

A Study on the Complex-Channel Blind Equalization Using ITL Algorithms

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ABSTRACT

For complex channel blind equalization, this study presents the performance and characteristics of two complex blind information theoretic learning algorithms (ITL) which are based on minimization of Euclidian distance (ED) between probability density functions compared to constant modulus algorithm which is based on mean squared error (MSE) criterion. The complex-valued ED algorithm employing constant modulus error and the complex-valued ED algorithm using a self-generated symbol set are analyzed to have the fact that the cost function of the latter forces the output signal to have correct symbol values and compensate amplitude and phase distortion simultaneously without any phase compensation process. Simulation results through MSE convergence and constellation comparison for severely distorted complex channels show significantly enhanced performance of symbol-point concentration with no phase rotation.

Key Words : Blind Equalizer, Complex Channel, Euclidian Distance, Constant Modulus Error, Phase Distortion, ITL

I. Introduction

Broadcasting system, multipoint networks and the wireless/mobile networks usually employ blind equalization techniques to mitigate multipath fading and inter-symbol interference (ISI) because they do not require a training sequence to start up or to restart after a communications breakdown^[1,2]. Most blind equalization algorithms utilize nonlinearity of the equalizer output for weights updates. Constant modulus algorithm (CMA) minimizes the error between output power and source signal constant modulus based on mean squared error (MSE) criterion^[3].

Unlike the MSE criterion, information theoretic learning (ITL) methods introduced by Principe^[4] are based on a combination of a nonparametric probability density function (PDF) estimator and a procedure to compute information potential (IP). The study in [5] demonstrated that the error samples of the ITL -trained systems exhibit a more concentrated

density function and the distribution of the produced outputs are also closer to that of the desired signals compared to MSE. As one of the ITL criteria, the Euclidian distance (ED) between two PDFs that contains only quadratic terms to utilize the tools of information potential was applied successfully to the biomedical classification problem^[6] and real-valued blind equalization^[7].

In some applications, however, signals are complex-valued and processing is done in complex multi-dimensional space such as QAM signal space. Then some concealed problems in real signal processing such as symbol-phase rotation are exposed and left as important problems to be solved.

This study analyzes and presents the performance of two complex blind equalizer algorithms based on ITL especially in complex channel environments that cause ISI and phase rotation to symbol space. The first ITL based algorithm deals with constant modulus error (CME), and the second one is a complex-valued ITL algorithm based on a self-

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generated symbol set which is an extension of the real-valued ITL algorithm^[7].

II. CMA based on MSE Criterion

For the equalizer output y_k and source signal constant modulus R_2 , the CME is defined as

$$e_{CME} = |y_k|^2 - R_2 \quad (1)$$

Then the cost function P_{CMA} to be minimized is

$$P_{CMA} = E[(|y_k|^2 - R_2)^2] \quad (2)$$

where $R_2 = E[|d_k|^4]/E[|d_k|^2]$ and d_k is the transmitted symbol at time k .

With weight vector W_k composed of L weights and input vector $X_k = [x_k, x_{k-1}, \dots, x_{k-L+1}]^T$, the output at symbol time k can be produced as $y_k = W_k^T X_k$. To adjust the blind equalizer coefficients, we derive the following algorithm^[3] by differentiating P_{CMA} with respect to W , employing steepest descent method, and dropping the expectation operation.

$$W_{k+1} = W_k - 2\mu_{CMA} \cdot X_k^* \cdot y_k \cdot (|y_k|^2 - R_2) \quad (3)$$

Employing M -ary PAM signaling systems, the level value A_m takes the following discrete values

$$A_m = 2m - 1 - M, \quad m = 1, 2, \dots, M \quad (4)$$

Then the constant modulus R_2 becomes

$$R_2 = E[|A_m|^4] / E[|A_m|^2] \quad (5)$$

In the following section III and IV, we propose two complex blind equalizer algorithms based on ITL. The first one deals with constant modulus error and ITL method. And the other one that will be introduced in section IV is based on a self-generated

symbol set and ITL.

III. Complex-valued Blind Equalization based on ED Minimization and CME

In supervised ED criterion, we minimize the Euclidian distance $ED[f_E(e), \delta(e)] = \int (f_E(e) - \delta(e))^2 de$ between the error signal PDF $f_E(e)$ and Dirac-delta function $\delta(e)$.

