Simple Demapping Methods for Turbo Coded Noncoherent MFSK Systems Under Rayleigh Fading Channels

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ABSTRACT

This paper proposes new suboptimum, yet very efficient demapping methods for Turbo coded noncoherent MFSK systems under AWGN and Rayleigh fading channels with channel state information. Simulation results show that the proposed methods achieve performance within 0.2 dB of the maximum-likelihood decoder and the previously proposed suboptimum demapping schemes with a fraction of the complexity.

Key Words : Demapping, Frequency Shift Keying, Turbo Codes

Ⅰ. Introduction

Many communication channels are characterized by a rapid time-varying phase response with AWGN. Under such conditions, accurate phase estimation could be difficult if not impossible. To avoid problems involving accurate phase estimation, noncoherent demodulation schemes such as MFSK may be employed[1-4].

On the other hand, Turbo codes have been shown to offer impressive performance on the AWGN and fully-interleaved Rayleigh fading channels with coherent modulation schemes such as quadrature amplitude modulation[5],[6]. Optimum demapping of Turbo codes with noncoherent MFSK under AWGN or Rayleigh fading channels with channel state information (CSI) requires the computation of the zeroth order modified Bessel functions of the first kind in computing the channel log-likelihood ratios (LLR)[7]-[9]. Here, CSI refers to the channel amplitude response. In order to bypass the direct evaluation of the complex Bessel functions, Hall and Wilson in [7] proposed an approximation to the exact channel LLR. This approximation shows negligible performance loss but still requires significant complexity requiring the evaluation of exponential functions. In [8], Valenti and Cheng, additionally proposed applying the Jacobian approximation[10] to further reduce the decoder complexity.

In this paper, we propose very simple, yet efficient channel LLR approximations under AWGN and Rayleigh fading channels with CSI for Turbo coded noncoherent MFSK systems. The proposed approximations offer performance within 0.2 dB of the maximum-likelihood (ML) decoder and the previously proposed suboptimum demapping schemes with a fraction of the complexity.

The remainder of the paper is organized as follows. In Section II, we introduce the system model and in Section III, we briefly overview the optimum ML decoder and previously proposed suboptimum demapping schemes and propose new channel LLR approximations. In Section IV, simulation results are presented, demonstrating the effectiveness of the proposed methods. Finally, conclusions are drawn in Section V.

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Ⅱ. System Model

The discrete-time model for the system under consideration is shown in Fig. 1. A binary message vector u of length K is encoded by a binary Turbo encoder to produce a codeword vector b of length N, assumed to be an integer multiple of $\log_2 M$. The codeword vector is then passed onto the channel interleaver resulting in an interleaved codeword vector $\tilde{b}$ which is then modulated using an MFSK modulator. The output of the MFSK modulator is represented by an $M\times L$ matrix $S\equiv[s_0\, s_1 \cdots s_{L-1}]$ consisting of $L\equiv N/\log_2 M$ MFSK modulated symbols where s_i is one of the M possible M-dimensional elementary column vectors. The MFSK modulated signal is then transmitted through a frequency nonselective Rayleigh fading channel represented by a complex fading coefficient vector $c\equiv[c_0\, c_1 \cdots c_{L-1}]$. The ith fading coefficient c_i is given by $c_i = a_i \exp(j\theta_i)$ where a_i and θ_i are the real-valued amplitude and phase responses of the channel corresponding to s_i. Here, we focus on the AWGN and the independent (fully interleaved) Rayleigh fading channel. For the AWGN channel $a_i = 1$, $i=0, 1,\cdots, L-1$ and the θ_i are independent and identically distributed (i.i.d.) RVs uniformly distributed on $[0, 2\pi)$. For the Rayleigh fading channel, the c_i are i.i.d. zero-mean complex Gaussian RVs with a variance of 1/2 in each dimension.

Let $y_i^T\equiv[y_i^0, y_i^1,\cdots,y_i^{M-1}]$ where y_i^l denotes the l th matched filter output corresponding to the symbol s_i. Then, the matrix of the receiver matched filter outputs for the code frame can be represented by $Y\equiv[y_0\, y_1 \cdots y_{L-1}]$ where $y_i = c_i s_i + n_i$. The noise vector n_i is the ith column of an $M\times L$ noise matrix N whose entries consist of uncorrelated zero-mean complex Gaussian RVs with variance $\sigma^2 = 1/(2\gamma)$ in each dimension where $\gamma\equiv\overline{E_s}/N_0$. Here, $\overline{E_s}$ is the average received energy per MFSK modulated signal and $N_0/2$ is the two-sided noise spectral density of the background thermal noise. Based on each y_i, the receiver computes the channel LLRs of the $\log_2 M$, interleaved binary code symbols contained in each y_i. The LLRs of the interleaved binary code symbols, $\tilde{z}=[\tilde{z}_0\, \tilde{z}_1 \cdots \tilde{z}_{N-1}]$ are then deinterleaved to yield $z=[z_0\, z_1 \cdots z_{N-1}]$, containing the channel LLR values of the code frame which is passed onto the iterative Turbo decoder to finally produce the estimated binary message vector $\hat{u}$[11].

