times \mathbf{s}_3(i)$, respectively; index i is increased by 1; other transmitters keep silent and send nothing;
7. base station receives the ith measurement $y[i] = x_{11} \times \mathbf{s}_1(i) \circ \mathbf{h}_1 + x_{30} \times \mathbf{s}_3(i) \circ \mathbf{h}_3 + \mathbf{n}(i)$; combined with previously received measurements, information symbols are reconstructed (correct recoveries are x_{11}, x_{30});
8. repeat steps (6) and (7) until two consequent recoveries are the same. Then, at time slot 12, *frame 2*'s *stop* signal is broadcasted by base station to inform Tx1, Tx3 to stop *frame 2*'s transmission.

C. Transmitter Synchronization

As we have mentioned, the transmitters are assumed to be perfectly synchronized in time. In practice, the synchronization can be achieved in the following traditional ways:

- If each transmitter is equipped with a GPS and operates in outdoor environments, their timing can be almost perfectly synchronized.

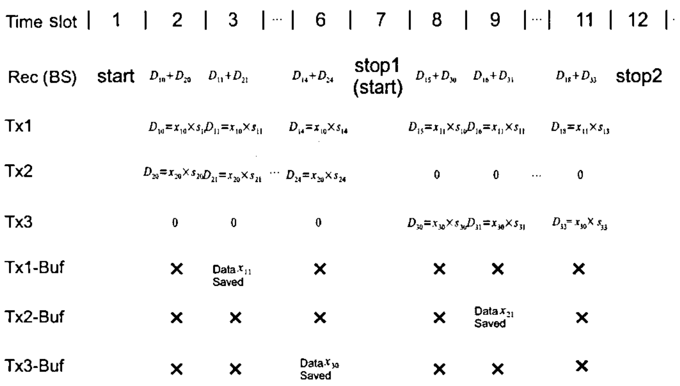

Fig. 2. The illustration of compressed multiple access transmission.

- The base station can broadcast a time synchronization signal periodically such that all transmitters can keep track the correct timing information.

In some cases, the perfect time synchronization cannot be achieved, e.g., each transmitter keeps in the sleeping mode for most of the time and cannot frequently listen to the time synchronization signal. In this situation, we can adapt the reconstruction algorithm to a asynchronous manner. In this subsection, we consider adapting the OMP algorithm to an asynchronous one.

First, we assume that each time slot is divided into $T_c + \tau_{\max}$ smaller chips, where $\tau_{\max}$ is the maximal time offset and T_c is the number of chips for transmission within each time slot. The time offsets of active transmitters $k_1, \cdots, k_K$ are given by $\tau_{k_1}, \cdots, \tau_{k_K}$ (measured in chips), respectively ($\tau_{k_i} \leq \tau_{\max}$).

Then, the algorithm of asynchronous OMP is given in procedure 3. The performance will be evaluated in the numerical simulations.

Algorithm 3 Asynchronous OMP algorithm

> Receive signal $\mathbf{r}$ of multiple time slots.
> Set the active set as empty and set the candidate set as $\{1, 2, \cdots, m\}$.
> **for** Residual signal is still large **do**
>   **for** All elements in the candidate set **do**
>   **for** All possible time offsets **do**
>   Shift the signature waveform according to the time offset.
>   Compute the projection of the signature waveform over the received signal.
>   **if** The projection is large **then**
>   Put the element into the active set.
>   Delete it from the candidate set.
>   Remove the corresponding signal from the received signal $\mathbf{r}$ and obtain the residual signal.
>   **end if**
>   **end for**
>   **end for**
> **end for**

D. *Conditional Equivalence of 0-Norm and 1-Norm Optimizations*

D.1 Noise-Free Case

To assure the performance of BP algorithm adopted at the receiver, we need to assure the equivalence of the following two optimization problems (denoted by P0 and P1, respectively) [2] when there is no noise and the signal is sufficiently sparse:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_0, \qquad \text{s.t. } \mathbf{r} = \mathbf{\Phi}\mathbf{x} \tag{9}$$

and

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1, \qquad \text{s.t. } \mathbf{r} = \mathbf{\Phi}\mathbf{x}. \tag{10}$$

Note that we discuss the received signal in only one time slot, for notational simplicity. The conclusion can be extended to observations in multiple time slots straightforwardly.