Rewriting ED between the two PDFs as

$$ED[f_E(e), \delta(e)] = \int f_E^2(\xi) d\xi + \int \delta^2(\xi) d\xi - 2 \int f_E(\xi) \delta(\xi) d\xi \quad (6)$$

where the term $\int f_E^2(\xi) d\xi$ in (6) is defined as information potential IP_e for error signal^[5], we obtain

$$ED[f_E(e), \delta(e)] = IP_e + c - 2f_E(0) \quad (7)$$

The term $\int \delta^2(\xi) d\xi$ in (6) can be treated as a constant c and (7) can be reduced to the following cost function for supervised learning.

$$ED[f_E(e), \delta(e)] = IP_e - 2f_E(E=0) \quad (8)$$

Now we can expand this concept to constant modulus error, then the cost function ED_{CME} for CME tries to create a concentration of constant modulus error samples near zero.

$$ED_{CME} = IP_{CME} - 2f_E(e_{CME} = 0) \quad (9)$$

For convenience sake, $f_E(e_{CME} = 0)$ in (9) will be referred to as PE in this section.

In order to calculate the error PDF $f_E(e_{CME})$ non-parametrically, we need the Parzen estimator^[11] using Gaussian kernel and a block of N error samples as follows

$$f_E(e_{CME}) = \frac{1}{N} \sum_{i=1}^N G_\sigma(e_{CME} - e_{CME_i})$$

$$= \frac{1}{N} \sum_{i=1}^N \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(e_{CME} - e_{CME_i})^2}{2\sigma^2}\right] \quad (10)$$

Using (10) and $e_{CME} = |y_k|^2 - R_2$ we obtain the terms in (9) as

$$IP_{CME} = \int f_E^2(\xi) d\xi = \frac{1}{N} \sum_{i=1}^N \sum_{i=1}^N G_{\sigma\sqrt{2}}(|y_i|^2 - |y_i|^2) \quad (11)$$

$$PE = \frac{1}{N} \sum_{i=1}^N G_\sigma(|y_i|^2 - R_2) \quad (12)$$

By differentiating ED_{CME} with respect to W , we obtain the following gradient:

$$\frac{\partial ED_{CME}}{\partial W} = \nabla_{IP_{CME,Re}} + j\nabla_{IP_{CME,Im}}$$

$$- 2(\nabla_{PE,Re} + j\nabla_{PE,Im}) \quad (13)$$

where subscripts Re and Im indicate real part and imaginary part of a complex number .

$$\nabla_{IP_{CME,Re}} = \frac{1}{2N^2\sigma^3\sqrt{\pi}} \sum_{l=1}^N \sum_{i=1}^N \exp\left[\frac{(|y_l|^2 - |y_i|^2)}{-4\sigma^2}\right]$$

$$\cdot (|y_l|^2 - |y_i|^2) \cdot [y_{l,Re} \cdot X_{l,Re} + y_{l,Im} \cdot X_{l,Im}$$

$$- (y_{i,Re} \cdot X_{i,Re} + y_{i,Im} \cdot X_{i,Im})] \quad (14)$$

$$\nabla_{IP_{CME,Im}} = \frac{1}{2N^2\sigma^3\sqrt{\pi}} \sum_{l=1}^N \sum_{i=1}^N \exp\left[\frac{(|y_l|^2 - |y_i|^2)}{-4\sigma^2}\right]$$

$$\cdot (|y_l|^2 - |y_i|^2) \cdot [y_{l,Im} \cdot X_{l,Re} - y_{l,Re} \cdot X_{l,Im}$$

$$- (y_{i,Im} \cdot X_{i,Re} - y_{i,Re} \cdot X_{i,Im})] \quad (15)$$

$$\nabla_{PE,Re} = \frac{2}{N\sigma^3\sqrt{2\pi}} \sum_{i=1}^N \exp\left[\frac{(y_{i,Re}^2 + y_{i,Im}^2 - R_2)}{-2\sigma^2}\right]$$

$$\cdot (y_{i,Re}^2 + y_{i,Im}^2 - R_2) \cdot (y_{i,Re} \cdot X_{i,Re} + y_{i,Im} \cdot X_{i,Im}) \quad (16)$$

$$\nabla_{PE,Im} = \frac{2}{N\sigma^3\sqrt{2\pi}} \sum_{i=1}^N \exp\left[\frac{(y_{i,Re}^2 + y_{i,Im}^2 - R_2)}{-2\sigma^2}\right]$$

$$\cdot (y_{i,Re}^2 + y_{i,Im}^2 - R_2) \cdot (y_{i,Im} \cdot X_{i,Re} - y_{i,Re} \cdot X_{i,Im}) \quad (17)$$