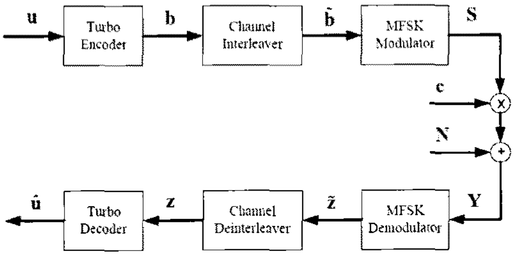

Fig. 1. Discrete-time system model of the Turbo coded noncoherent MFSK system

Ⅲ. Proposed Channel LLR Approximations

In this section, we propose very simple, yet efficient suboptimum approximations to the channel LLR values required by the iterative Turbo decoder. First, we briefly overview the exact channel LLR and previously proposed approximations. Since the demodulator operates on a symbol-by-symbol basis, we may drop the subscripts of y and a without ambiguity.

Under the Rayleigh fading channel with CSI, the exact channel LLR for the kth bit contained in an MFSK symbol is given as follows[9]:

$$\tilde{z}_k = \frac{\sum_{l\in A_0^k} I_0(2a\gamma|y^l|)}{\sum_{l\in A_1^k} I_0(2a\gamma|y^l|)} \tag{1}$$

where $k=0,1,\cdots,\log_2 M-1$, A_d^k is the set of MFSK symbols whose corresponding kth bit is

$d \in 0,1$ and $I(\cdot)$ is the zeroth order modified Bessel function of the first kind[12]. The channel LLR for the AWGN channel is also given by (1) with $a = 1$. As can be seen from (1), computing the exact channel LLRs for the AWGN and the Rayleigh fading channels with CSI is quite complex, requiring the evaluation of modified Bessel functions. Hall and Wilson in [7] utilized the following approximation in order to bypass direct evaluation of the modified Bessel function:

$$I_0(x) \approx \begin{cases} 1+0.25x^2, & \text{if } 0 \le x \le 1.8272 \\ e^x/\sqrt{2\pi x}, & otherwise \end{cases} \tag{2}$$

However, this approximation is still quite complex due to the exponential, square-root and division operations. On the other hand, Valenti and Cheng in [8] employed the Jacobian (Max-Log) approximation[10] to (1) resulting in

$$\begin{aligned} \tilde{z}_k \approx & \max_{l \in A_0^k}(\ln I_0(2a\gamma|y_l|)) \\ & - \max_{l \in A_1^k}(\ln I_0(2a\gamma|y_l|)) \end{aligned} \tag{3}$$

However, this still does not solve the problem of having to evaluate the modified Bessel function. Valenti and Cheng in [9] proposed using (2) or employing a lookup table for the evaluation of the function $\ln(I_0(x))$.

Fig. 2 shows a plot of the function $y = \ln(I_0(x))$. Notice that for sufficiently large values of x, the function, $y = \ln(I_0(x))$ may be well approximated simply by $y = x$. Hence, applying $\ln(I_0(x)) \approx x$ to (3), we have,

$$\tilde{z}_k \approx 2a\gamma(\max_{l \in A_0^k}(|y_l|) - \max_{l \in A_1^k}(|y_l|)). \tag{4}$$

On the other hand, for small values of x, i.e., low SNR, $\ln(I_0(x))$ can be well approximated by $x^2/4$ [13]. Applying the approximation $\ln(I_0(x)) \approx x^2/4$ to (3), we have,

$$\begin{aligned} \tilde{z}_k \approx & a^2\gamma^2(\max_{l \in A_0^k}(|y_l|^2) \\ & - \max_{l \in A_1^k}(|y_l|^2)) \end{aligned} \tag{5}$$

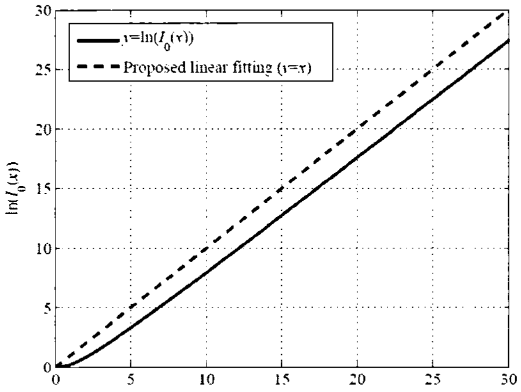

Fig. 2. The proposed linear fitting for $\ln(I_0(x))$.

Here, approximation (4) provides a high SNR approximation while (5) provides a low SNR approximation to (3). Note that the computational complexities of the approximations (4) and (5) are clearly orders of magnitude lower compared to (1) or the approximations suggested in [7] and [9]. Since the complexity of calculating the norm squared value of a complex value is smaller than the complexity of calculating its norm, (5) is somewhat simpler than (4). However, simple but accurate approximations for computing the norm of a complex number are widely available in the literature[14-16]. In the following section, we show that the application of these approximations to (4) results in negligible performance degradations.