When the elements of $\mathbf{\Phi}$ are identically distributed, the equivalence has been proved in [2]. However, in our case, the elements may not be identically distributed since they are modulated by channel gains, which could be non-uniform, and the conclusions in [2] cannot be applied directly. Fortunately, the following proposition assures the equivalence under certain conditions, whose proof is given in the appendix. Note that the quantity $\frac{m}{n}$ measures the sparsity of the signal. Therefore, the condition of the proposition is essentially the limit on the sparsity.

Proposition 1: Define event $E(\Phi, \rho)$ as that, $\forall \|\mathbf{x}\|_0 \leq \rho m$, (P0) and (P1) yield the same unique solution equaling $\mathbf{x}$. Suppose that $\frac{m}{n} \leq C$, there exists a $\rho(C)$ such that

$$P\left(E(\Phi, \rho(C))\right) \to 1 \tag{11}$$

as $m, n \to \infty$, where the randomness is over the selection of $\mathbf{\Phi}$.

D.2 Noisy Channel Case

When noise exists, it is almost impossible to recover the original signal precisely. Therefore, we can apply the noise-aware version of (P1) in [14] to recover the original signal. The corresponding optimization problem is given by

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1, \qquad \text{s.t. } \|\mathbf{r} - \mathbf{\Phi}\mathbf{x}\|_2 \leq \delta \tag{12}$$

where δ is a controlling constant. When $\delta = 0$, (12) degenerates to (10). The stability of optimization problem in (12) has been discussed in [14], based on the assumption that the column vectors in $\mathbf{\Phi}$ have unit norm. We extend the conclusions (Theorem 3.1) in [14] to the channel gain dependent random matrix in this paper. We assume that noise $\mathbf{n}$ has bounded norm, namely

$$\|\mathbf{n}\|_2 \leq \epsilon. \tag{13}$$

For unbounded noise, $\|\mathbf{n}\|_2 > \epsilon$, we can claim outage (or erasure) of the communication system. Then, ϵ can be determined by tolerable outage probability and noise distribution. Similar to [14], we define the *coherence* for matrix $\mathbf{\Phi}$ as

$$M(\mathbf{\Phi}) \triangleq \max_{i \neq j} \frac{|\phi_i^T \phi_j|}{\|\phi_i\|_2 \|\phi_j\|_2} \tag{14}$$

which measures the linear dependency of columns in $\mathbf{\Phi}$. Based on the definition of coherence, we have the following proposition (the proof is given in Appendix B)

Proposition 2: When the sparsity of data burst satisfies (recall that f is the ratio of $h_{\min}^2$ and $h_{\max}^2$)

$$\|\mathbf{x}\|_0 \le \frac{f+M}{4M}, \tag{15}$$

we have

$$\|\hat{\mathbf{x}}_{\delta,\epsilon} - \mathbf{x}\|_2 \le \frac{1}{\gamma_{\max}(f - M(4N-1))} \tag{16}$$

where $\hat{\mathbf{x}}_{\delta,\epsilon}$ is the recovered signal obtained from (12) and

$$\gamma_{\max} \triangleq \frac{h_{\max}^2}{(\epsilon+\delta)^2}.$$

IV. NUMERICAL RESULTS

Numerical simulations have been carried out to evaluate the performance of our proposed compressed multiple access. We assume that the channel amplitude gain satisfies a Rayleigh distribution within the interval $[0.1, 10]$, i.e., $h_{\min} = 0.1$ and $h_{\max} = 10$.

Compressive sensing reconstruction algorithm OMP[1] is used for compressed multiple access' signal reconstruction process. Because OMP is usually faster than other reconstruction algorithms such as BP [7]; moreover, it can handle noisy compressive measurements efficiently [15].[2] We choose the simple slotted CSMA as a baseline, which employs truncated binary exponential back-off mechanism with the maximum delay of 1023 time slots [16]. We assume that 256 transmitters are associated with a base station. Each transmitter generates data packet independently, and interval between data packets of every transmitter is exponentially distributed. By changing the rate parameter (e.g., 0.43 data/slot means that there are averagely 0.43 active transmitters in each time slot), effects of different data generation rates of traffic are tested. In each realization, 10000 time slots are simulated. We use 100 realizations to obtain the transmission delay statistics, including transmission delay's mean value and standard variance. The cumulative distribution function (CDF) of the number of active transmitters is provided in Fig. 3, which shows that the number of active transmitters, as a random variable, varies significantly.