Replacing index i with time index $k - i + 1$, we can update the weights of the complex blind equalizer (we will call this MED-CME in this paper).

$$W_{k+1} = W_k - \mu_{MED-CME} \frac{\partial ED_{CME}}{\partial W} \quad (18)$$

IV. Complex-valued Blind Equalization based on ED Minimization and A Self-Generated Symbol Set

The Euclidian distance between the transmitted symbol PDF f_D and the equalizer output PDF f_Y can be expressed as

$$ED[f_D, f_Y] = IP_Y + IP_D - 2 \cdot IP_{DY} \quad (19)$$

where $IP_{DY} = \int f_D(\xi) f_Y(\xi) d\xi$. The term IP_D can be treated as a constant. For IP_{DY} the receiver generates 16 constellation symbol points $d_i = d_{i,Re} + jd_{i,Im}$ which are equally likely as

The real and imaginary parts of the transmitted 16 QAM symbols are generated. Now the terms IP_Y and IP_{DY} in (19) are expressed non-parametrically using the Parzen window method as

$$IP_Y = \frac{1}{N^2} \sum_{i=1}^N \sum_{l=1}^N G_{\sigma\sqrt{2}}(y_l - y_i), \quad (20)$$

$$IP_{DY} = \frac{1}{N^2} \sum_{i=1}^N \sum_{l=1}^N G_{\sigma\sqrt{2}}(d_i - y_l). \quad (21)$$

For equalizer weight update $W_{new} = W_{old} - \mu_{CMEDI} \frac{\partial ED[f_D, f_Y]}{\partial W}$, the complex valued gradient can be obtained as follows:

$$\frac{\partial ED[f_D, f_Y]}{\partial W} = \nabla_{IP_{Y,Re}} + j\nabla_{IP_{Y,Im}} - 2(\nabla_{IP_{DY,Re}} + j\nabla_{IP_{DY,Im}}) \quad (22)$$

where

$$\begin{aligned} \nabla_{IP_{Y,Re}} &= \frac{1}{4N^2\sigma^3\sqrt{\pi}} \\ &\cdot \sum_{l=1}^N \sum_{i=1}^N \exp\left[\frac{(y_{l,Re} - y_{l,Re})^2 + (y_{l,Im} - y_{l,Im})^2}{-4\sigma^2}\right] \\ &\cdot [(y_{i,Re} - y_{l,Re}) \cdot (X_{l,Re} - X_{i,Re}) \\ &+ (y_{i,Im} - y_{l,Im}) \cdot (X_{l,Im} - X_{i,Im})] \end{aligned} \quad (23)$$

$$\begin{aligned} \nabla_{IP_{Y,Im}} &= \frac{1}{4N^2\sigma^3\sqrt{\pi}} \\ &\cdot \sum_{l=1}^N \sum_{i=1}^N \exp\left[\frac{(y_{l,Re} - y_{l,Re})^2 + (y_{l,Im} - y_{l,Im})^2}{-4\sigma^2}\right] \\ &\cdot [(y_{i,Im} - y_{l,Im}) \cdot (X_{l,Re} - X_{i,Re}) \\ &+ (y_{i,Re} - y_{l,Re}) \cdot (X_{l,Im} - X_{i,Im})] \end{aligned} \quad (24)$$

$$\begin{aligned} \nabla_{IP_{DY,Re}} &= \frac{1}{4N^2\sigma^3\sqrt{\pi}} \\ &\cdot \sum_{l=1}^N \sum_{i=1}^N \exp\left[\frac{(d_{l,Re} - y_{l,Re})^2 + (d_{l,Im} - y_{l,Im})^2}{-4\sigma^2}\right] \\ &\cdot [(d_{i,Re} - y_{l,Re}) \cdot (X_{l,Re} - X_{i,Re}) \\ &+ (d_{i,Im} - y_{l,Im}) \cdot (X_{l,Im} - X_{i,Im})] \end{aligned} \quad (25)$$

$$\begin{aligned} \nabla_{IP_{DY,Im}} &= \frac{1}{4N^2\sigma^3\sqrt{\pi}} \\ &\cdot \sum_{l=1}^N \sum_{i=1}^N \exp\left[\frac{(d_{l,Re} - y_{l,Re})^2 + (d_{l,Im} - y_{l,Im})^2}{-4\sigma^2}\right] \\ &\cdot [(d_{i,Im} - y_{l,Im}) \cdot (X_{l,Re} - X_{i,Re}) \\ &+ (d_{i,Re} - y_{l,Re}) \cdot (X_{l,Im} - X_{i,Im})] \end{aligned} \quad (26)$$