Ⅳ. Simulation results

In this section, we present the simulation results for the frame error rate (FER) performance for the proposed approximations under the AWGN and the Rayleigh fading channel with CSI. imulations were performed using the Turbo code as defined in the third-generation partnership project (3GPP) standard[17] with Max-Log-MAP[18] decoding. A scaling factor of 0.7 was used for the extrinsic information values exchanged between the constituent decoders[19]. In all simulations, a frame length of $K = 144$ bits, a code rate of $r = 1/3$ and uniform channel interleaver is assumed. The *linear* and the *square* approximations denote the approximations based on (4) and (5), respectively.

The FER curves versus $\overline{E}_b/N_0$ ($\overline{E}_b = \overline{E}_s/r/\log_2 M$) under AWGN and Rayleigh fading channels with CSI are shown in Figs. 3-4. The approximations of Hall and Wilson[7] and Valenti and Cheng[9] show performance indistinguishable from that of the exact channel LLR under both channels. We observe that maximum performance losses for the *linear* and the *square* approximations are within 0.1 dB under the AWGN channel. Under the Rayleigh fading channel, the maximum losses for the two proposed approximations are within 0.2 dB. Note that for high SNRs, the *linear* approximation performs better than the *square* approximation and vice versa for low SNRs, as expected. An appropriate selection of either the *linear* or the *square* approximation depending on the modulation order M guarantees performance losses within 0.1 dB at extremely low complexity.

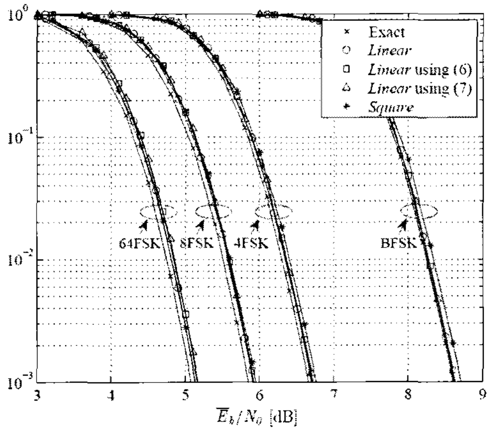

Fig. 3. FER versus $\overline{E}_b/N_0$ for various demapping methods under the AWGN channel. Turbo code, Max-log-MAP decoding, scaling factor=0.7, $K=144$ bits, $r=1/3$.

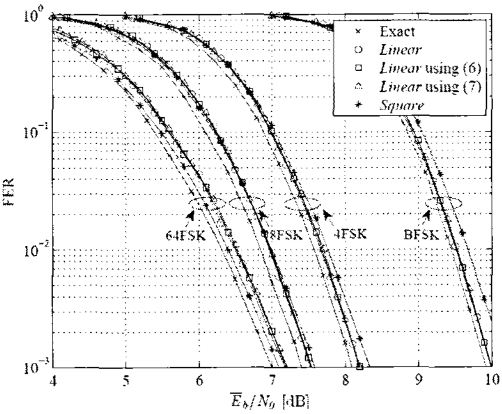

Fig. 4. FER versus $\overline{E}_b/N_0$ for various demapping methods under the Rayleigh fading channel. Turbo code, Max-log-MAP decoding, scaling factor=0.7, $K=144$ bits, $r=1/3$.

The FER curves for the *linear* approximation of (4) are also shown in Figs 3 - 4 when the following approximation suggested in [16] is applied to computing $|y^l|$, i.e.,

$$|y^l| \approx \alpha \max(y^l_{re}, y^l_{im}) + \beta \min(y^l_{re}, y^l_{im}). \quad (6)$$

Here, α and β are positive real values chosen so that the mean-squared error is minimized and $y^l_{re} = |Re(y^l)|$ and $y^l_{im} = |Im(y^l)|$. When y^l are zero-mean i.i.d., complex Gaussian RVs, it is easily found that $\alpha = 0.96$ and $\beta = 0.42$. The following simpler approximation to y^l is also given in [15]

$$|y^l| \approx \max(y^l_{re}, y^l_{im}) + 0.5\min(y^l_{re}, y^l_{im}). \quad (7)$$

Notice that both approximations (6) and (7) for y^l do not incur noticeable degradation in performance. Though not shown, we have also verified that simulation results obtained for other decoding scheme (Log-MAP[18]), modulation orders, frame lengths and code rates all indicate similar conclusions.

V. Conclusion

New suboptimum, yet very efficient demapping methods for Turbo coded noncoherent MFSK systems under AWGN and Rayleigh fading channels with CSI were proposed. Simulation results indicate that, with a fraction of the complexity, the proposed demapping methods achieve performance within 0.2 dB of the ML decoder and the previously proposed suboptimum demapping schemes.

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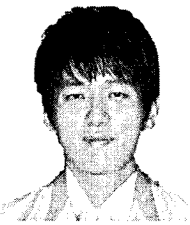

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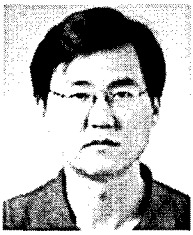

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