The length of each data packet sent by transmitters is set to 16 bits (e.g., temperature monitoring sensors report to control center). And for CSMA, we add a header for identifying the reporting transmitters. Since 256 transmitters are deployed, we add extra 8 bits for the transmitter ID. We assume that the PAM-16 modulation is used, thus 4 bits are transmitted in every successful transmitting time slot. Meanwhile, we assume that the dimension of the vector channel is 2. For a fair comparison, we assume that the CSMA approach can transmit two data symbols over the two dimensions simultaneously. Therefore, in the CSMA approach, all degrees of freedom in the vector channel are used for multiplexing, while they are used for multiple access in the compressed multiple access scheme since the same data symbol is transmitted over all dimensions of the vector channel. As a result, for the compressed multiple access, a data needs 4 frames to transmit, while for CSMA, it takes 3 transmitting time slots, when there are no competing transmitters. For instance, if the average interval between two data packets of every transmitter is 1000 time slots, then for each dimension of the vector channel, its data rate is 0.77, i.e., 0.77 successful transmission is needed for each time slot $((16+8)/(4 \times 2)) \times 256/1000$, "$16+8$" is the number of bits of a data packet for CSMA, 16 bits data and 8 bits ID, "4×2" is the number of bits transmitted each time slot, PAM-16 transmits 4 bits per time slot and there are 2 dimensions, 256 transmitters and 1000 is average interval). The signature waveform uses random binary variables. We also assume that the channel gains $\mathbf{H}$ are uniformly distributed between -3 dB and 3 dB. The assumption is reasonable if we consider the power control of transmitters. The transmit power can be incorporated into the

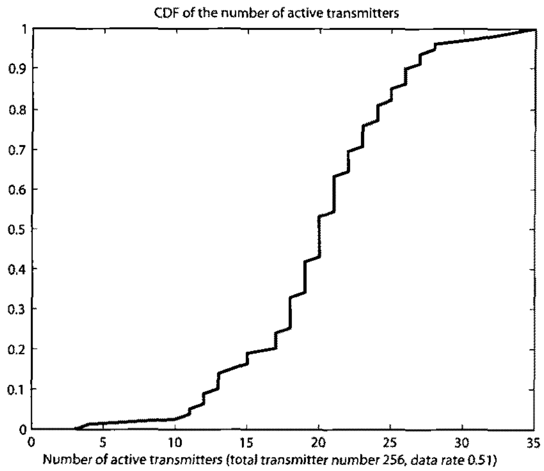

Fig. 3. CDF of the number of active transmitters.

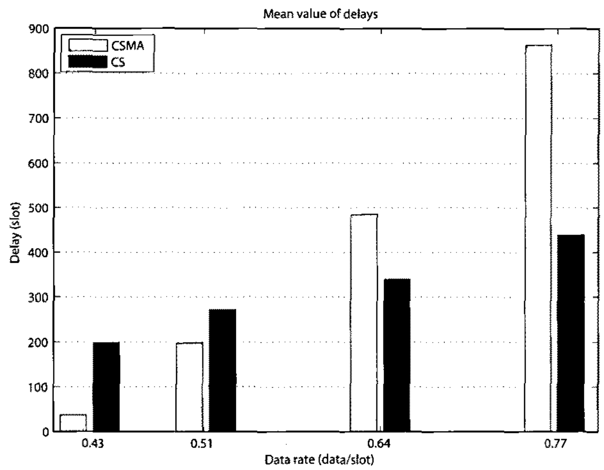

Fig. 4. Mean values of delay under different data rates.

[1]Note that OMP tries to directly solve the 0-norm optimization problem instead of solving the 1-norm optimization problem.

[2]Note that BP is also modified to combat noise [19] However, the computational cost is much higher.