For convenience sake, this method shall be referred to here as complex valued minimum ED 1

Table 1. Desired symbol assignment

i	$1, \dots, N/4$	$N/4+1, \dots, N/2$	$N/2+1, \dots, 3N/4$	$3N/4, \dots, N$
$d_{i,Re}$	+3	+1	-1	-3
$d_{i,Im}$	+3	+1	-1	-3

(CMED1) algorithm.

To investigate the robustness of the proposed algorithm CMED1 to channel phase distortions over CMA, we rewrite the term IP_{DY} as a set of partitioned functions. Considering 16 QAM signaling, as used in our simulation in section V, the set of outputs y_i can be partitioned according to the transmitted symbol set into 16 subsets as

$$R^{(p,q)} = \{y_i, A_m = p + jq\} \text{ for } \begin{cases} p = -3, -1, +1, +3 \\ q = -3, -1, +1, +3 \end{cases} \quad (27)$$

Then the information potential IP_{DY} in (21) can be expressed as

$$\begin{aligned} IP_{DY} &= \sum_{i \in R^{(+1,+1)}} G_{\sigma\sqrt{2}}(1+j-y_i) + \sum_{i \in R^{(+1,-1)}} G_{\sigma\sqrt{2}}(1-j-y_i) \\ &+ \sum_{i \in R^{(-3,+3)}} G_{\sigma\sqrt{2}}(-3+3j-y_i) + \sum_{i \in R^{(-3,-3)}} G_{\sigma\sqrt{2}}(-3-3j-y_i) + \dots \end{aligned} \quad (28)$$

Noticing that each term in (28) is maximized when $y_i = 1 + j$ for $i \in R^{(+1,+1)}$, $y_i = 1 - j$ for $i \in R^{(+1,-1)}$, ..., $y_i = -3 - 3j$ for $i \in R^{(-3,-3)}$, respectively. This can be interpreted that the cost function forces the output signal to have correct symbol values through adjusting weights to compensate amplitude and phase distortion induced from channel.

On the other hand, the CMA cost function (1) can be partitioned using a sample mean estimator as

$$\begin{aligned} P_{CMA} &= \sum_{i \in R^{(+1,+1)}} (R_2 - |y_i|^2)^2 + \sum_{i \in R^{(+1,-1)}} (R_2 - |y_i|^2)^2 \\ &+ \dots + \sum_{i \in R^{(-3,+3)}} (R_2 - |y_i|^2)^2 + \sum_{i \in R^{(-3,-3)}} (R_2 - |y_i|^2)^2 \end{aligned} \quad (29)$$

where $R_2 = E[A_m^4] / E[A_m^2] = 13.2$. Clearly each term in (29) is minimized when $|y_i|^2 = 13.2$ for all

symbol regions: $i \in R^{(+1+j)}, i \in R^{(+1-j)}, \dots, i \in R^{(-3-3j)}$. This implies that the cost function of CMA pushes output samples to have a constant power 13.2 regardless of symbol classes. Besides this signal magnitude problem, channel phase problem is another significant drawback to CMA. In many cases of complex channel inducing channel phase distortion with some angle θ , the output y_i for transmitted symbol A_m is rotated as

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} A_{m,Re} \\ A_{m,Im} \end{bmatrix} = \begin{bmatrix} y_{i,Re} \\ y_{i,Im} \end{bmatrix} \quad (30)$$

It is clear that regardless of the phase shift θ in (30), $|y_i|^2 = |A_m|^2$.

V. Simulation Results and Discussion

The transmitted symbol is assumed to be i.i.d, taking the equally probable values from $\{\pm 1 \pm j, \pm 3 \pm j, \pm 1 \pm 3j, \pm 3 \pm 3j\}$. The three complex blind algorithms are considered: the CMA in (3), MED-CME in (18), CMED1 in (22). The complex channel models $H_1(z)$ and $H_2(z)$ ^[8] in this simulation are

$$\begin{aligned} H_{1,Re}(z) &= -0.005 + 0.009z^{-1} - 0.024z^{-2} \\ &+ 0.854z^{-3} - 0.218z^{-4} - 0.049z^{-5} - 0.016z^{-6} \\ H_{1,Im}(z) &= -0.004 + 0.030z^{-1} - 0.104z^{-2} \\ &+ 0.520z^{-3} + 0.273z^{-4} - 0.074z^{-5} + 0.020z^{-6} \end{aligned} \quad (31)$$

$$\begin{aligned} H_{2,Re}(z) &= -0.141z^{-1} + 0.95z^{-2} \\ &+ 0.27z^{-3} - 0.078z^{-4} \\ H_{2,Im}(z) &= -0.004z^{-1} - 0.919z^{-2} \\ &+ 0.37z^{-3} - 0.089z^{-4} \end{aligned} \quad (32)$$

The variance of AWGN was 0.001. μ_{CMA}

$\mu_{CMA-CME}$ μ_{CMED1} are set to be 0.0000005, 0.005, and 0.001, respectively. The kernel sizes for ITL algorithms are 15.0 for MED-CME and 0.5 for CMED1. The convergence results are illustrated in Fig. 2 for channel model H1, and in Fig. 6 for channel model H2. For H1 in Fig. 2, the CME based CMA and MED-CME have produced very poor minimum MSE performance but CMED1 converges well showing significant performance enhancement by over 18dB comparing to CME based algorithms. In Fig. 3-5, though ED based blind algorithm MED-CME produces very concentrated output points distribution comparing to

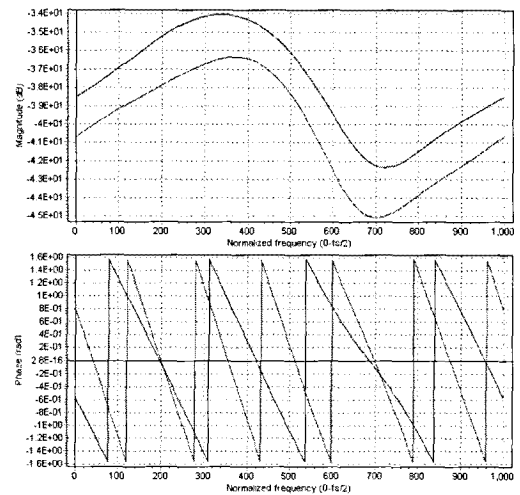


Fig. 1. Channel magnitude and phase characteristics (red: $H_1(z)$, blue : $H_2(z)$)