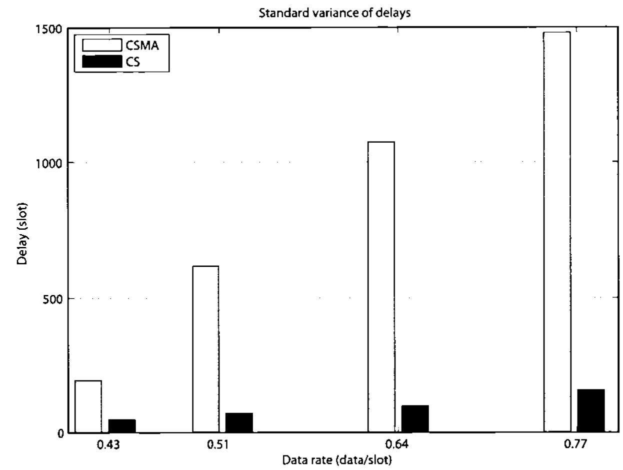

Fig. 5. Standard variances of delay under different data rates.

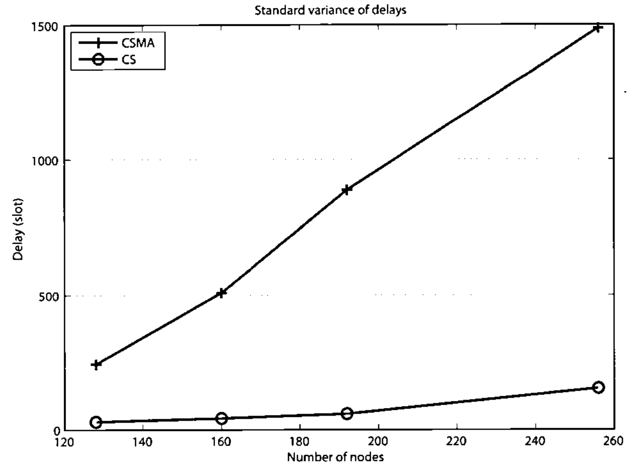

Fig. 7. Standard variance of delays with different number of nodes.

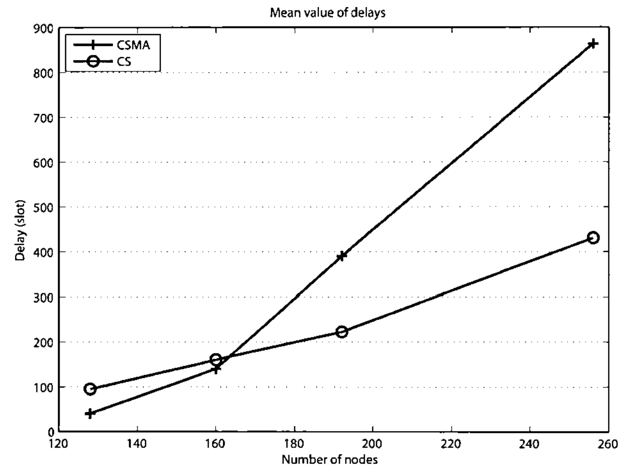

Fig. 6. Mean value of delays with different number of nodes.

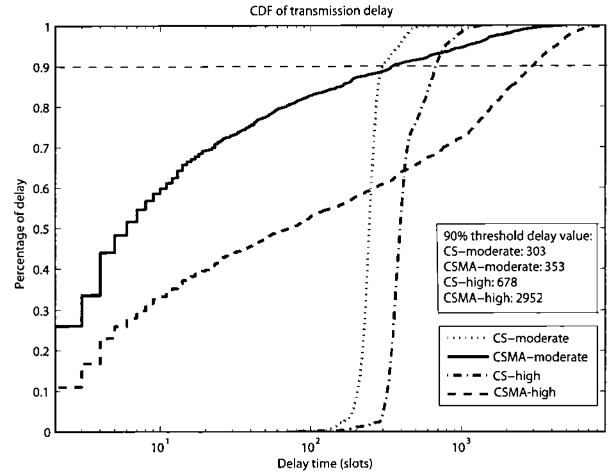

Fig. 8. CDF of delays.

channel gains. Then, the randomness of the channel gain is from the granularity of the power control (we assume that the transmit power does not change continuously).

We first compare the performance of proposed compressed multiple access and slotted CSMA under the situation that no noise is presented. Under a very low data rate, CSMA has a shorter average delay than the compressed multiple access. However, when data rate changes from very low (averagely, 0.43 data generated per time slot) to medium or high data rate (0.64 or 0.77 data per time slot), CSMA's average delay increases much faster than compressed multiple access, as observed in Fig. 4. Another key observation is that, regardless of the traffic load, the proposed compressed multiple access scheme always achieves much smaller variance of transmission delay (or, equivalently jitter) than CSMA. This implies that the compressed multiple access scheme is suitable for real-time traffic which has a rigorous requirement for the delay.