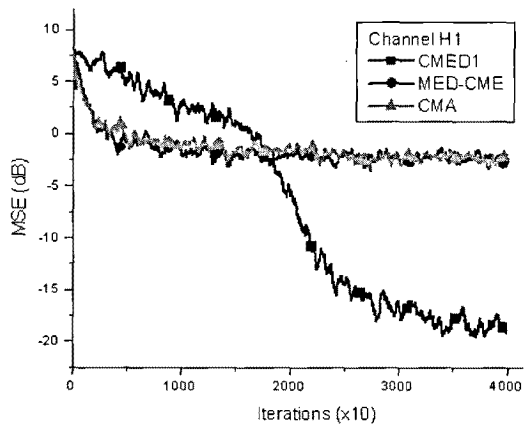


Fig. 2. MSE convergence comparison for channel H1

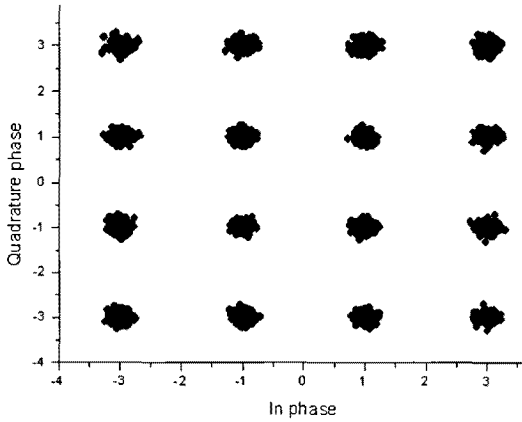


Fig. 3. Constellation performance of CMED1 for H1

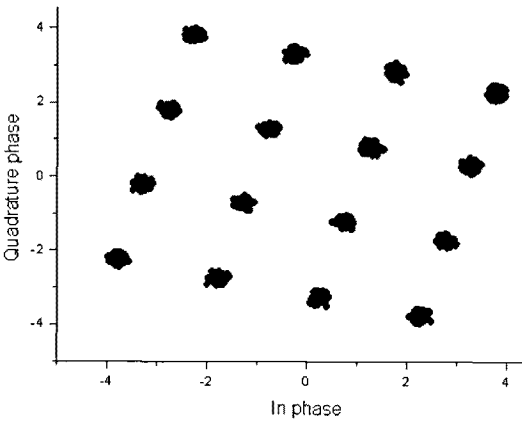


Fig. 4. Constellation performance of MED-CME for H1

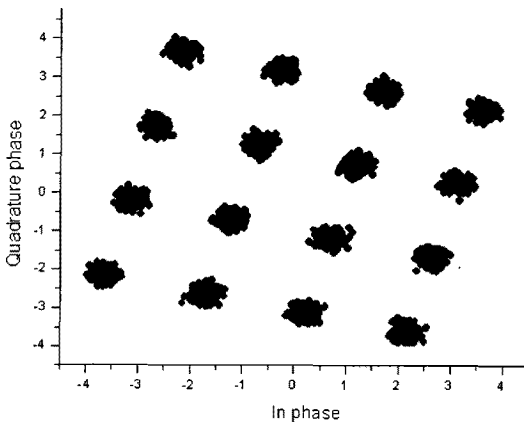


Fig. 5. Constellation performance of CMA for H1

CMA, the two algorithms can not cope with the channel phase distortion. On the other hand, the

complex valued CMED1 based on ED minimization criterion and a self-generated symbol set produces output points that are well concentrated to the exact constellation symbol points without any aid of phase compensation.

To prove these advantages, we carried out the simulation in a severer channel model H2 in (31). As in H1, the CME-based MED-CME and CMA have yielded inferior MSE learning performance as in Fig. 6, but MED-CME is better than CMA by about 3 dB. This indicates that ED-based blind algorithms have better performance than MSE-based criterion. As predicted, the proposed CMED1 shows superior performance enhanced by around 13dB comparing to CMA.

Constellation performance is depicted in Fig. 7-9.