We also test the influence of changing the number of transmitters associated with the base station. The results are shown in Figs. 6 and 7. By varying the number of nodes from 128 to 256, the mean value and standard variance of delays for both the compressed multiple access and CSMA increase. Again, the mean value and standard variance of delays of CSMA increase much faster than the compressed multiple access. This implies an advantage of compressed multiple access, i.e., its delays are concentrated in a much smaller range, which is critical for system stability, as observed in Figs. 5 and 7. The CDF curves of the delays are shown in Fig. 8 for both the compressed multiple access and CSMA, as well as for both moderate and high traffic loads (0.51 and 0.77 data packet per time slot). We can observe that the compressed multiple access has much smaller 90% values of the delay (the 90% values are labeled in the figure). Quite often, a system's transmission bottleneck is the tail of long delays which may determine the whole system's performance. Therefore, the compressed multiple access is more suitable to provide quality of service (QoS) and is more stable when dealing with heavy traffic.

When noise presents, both CSMA and compressed multiple access suffer, however, compressed multiple access preserves its advantages of smaller variance and smaller 90% value of transmission delays over CSMA. Fig. 9 shows that under both data rates (*high*: 0.77 packet per time slot and *moderate*: 0.51

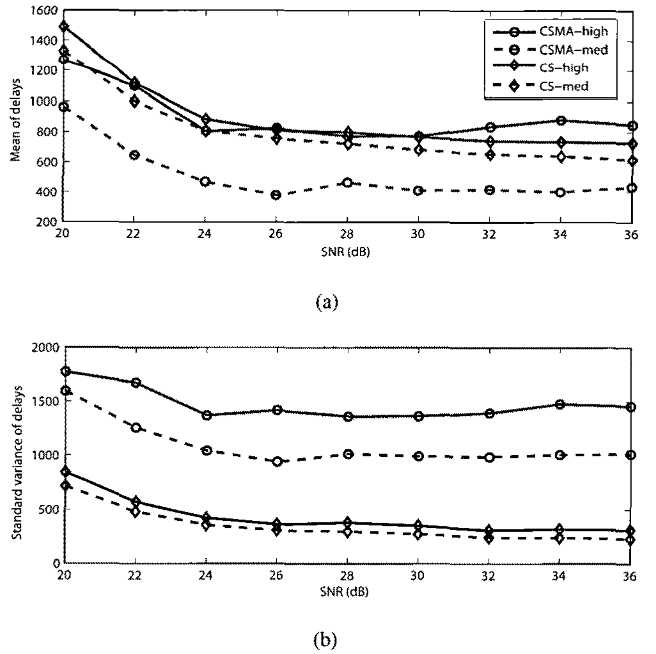

Fig. 9. (a) Mean value of delays and (b) standard variance of delays (with noise).

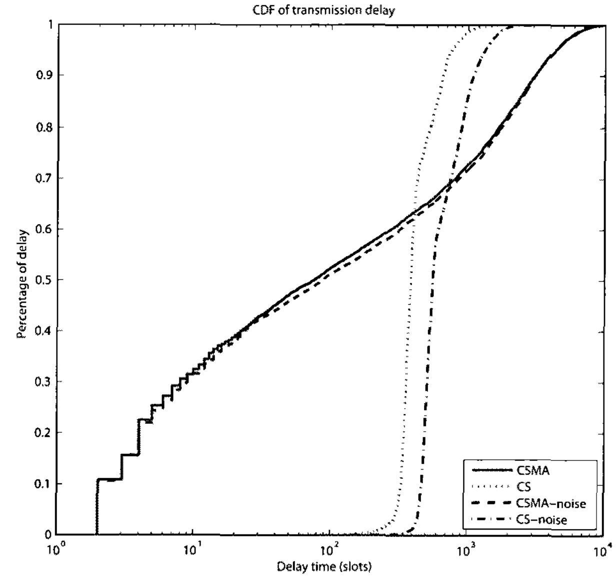

Fig. 10. CDF of delays (with noise).

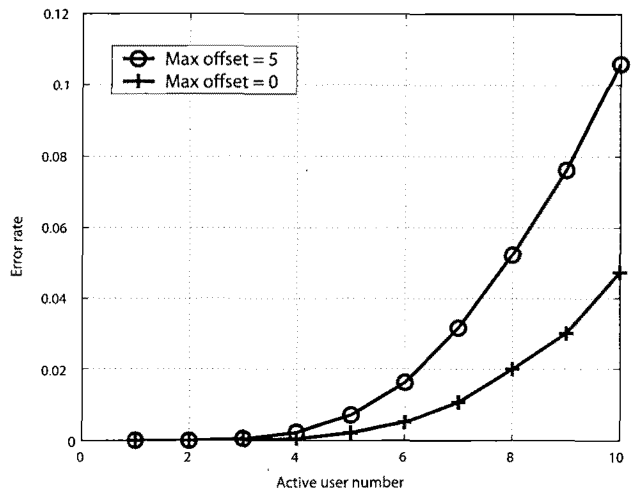

Fig. 11. Error rate of asynchronous OMP.

packet per time slot), compressed multiple access always has smaller variance of delays. As to mean value, under high data rate and low SNR, compressed multiple access provides competitive mean value of delays as CSMA does; under high data rate and high SNR, compressed multiple access has smaller mean value of delays than that of CSMA, therefore outperforms CSMA. When data rate is moderate, CSMA has smaller mean value of delays. But as SNR increases, the difference between mean value of delays associated with CSMA and mean value associated with compressed multiple access becomes smaller. CDF curves of the delays with noise are shown in Fig. 10. This figure is obtained when SNR is 35 dB and data rate is high (0.77 packet per time slot). As observed from Fig. 10, noise presents challenges to compressed multiple access due to increased difficulty in reconstructing signal from measurements polluted by noise [15]. But when SNR is high enough (above 30 dB), even performance of compressed multiple access does deteriorate by certain degree, it still has smaller 90% value of transmission delays than that of CSMA.

To demonstrate the validity of the asynchronous OMP algorithm in procedure 3, we have carried out simulations for a single frame with 50 time slots. We assume that there are 20 users and the number of active users change from 1 to 10. We assume $T_c = 20$ and $\tau_{\max} = 0, 5$. When $\tau_{\max} = 0$, the transmitters are perfectly synchronous. We define an error as the event that the estimated set of active users is wrong. Then, the error rates are shown in Fig. 11 for various numbers of active users. We observe that the asynchronicity incurs some performance degradation; however, the error rate is still low even when the sparsity is around 50% (when there are 10 active users). This significantly demonstrates the validity of the proposed asynchronous OMP algorithm. More detailed simulations will be carried in the future.

V. CONCLUSIONS

We have applied the technique of compressed sensing to compress the data transmission and transmitter identification in a multiple access system with random data traffic. Collision is allowed for the multiple access in a way similar to CDMA. A protocol has been proposed to accomplish the proposed algorithm. Numerical results have demonstrated the small average and variance of delay for the proposed scheme, especially under heavy traffic situation, compared with the traditional slotted CSMA. This implies that our proposed multiple access scheme is useful for realtime (soft) data traffic with QoS requirements.

APPENDICES

I. Proof of Proposition 1

Proof: It has been shown in [2] and [3] that optimizations P1 and P0 are equivalent if the following conditions about $\mathbf{\Phi}$ are satisfied, where J is an arbitrary subset of $(1, \cdots, m)$

- C1: The minimum singular value of $\mathbf{\Phi_J}$ is larger than η_1 uniformly for J satisfying $|J| < \rho_1 n$.
- C2: $\|\mathbf{v}\|_1 \geq \eta_2\sqrt{n}\|\mathbf{v}\|_2$ uniformly for J satisfying $|J| < \rho_2 n$, where $\mathbf{v} = \mathbf{\Phi}_J \mathbf{x}_J$.
- C3: $\|\mathbf{x}_{J^c}\|_1 \geq \eta_3\|\mathbf{v}\|_1$ uniformly for J satisfying $|J| < \rho_3 n$, where $\mathbf{v} = -\mathbf{\Phi}_{J^c}\mathbf{x}_{J^c}$.

We check the conditions C1–C3, separately. Throughout the proof, we assume that $J = \{1, 2, \cdots, k\}$, without loss of generality.