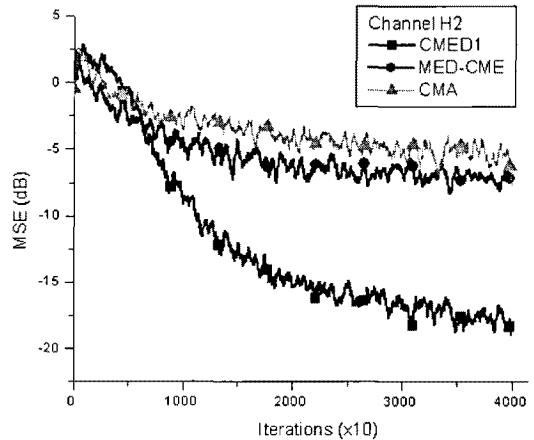


Fig. 6. MSE convergence comparison for channel H2

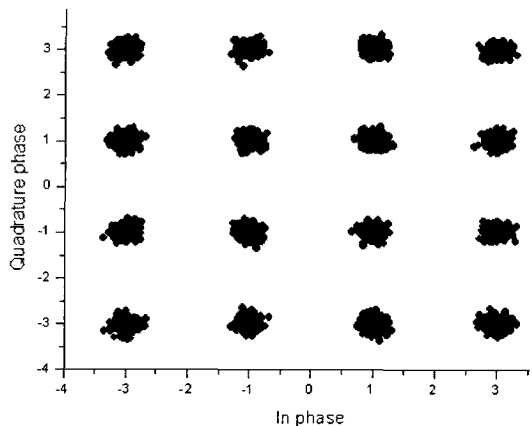


Fig. 7. Constellation performance of CMED1 for H2

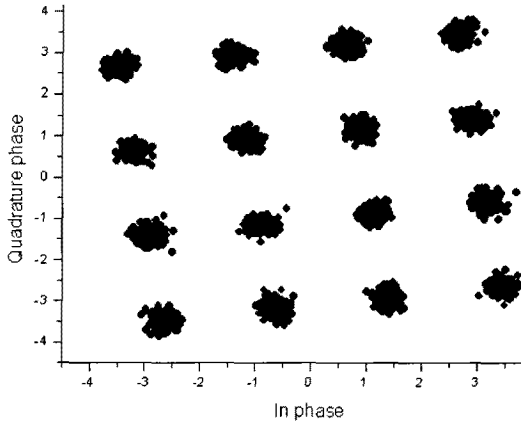


Fig. 8. Constellation performance of MED-CME for H2

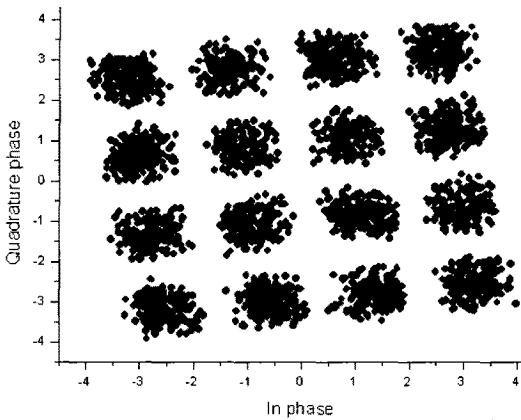


Fig. 9. Constellation performance of CMA for H2.

The CMA in this channel model results in very dispersed and phase-distorted constellation in Fig. 9. The ED-based MED-CME shows more concentrated constellation than CMA but it still does not solve the channel phase problem. The 3dB enhancement of MED-CME is considered to be caused by ED-based criterion. The constellation result of CMED1 based on ED and a self-generated symbol set for the channel model H2 still shows output points that are closely concentrated to the exact constellation symbol points.

VI. Conclusions

This study presents the performance and characteristics of two complex blind ITL algorithms

which are based on minimization of PDF Euclidian distance for complex channel blind equalization. One is complex-valued MED-CME employing constant modulus error and the other is complex-valued CMED1 using a self-generated symbol set.

In the analysis of the robustness of the proposed algorithm CMED1 to channel phase distortions over CMA rewriting the information potential IP_{DY} as a set of partitioned functions, it is revealed that the cost function forces the output signal to have correct symbol values and compensate amplitude and phase distortion simultaneously without any phase compensation process, whereas the cost function of CMA pushes output samples to have a constant power regardless of symbol classes and the phase difference between the correct symbol and output can not be detected in CME-based algorithms.

Simulation results for severely distorted complex channels proved those characteristics through MSE convergence and constellation comparison. The CMED1 yielded output points that are closely concentrated to the exact constellation symbol points for both complex channel models. Therefore we can conclude that the characteristics of CMED1 retaining significantly enhanced performance of symbol-point concentration and no need to solve channel phase problems can be very promising in complex channel blind equalization field.

References

- [1] L. M. Garth, "A dynamic convergence analysis of blind equalization algorithms," IEEE Trans. on Comm., Vol.49, April. 2001, pp.624-634.
- [2] F. Mazzenga, "Channel estimation and equalization for M-QAM transmission with a hidden pilot sequence," IEEE Trans. on Broadcasting, Vol.46, June. 2000, pp.170-176.
- [3] J. R. Treichler and B. Agee, "A new approach to multipath correction of constant modulus signals," IEEE Trans. Acoust., Speech, Signal Process, Vol.ASSP-31, Nov. 1983, pp.349-372.

- [4] J. C. Principe, D. Xu and J. Fisher, Information Theoretic Learning in: S. Haykin, Unsupervised Adaptive Filtering, Wiley, (New York, USA), 2000, pp.265-319.
- [5] D. Erdogmus, and J.C. Principe, "An Entropy Minimization algorithm for Supervised Training of Nonlinear Systems," IEEE Trans. Signal Processing, Vol.50, July, 2002, pp.1780-1786.
- [6] K. H. Jeong, J. W. Xu, D. Erdogmus, and J. C. Principe, "A new classifier based on information theoretic learning with unlabeled data," Neural Networks, 18,2005, pp.719-726.
- [7] N. Kim, K. H. Jeong, and K. Kwon, "A Study on the Weighting Effect on Information Potentials in Blind Equalizers for Multipoint Communication," International Conference on Information Science and Technology, APIC-IST 2008, (Indang, Philippines), Dec. 18-19, 2008, pp.103-108.
- [8] V. Weerackody and S. A. Kassam, "Dual-Mode Type Algorithms for Blind Equalization," IEEE Trans. on Comm., Vol.42, Jan. 1994, pp.22-28.

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