We first check condition C1. The proof follows the argument of Lemma 3.1 in [3]. We define

$$R_i = \left(\frac{1}{n}\sum_{j=1}^{n} Z_{ij}^2\right)^{\frac{1}{2}}$$

where $\{Z_{ij}\}$ are independent and identically distributed standard Gaussian random variables, being independent of $\mathbf{\Phi}$. Let $\mathbf{R} = \text{diag}\left(\left\{\frac{R_i}{\|h_i\|}\right\}\right)$ and $\mathbf{X} = \mathbf{\Phi}\mathbf{R}^{-1}$. Then, we have

$$\lambda_{\min}\left(\mathbf{\Phi}_J^T\mathbf{\Phi}_J\right) \geq h_{\min}^2 \lambda_{\min}\left(\mathbf{X}_J^T\mathbf{X}_J\right)\left(\max_i R_i\right)^{-2}$$

where we applied the assumption that $\|h_i\| \geq h_{\min}$. The subsequent argument is the same as that of [3].

Next, we check condition C2. By applying the assumption that $h_{ij} \geq h_{\min}$, for any vector $\alpha \in \mathbb{R}^{|J|}$, we have

$$\begin{aligned}
\|\mathbf{\Phi}_J\alpha\|_1 &= \|\mathbf{H}_J \circ \mathbf{S}_J\alpha\|_1 \\
&= \sum_i \left|\sum_j \mathbf{H}_{ij}\left(\mathbf{S}_J\right)_{ij}\alpha_j\right| \\
&\geq h_{\min}\sum_i \left|\sum_j \left(\mathbf{S}_J\right)_{ij}\alpha_j\right| \\
&= h_{\min}\|\mathbf{S}_J\alpha\|_1 .
\end{aligned}$$

Following the same argument as in [3], we can show that C2 also holds.

Finally, we check condition C3. Similar to (5.4) in [3], we set the following linear programming problem:

$$\min_{\delta_{J^c}} \|\mathbf{h}_{J^c} \circ \delta_{J^c}\|, \text{ s.t. } \mathbf{\Phi}_{J^c}\delta_{J^c} = -\mathbf{v}. \tag{17}$$

Applying the assumption that $h_{ij} \leq h_{\max}$, we have

$$\|\delta_{J^c}\| \geq \frac{1}{h_{\max}}\|\mathbf{h}_{J^c} \circ \delta_{J^c}\|. \tag{18}$$

It is easy to check that the dual linear programming of (17) is the same as that in [3].

Then, by applying Lemma 5.1 in [3], we have shown that condition C3 holds with probability 1 as $n, m \to \infty$. This concludes the proof. □

II. Proof of Proposition 2

Proof: The proof is the same as that of Theorem 3.1 in [14] before the optimization problem (3.9) in [14]. We define $\mathbf{G} = \mathbf{\Phi^T\Phi}$. The constraint $\|\mathbf{\Phi w}\|_2^2 \leq \mathbf{\Delta^2}$, where $\Delta \triangleq \delta + \epsilon$, implies

$$\begin{aligned}
\frac{1}{\gamma_{\max}} &= \frac{\Delta^2}{h_{\max}^2} \\
&\geq \frac{\mathbf{w}^T\mathbf{G}\mathbf{w}}{h_{\max}^2} \\
&= \|\mathbf{w}\|_2^2 - \mathbf{w}^T\left(\frac{1}{h_{\max}^2}\mathbf{G} - \mathbf{I}\right)\mathbf{w} \\
&\geq \|\mathbf{w}\|_2^2 - |\mathbf{w}|^T\left|\frac{1}{h_{\max}^2}\mathbf{G} - \mathbf{I}\right||\mathbf{w}| \\
&\geq \|\mathbf{w}\|_2^2 - M|\mathbf{w}|^T|\mathbf{1} - \mathbf{I}||\mathbf{w}| \\
&\quad - |\mathbf{w}|^T(1-f)\mathbf{I}|\mathbf{w}| \\
&= (M+f)\|\mathbf{w}\|_2^2 - M\|\mathbf{w}\|_1^2 .
\end{aligned} \tag{19}$$

The following argument remains the same as that in [14]. Then, the condition (3.17) in [14] becomes

$$(f+M)V - M\mu V \leq \frac{1}{\gamma_{\max}}. \tag{20}$$

This concludes the proof. □

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